## Diatomic Molecules Part II

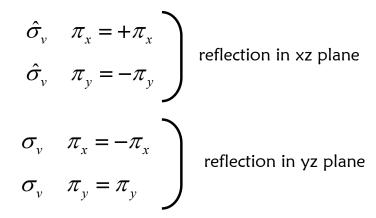
Chem 2430

 $\pm$  Symmetry: Behavior under  $\hat{\sigma}_v$ 

 $\sigma_{g},\sigma_{u}\,$  are + since they have no  $\phi$  dependence

 $\pi_+$  and  $\,\pi_-$  are not eigenfunctions of  $\hat{\sigma}_{_{\!\mathcal{V}}}$ 

but  $\pi_{\rm +}+\pi_{\rm -}$  and  $\pi_{\rm +}-\pi_{\rm -}$  which give  $\pi_{\rm x},\pi_{\rm y}$  are



$$\pi_{+}^{2} \rightarrow {}^{1}\Delta$$

$$(\pi_{+}\pi_{-} + \pi_{-}\pi_{+})(\alpha\beta - \beta\alpha) {}^{-1}\Sigma_{g}^{+}$$

$$(\pi_{+}\pi_{-} - \pi_{-}\pi_{+})\alpha\alpha {}^{3}\Sigma_{g}^{-}$$

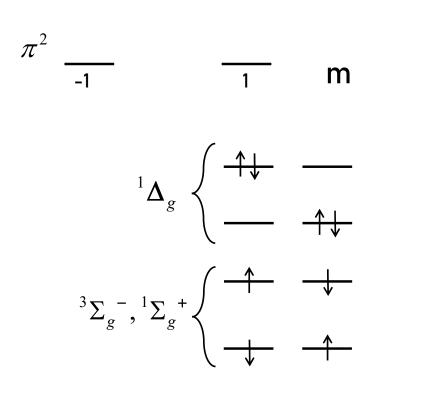
$$(\pi_{+}\pi_{-} + \pi_{-}\pi_{+}) = (\pi_{x} + i\pi_{y})(\pi_{x} - i\pi_{y}) + (\pi_{x} - i\pi_{y})(\pi_{x} + i\pi_{y})$$

$$= \pi_{x}^{2} + \pi_{y}^{2} + i(\pi_{y}\pi_{x} - \pi_{x}\pi_{y}) + \pi_{x}^{2} + \pi_{y}^{2} - i(\pi_{y}\pi_{x} - \pi_{x}\pi_{y})$$

$$= \pi_{x}^{2} + \pi_{y}^{2}$$
Hence  $\sigma_{v}(\pi_{+}\pi_{-} + \pi_{-}\pi_{+}) = +(\pi_{+}\pi_{-} + \pi_{-}\pi_{+})$ 

$$(\pi_{+}\pi_{-} - \pi_{-}\pi_{+}) = \pi_{x}\pi_{y} - \pi_{y}\pi_{x}$$

So 
$$\sigma_v(\pi_+\pi_--\pi_-\pi_+) = -(\pi_+\pi_--\pi_-\pi_+)$$



each electron has  $m = \pm 1$  so M = 2, -2, 0

 $\pi$  electron  $m = \pm 1$   $M = \pm 3, \pm 1$   ${}^{3}\Pi, {}^{1}\Pi, {}^{3}\Phi, {}^{1}\Phi$  $\delta$  electron  $m = \pm 2$ 

$$^{1}\Sigma, ^{3}\Sigma, ^{1}\Sigma, ^{3}\Sigma, ^{1}\Delta, ^{3}\Delta$$

 $\pi^2$ 

 $\pi\delta$ 

 $1\pi 2\pi$ 

16 arrangements  $(4 \times 4 = 16)$ 

 $L_z \rightarrow \lambda$  for one el. diatomic  $L_z \rightarrow \Lambda$  for a diatomic with  $\geq 2$ electrons

