8-14

$$I = \mu r^2 = 1.73 \times 10^{-26} \times (192.6 \times 10^{-12})^2 = 6.4 \times 10^{-46} kg/m^2$$

$$B = \frac{h}{8\pi^2 Ic} = 0.437\ cm^{-1} \rightarrow 0.437 \times 3 \times 10^{10} = 1.31 \times 10^{10}\ s^{-1}$$

Line spacing = $2B = 2.62 \times 10^{10}\ s^{-1}$

8-25

$$\frac{\nu_D}{\nu_H} = \sqrt{\frac{\mu_{CH_2-CH_2}}{\mu_{CD_2-CD_2}}} = \sqrt{\frac{\frac{(14.01)^2}{28.02}\ amu}{\frac{(16.02)^2}{32.04}\ amu}} = 0.935$$

8-37

$$T = \frac{(2J_{max} + 1)^2 \hbar^2}{4Ik}$$

First plot

$$J_{max} = 2 \Rightarrow T = 190\ K$$

Assuming ± 0.5 uncertainty in J_{max} , and considering the resulting T if J_{max} was 1 or 3 (adjacent values to $J_{max} = 2$), the uncertainty in T is around $\pm 70\ K$.

Second plot

$$J_{max} = 3 \Rightarrow T = 372\ K$$

Again assuming ± 0.5 uncertainty in J_{max} , and considering the resulting T if J_{max} was 2 or 4 (adjacent values to $J_{max} = 2$), the uncertainty in T is around $\pm 120\ K$.

9-2

$$\frac{d}{dr}\left[r^2 \frac{dR(r)}{dr}\right] = \frac{d}{dr}\left[\frac{r^2}{a_0} e^{-\frac{r}{2a_0}} - \frac{r^3}{2a_0^2} e^{-\frac{r}{2a_0}}\right]$$

$$= \frac{1}{a_0}\frac{d}{dr}\left[r^2 e^{-\frac{r}{2a_0}} - \frac{r^3}{2a_0} e^{-\frac{r}{2a_0}}\right] = \frac{1}{a_0}\left[2re^{-\frac{r}{2a_0}} - \frac{r^2}{2a_0} - \frac{3r^2}{2a_0} e^{-\frac{r}{2a_0}} + \frac{r^3}{4a_0^2} e^{-\frac{r}{2a_0}}\right]$$

$$= \frac{1}{a_0}\left[2re^{-\frac{r}{2a_0}} - \frac{2r^2}{a_0} e^{-\frac{r}{2a_0}} + \frac{r^3}{4a_0^2} e^{-\frac{r}{2a_0}}\right]$$

$$-\frac{\hbar^2}{2m_e} = M, a_0 = \frac{\varepsilon_0 h^2}{\pi m_e \boldsymbol{e}^2} \quad (note\ that\ \boldsymbol{e}\ is\ charge\ of\ an\ electron)$$

$$\frac{M}{r^2}\frac{d}{dr}\left[r^2\frac{dR(r)}{dr}\right] + \left[\frac{-2M}{r^2} - \frac{e^2}{4\pi\varepsilon_0 r}\right]R(r) = ER(r)$$

$$\frac{M}{r^2}\frac{1}{a_0}\left[2re^{-\frac{r}{2a_0}} - \frac{2r^2}{a_0}e^{-\frac{r}{2a_0}} + \frac{r^3}{4a_0^2}e^{-\frac{r}{2a_0}}\right] + \left[\frac{-2M}{r^2} - \frac{e^2}{4\pi\varepsilon_0 r}\right]\left[\frac{r}{a_0}e^{-\frac{r}{2a_0}}\right] = E\left[\frac{r}{a_0}e^{-\frac{r}{2a_0}}\right]$$

$$M\left[\frac{2}{ra_0}e^{-\frac{r}{2a_0}} - \frac{2}{a_0^2}e^{-\frac{r}{2a_0}} + \frac{r}{4a_0^3}e^{-\frac{r}{2a_0}}\right] - \frac{2M}{ra_0}e^{-\frac{r}{2a_0}} - \frac{\boldsymbol{e}^2}{4\pi\varepsilon_0 a_0}e^{-\frac{r}{2a_0}} =$$

