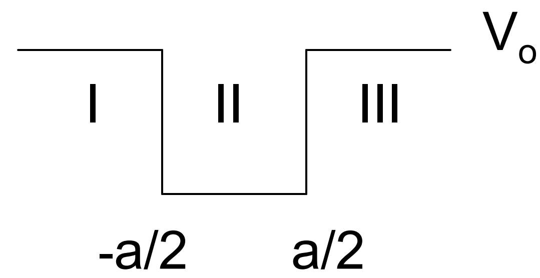
Chapter 5

Particle in finite box

$V(x) = 0$, $-a/2 < x < a/2$
$= V_o$, for $x \geq a/2$, $x \leq -a/2$



inside box: $$\frac{d^2\psi}{dx^2} = \frac{-2mE}{\hbar^2}\psi$$

outside box: $$\frac{d^2\psi}{dx^2} = \frac{2m(V_o - E)}{\hbar^2}\psi$$

I $$\psi(x) = A'e^{-kx} + B'e^{kx}$$

$$k = \sqrt{\frac{2m(V_o - E}{\hbar^2}}$$

III $$\psi(x) = Ae^{-kx} + Be^{kx}$$

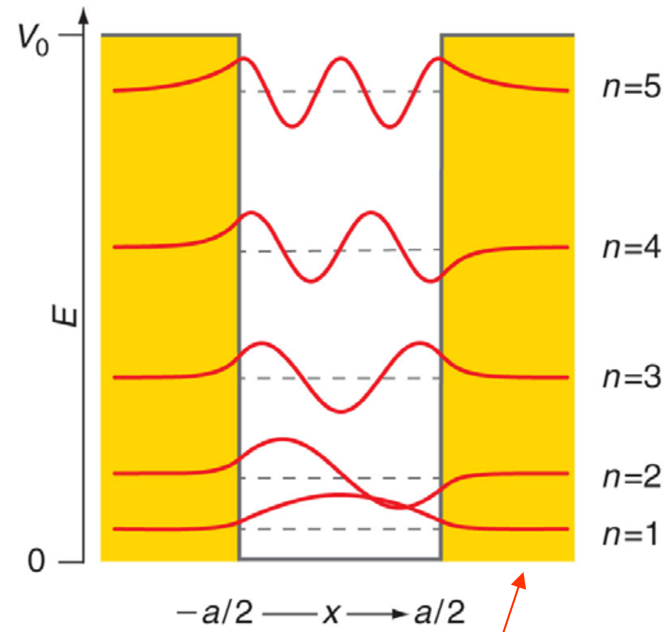
0

$$\text{I} \qquad \psi_I = B'e^{kx}$$

$$\text{III} \qquad \psi_{III} = Ae^{-kx}$$

exponentially decaying

$$\text{II} \qquad \psi_{II} = C\sin kx + D\cos kx, \qquad \text{k}=\sqrt{\frac{2mE}{\hbar^2}}$$

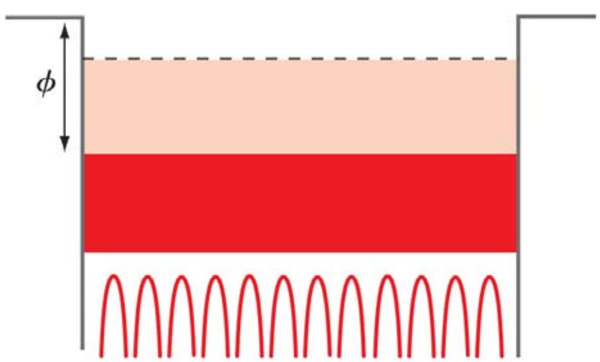


require that ψ and $\dfrac{d\psi}{dx}$ continuous match at the boundaries

tunneling

$$\psi_I(-a/2) = \psi_{II}(-a/2) \qquad \psi_{II}(a/2) = \psi_{III} = (a/2)$$

$$\psi_I'(-a/2) = \psi_{II}'(-a/2) \qquad \psi_{II}'(a/2) = \psi_{III}' = (a/2)$$