 $\Lambda \neq 0, \quad m_i = \pm \Lambda$ 

Species/Config	Bond Order	Term Symbol	
$H_2^+ 1\sigma_g$	$\frac{1}{2}$	$^{2}\Sigma_{g}^{+}$	
$H_2 1 \sigma_g^2$	1	$1 \sum_{g}^{\circ}$	
$H_2^{-1}\sigma_g^{-2}1\sigma_u$	$\frac{1}{2}$	$^{2}\Sigma_{u}^{+}$	
$He_2 1\sigma_g^2 1\sigma_u^2$	0	${}^{1}\Sigma_{g}^{+}$	
$Li_{2} 1\sigma_{g}^{2} 1\sigma_{u}^{2}$	$2\sigma_g^2$ 1	${}^{1}\Sigma_{g}^{+}$	
$Be_22\sigma_g^2 2\sigma_g$	$\sigma_{u}^{2} = 0$	${}^{1}\Sigma_{g}^{+}$	
$B_2 3 \sigma_g^2$	1	${}^{1}\Sigma_{g}^{+}$	
$1\pi_{u}^{2}$	1	${}^{3}\Sigma_{g}^{-},{}^{1}\Sigma_{g}$	$\Sigma_g^+, {}^1\Delta_g$
$3\sigma_{g}1\pi_{u}$	1	${}^{3}\Pi_{u}, {}^{1}\Pi$	$T_u$

$$C_{2}...3\sigma_{g}^{2}1\pi_{u}^{2} 2 \qquad {}^{3}\Sigma_{g}^{-}, {}^{1}\Sigma_{g}^{+}, {}^{1}\Delta_{g}$$

$$...1\pi_{u}^{4} 2 \qquad {}^{1}\Sigma_{g}^{+}$$

$$...3\sigma_{g}1\pi_{u}^{3} 2 \qquad {}^{3}\Pi_{u}, {}^{1}\Pi_{u}$$

$$N_{2}...3\sigma_{g}^{2}1\pi_{u}^{4} 3 \qquad {}^{1}\Sigma_{g}^{+}$$

$$O_{2}...3\sigma_{g}^{2}1\pi_{u}^{4}1\pi_{g}^{2} 2 \qquad {}^{3}\Sigma_{g}^{-}, {}^{1}\Sigma_{g}^{+}, {}^{1}\Delta_{g}$$

$$F_{2}...3\sigma_{g}^{2}1\pi_{u}^{4}1\pi_{g}^{4} 1 \qquad {}^{1}\Sigma_{g}^{+}$$

$$Ne_{2}...3\sigma_{g}^{2}1\pi_{u}^{4}1\pi_{g}^{4}3\sigma_{u}^{2} 0 \qquad {}^{1}\Sigma_{g}^{+}$$

Excited states

 $He_2: 1\sigma_g^2 1\sigma_u 2\sigma_g \quad BO=1$ 

$$O_{2} \pi_{u}^{4} \pi_{g}^{2} = {}^{1}\Sigma_{g}^{+} (1.6 \text{ eV})$$
$$= {}^{1}\Delta_{g} (0.98 \text{ eV})$$
$$= {}^{3}\Sigma_{g}^{-} (0 \text{ eV})$$

In spectroscopy the ground state is often labeled as X, and successive excited states (of a given multiplicity) by A, B, C,...

Total angular momentum: combine spin and  $\Lambda$ 

 $\Lambda + S, \Lambda + S - 1, \dots \Lambda - S$ 

Analog to the J quantum number for atoms

## $H_2$ molecule

$$\sigma_{g} = \frac{1}{\sqrt{2(1+S_{AB})}} (1s_{A} + 1s_{B})$$
$$\psi = \left|\sigma_{g}\overline{\sigma}_{g}\right|$$

With this treatment  $D_e = 3.5 \text{ eV vs. } 4.75$  (exact) Larger basis set:  $D_e = 3.64 \text{ eV}$ So correlation energy ~ -1.11 eV. Correlation energy of He atom = -1.14 eV

Roughly speaking correlation energy  $\approx$  -0.9 eV per pair of electrons

$$CI \qquad \Psi = c_1 \left| 1\sigma_g \overline{1\sigma_g} \right| + c_2 \left| 1\sigma_u^* \overline{1\sigma_u^*} \right|$$
$$D_e = 4.03 \text{ eV}.$$

33 configuration wave function  $D_e = 4.71 \text{ eV}$ 

For the Be atom, the correlation energy is 2.5 eV

Note if one were analyzing this in terms of pair correlation energies, there are 1s-1s, 2s-2s, and 1s-2s pairs