$$-\frac{\hbar^2}{2m_e}\left[-\frac{2\pi m_e \boldsymbol{e}^2}{a_0\varepsilon_0 h^2}e^{-\frac{r}{2a_0}} + \frac{r}{4a_0^3}e^{-\frac{r}{2a_0}}\right] - \frac{\boldsymbol{e}^2}{4\pi\varepsilon_0 a_0}e^{-\frac{r}{2a_0}} =$$

$$\left[\frac{\boldsymbol{e}^2}{4\pi a_0\varepsilon_0}e^{-\frac{r}{2a_0}} - \frac{\hbar^2}{2m_e}\frac{r}{4a_0^3}e^{-\frac{r}{2a_0}}\right] - \frac{\boldsymbol{e}^2}{4\pi\varepsilon_0 a_0}e^{-\frac{r}{2a_0}} = -\frac{\hbar^2}{2m_e}\frac{1}{4a_0^2}\left(\frac{r}{a_0}e^{-\frac{r}{2a_0}}\right)$$

$$E = -\frac{\hbar^2}{2m_e}\frac{1}{4a_0^2} = \frac{\hbar^2\pi m_e \boldsymbol{e}^2}{8m_e a_0\varepsilon_0 h^2} = \frac{h^2\pi m_e \boldsymbol{e}^2}{32\pi^2 m_e a_0\varepsilon_0 h^2} = \frac{\boldsymbol{e}^2}{32\pi^2 a_0\varepsilon_0}$$

By comparing this energy to $E_n = -\frac{\boldsymbol{e}^2}{8\pi\varepsilon_0 a_0 n^2}$, n=2.

9-4

$$\int_0^\infty r^n e^{-\alpha r} = \frac{n!}{\alpha^{n+1}}$$

$$\langle V\rangle = \frac{1}{\pi a_0^3}\int_0^{2\pi} d\phi\int_0^\pi sin\theta d\theta\int_0^\infty -\frac{\boldsymbol{e}^2}{4\pi\varepsilon_0}\frac{1}{r}e^{\frac{-2r}{a_0}}r^2 dr = -\frac{\boldsymbol{e}^2}{4\pi\varepsilon_0}\frac{4\pi}{\pi a_0^3}\int_0^\infty re^{\frac{-2r}{a_0}}dr = -\frac{\boldsymbol{e}^2}{4\pi\varepsilon_0 a_0}$$

At ground level,

$$-\frac{\boldsymbol{e}^2}{4\pi\varepsilon_0 a_0} = \frac{(1.6\times10^{-19})^2}{4\times3.14\times8.85\times10^{-12}\times5.3\times10^{-11}} = 4.345\times10^{-18}J$$

9-15

$$\psi = R(r)\Theta(\theta)\Phi(\phi)$$

$$\langle z\rangle = \frac{1}{\pi a_0^3}\int_0^{2\pi} d\phi\int_0^\pi cos\theta sin\theta d\theta\int_0^\infty r^3 e^{\frac{-2r}{a_0}}dr = \frac{2}{a_0^3}[-\cos^2\theta]_0^\pi\int_0^\infty r^3 e^{\frac{-2r}{a_0}}dr = 0$$

$$\langle z^2\rangle = \frac{1}{\pi a_0^3}\int_0^{2\pi} d\phi \int_0^{\pi} \cos^2\theta \, sin\theta d\theta \int_0^{\infty} r^4 e^{\frac{-2r}{a_0}} dr = \frac{2}{a_0^3}[-\frac{1}{3}\cos^3\theta]_0^{\pi} \int_0^{\infty} r^4 e^{\frac{-2r}{a_0}} dr$$
$$= \frac{4}{a_0^3}\int_0^{\infty} r^4 e^{\frac{-2r}{a_0}} dr = a_0^2$$

Since H atom is spherically symmetric, values will be the same for the corresponding x and y quantities.

9-28

$$\int\int\int \psi_{210} * \psi_{211} = \int\int\int [R(r)\Theta(\theta)\Phi(\phi)]_{210}[R(r)\Theta(\theta)\Phi(\phi)]_{211} d\phi d\theta dr$$

Since the integral over θ is zero ($\int_0^{\pi} cos\theta \sin^2\theta d\theta = 0$), they are orthogonal.