empty levels

filled levels

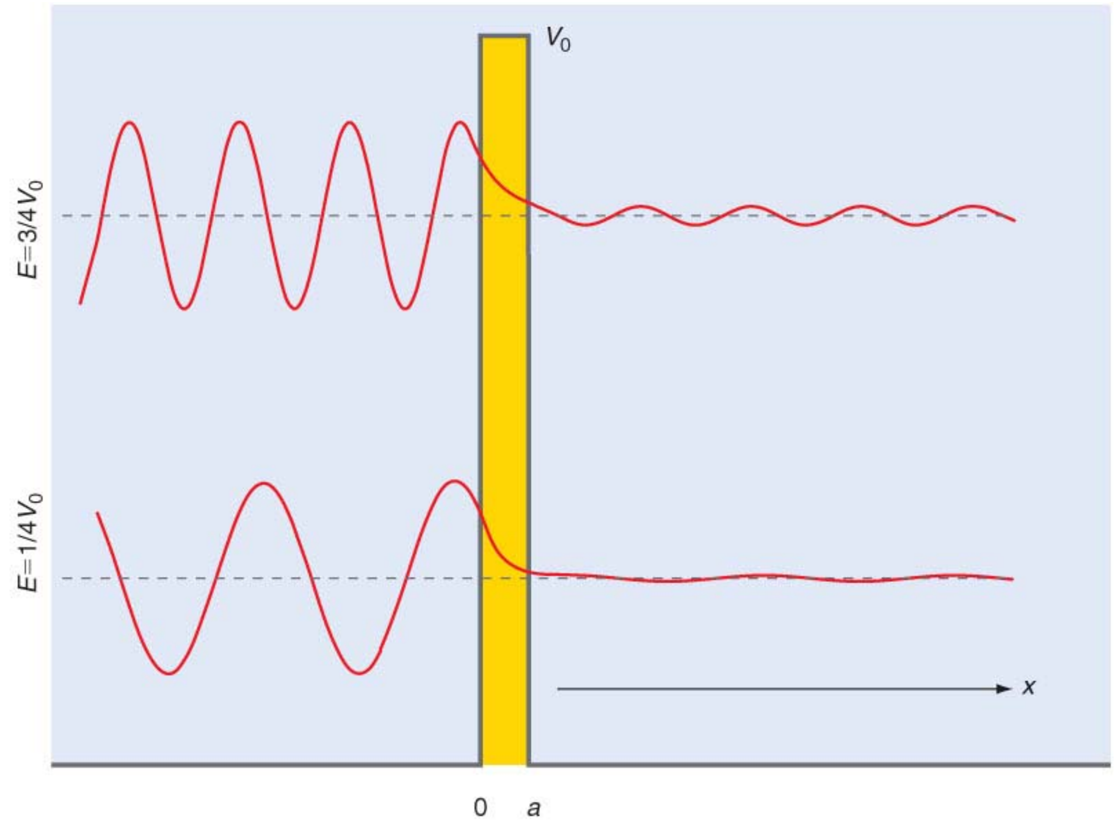
Na^+ cores

zero energy gap $\Rightarrow$ metal
large energy gap $\Rightarrow$ insulator

small gap $\rightarrow$ semiconductor

Apply electrical potential → charge will flow (for metal)