The HF wave function of  $H_2$  has both ionic and covalent components

$$\sigma_{g^{2}} = (1s_{a} + 1s_{b})(1s_{a} + 1s_{b}) = (1s_{a}^{2} + 1s_{b}^{2}) + (1s_{a}1s_{b} + 1s_{b}1s_{a})$$

This 50/50 mixture persists at all distances. So the dissociation limit corresponds to  $\frac{1}{2}$  (H+H) +  $\frac{1}{2}$  (H<sup>+</sup>+H<sup>-</sup>)

IP H = 13.6 eV

EA  $H \approx 0.7$  eV

So the ionic limit is  $\sim 14.2 \text{ eV}$  above the covalent limit

and the  $R \rightarrow \infty$  asymptote is  $\sim 7.1 \text{ eV}$  too high in energy

Note 
$$\sigma_u^2 = (1s_a - 1s_b)(1s_a - 1s_b) = (1s_a^2 + 1s_b^2) - (1s_a 1s_b + 1s_b 1s_a)$$

So by taking  $\psi = c_1 \sigma_g^2 + c_2 \sigma_u^2$ , we can vary the % of ionic character

At  $R = \infty$ ,  $c_2 = -c_1$ , and the dissociation is to the purely covalent limit

Valence Bond treatment of H<sub>2</sub>

$$\psi = (1s_a 1s_b + 1s_b 1s_a)(\alpha\beta - \beta\alpha)$$

For the simple VB wavefunction

 $D_e = 3.15 \text{ eV}$ , which is in poorer agreement with the exact value than is the HF result

Optimize the orbital exponents:  $D_e = 3.78 \text{ eV}$ 

Expand the basis set :  $D_e = 4.04 \text{ eV}$ 

Can mix in the ionic configuration

 $\psi = c_1 \left( 1s_a 1s_b + 1s_b 1s_a \right) + c_2 \left( 1s_a^2 + 1s_b^2 \right)$ 

Equivalent to the two-configuration wave function based on MOs that we considered earlier

$$H_{2}: 1\sigma_{g} \rightarrow 1\sigma_{u}^{*} \text{ excited states}$$

$$\psi_{S} = \left(1\sigma_{g}1\sigma_{u}^{*} + 1\sigma_{u}^{*}1\sigma_{g}\right)\left(\alpha\beta - \beta\alpha\right) \text{ Singlet}$$

$$\psi_{T} = \left(1\sigma_{g}1\sigma_{u}^{*} - 1\sigma_{u}^{*}1\sigma_{g}\right) \begin{cases} \alpha\alpha\\ (\alpha\beta + \beta\alpha)\\ \beta\beta \end{cases}$$

Consider the triplet state

$$\psi_{T} = (1s_{A} + 1s_{B})(1s_{A} - 1s_{B}) - (1s_{A} - 1s_{B})(1s_{A} + 1s_{B})$$
  
$$= (1s_{A}^{2} - 1s_{B}^{2}) + (1s_{B}1s_{A} - 1s_{A}1s_{B})$$
  
$$- (1s_{A}^{2} - 1s_{B}^{2}) + (1s_{B}1s_{A} - 1s_{A}1s_{B})$$
  
$$= 1s_{B}1s_{A} - 1s_{A}1s_{B}$$

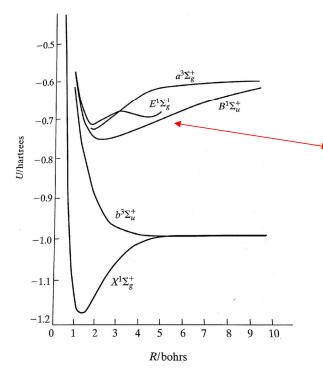
This goes to the same limit as the ground state (with ionic terms removed) as  $R = \infty$  i.e., to H atoms with the electrons in their 1s orbitals.

This makes sense sense at  $R = \infty$  we have two <sup>2</sup>S H atoms, that can couple to a singlet or triplet.

Now lets consider the excited  $\sigma_g \sigma_u$  singlet state

$$\psi_{s} = 1s_{A}^{2} - 1s_{B}^{2}$$

This is 100% ionic (H<sup>-</sup>H<sup>+</sup> and H<sup>+</sup>H<sup>-</sup>)



Potential energy curves of H<sub>2</sub> From Levine Notice the unusual shape of its potential energy curve which at long should go as R<sup>-1</sup> due to the ionic nature of the wave function.