Tunneling through barrier



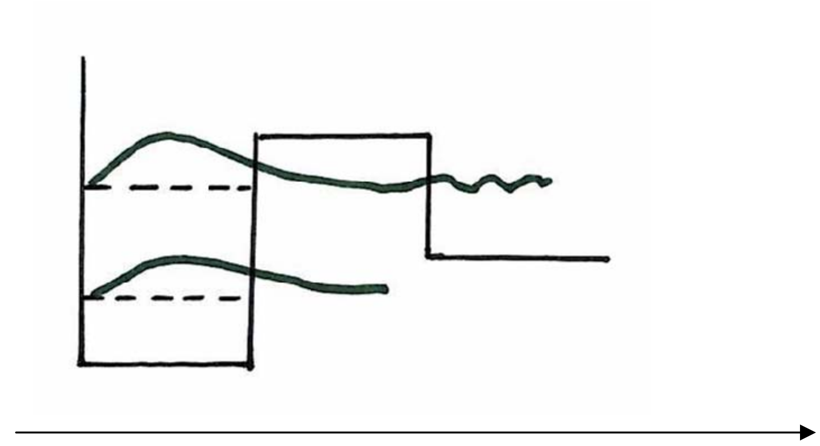
Can show tunneling is proportional to

$$e^{-2a\sqrt{2m(V_o-E)/\hbar^2}}$$

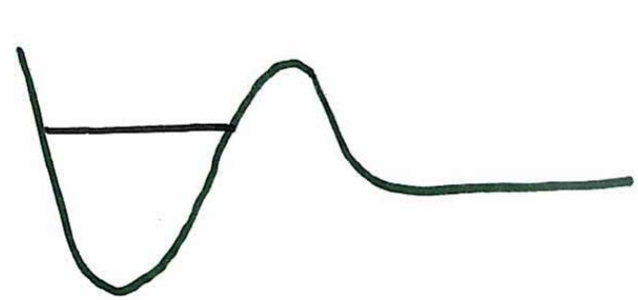
tunneling most important when:
- particle light
- energy near top of barrier
- barrier width is small

Note if $E > V_o$, the particle can be reflected by the barrier

resonances:
particle can escape by tunneling



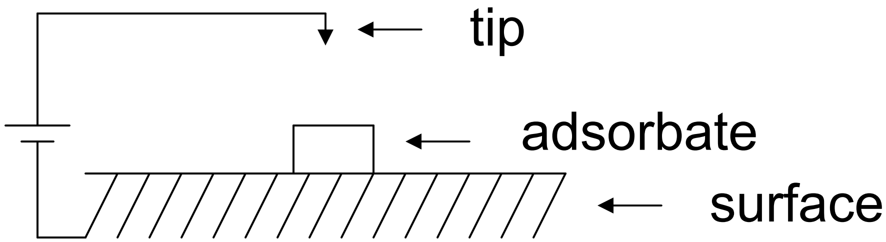
Examples:
- radioactive decay
- temporary anions: Be^-, N_2^-, benzene$^-$ electron falls off in $\approx 10^{-14}$ sec



How can one measure something with such a short lifetime?

Chapter 5, continued

Scanning tunneling microscopy (STM) – invented ~20 years ago at IBM Research Labs, Zürich



Apply voltage – measure current

often run so that as the tip is scanned over the surface, the height is varied so as to keep the current constant

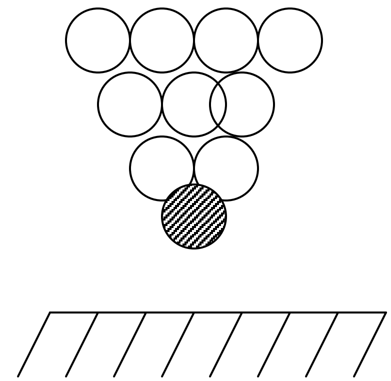
the tip does not actually touch the surface

electrons tunnel between tip and surface

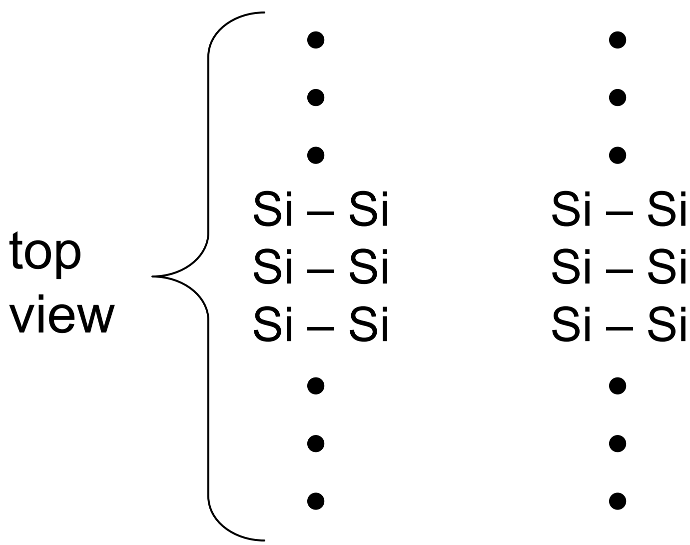
Atomic resolution

tunneling dominated by single atom
at the end of the tip

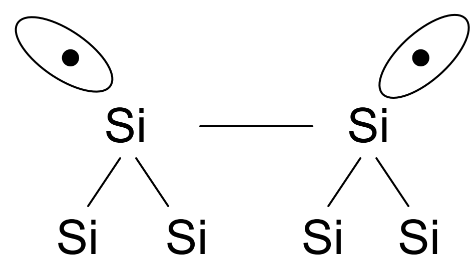
Why?



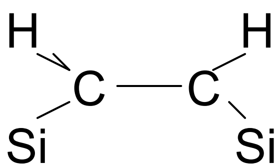
Used to study defects and adsorbed molecules on surfaces



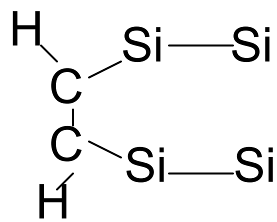
Si(100) surface –
– rows of silicon dimers



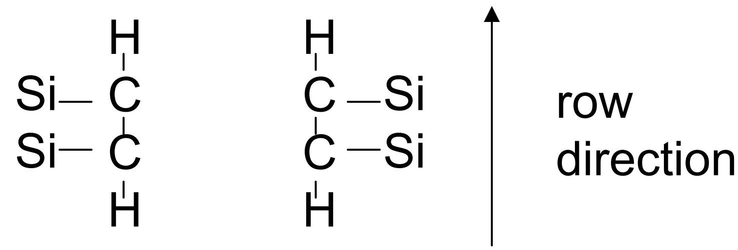
STM measurements (John Yates – Pitt) show 3 binding sites of acetylene on the surface



on-top dimer

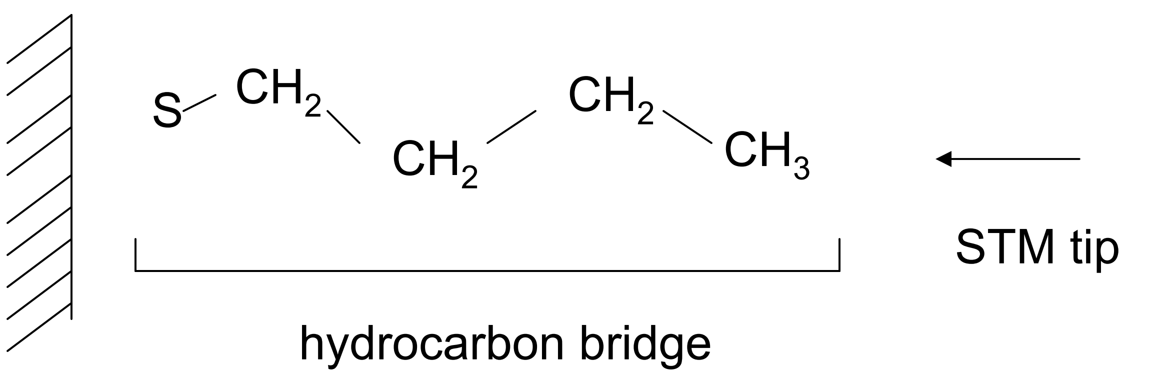


bridging two dimers



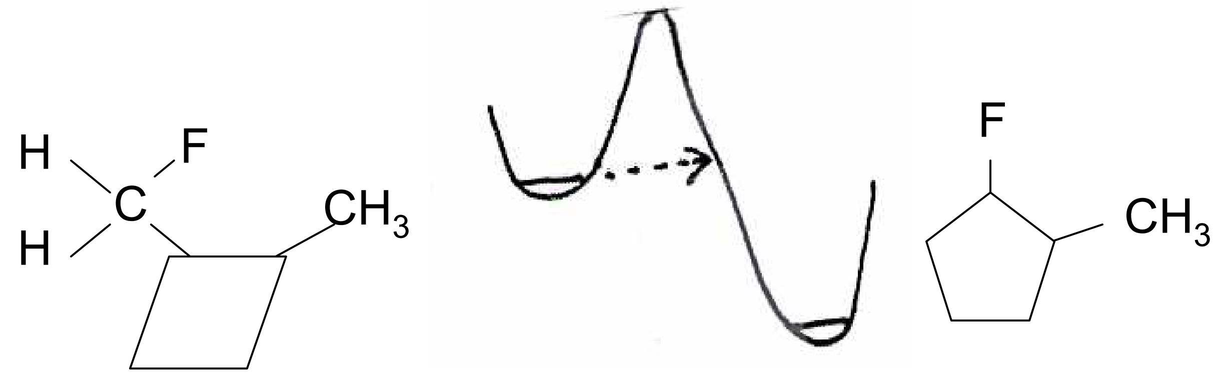
two acetylene molecules on two dimers

can also study tunneling through molecules



tunneling current falls off exponentially with distance

Reaction is 10^{152} times faster than expected at T = 10 K.



It proceeds *via* tunneling rather than going over the barrier

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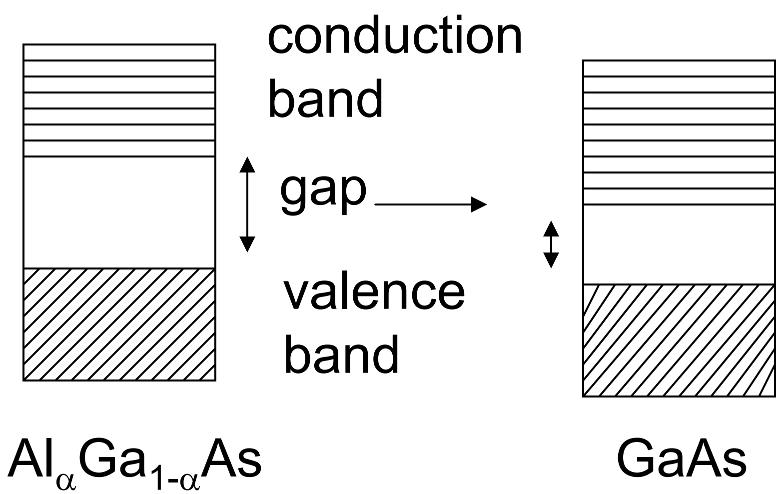
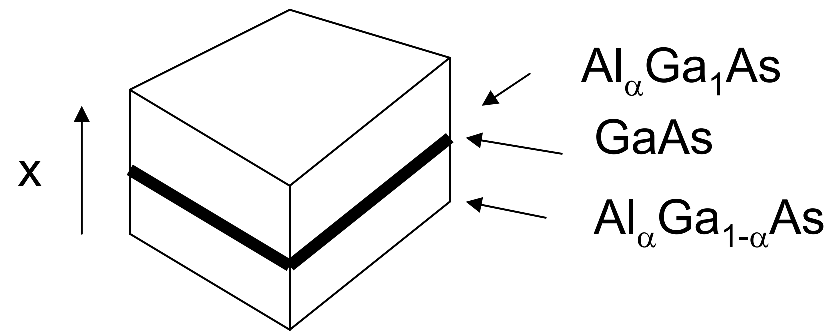
quantum wells



GaAs has a smaller band gap

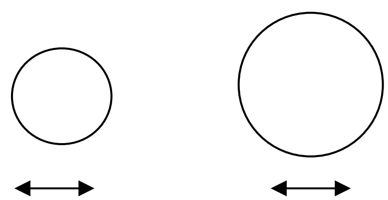
Make a layered device



Electron is free in y, z directions and confined in the x direction

Such systems can act as lasers

quantum dots



CdSe dots

2 nm 5 nm

emits green red fluorescence

used as tags to study processes in cells