4-32

To be an acceptable wave function for particle in the box:

ψ *and its partial derivatives must be continuous.*

$\int \psi * \psi$ *must exists* (*i.e.* ψ *must be quadratically integrable*).

$\int \psi * \psi$ *is the probability density so it must be single valued.*

ψ *cannot be infinite over a finite region of space.*

ψ *must satisfy the boundary conditions* ($\psi(0) = \psi(a) = 0$).

a) *is not acceptable, since* $\psi(0) \neq 0$
b) *is not acceptable, since* $\psi(a) \neq 0$
c) *is acceptable for the part of* ψ *in the box and can be normalized.* ψ outside the box must be zero.
d) *is not acceptable, since* $\psi(0) \neq 0$ *and* ψ *at* $x = 0$ *is not continuous*

5-1

$$\left|\frac{F}{A}\right|^2 = \frac{1}{1 + \frac{(k^2+\kappa^2)^2}{4(\kappa k)^2}\sinh^2 \kappa a}$$

$$\sinh \kappa a = \frac{e^{\kappa a} - \frac{1}{e^{\kappa a}}}{2} \qquad and \;\; \kappa = \sqrt{\frac{2m}{\hbar^2}(V_0 - E)}$$

$$\Rightarrow \sinh \kappa a \approx \frac{e^{\kappa a}}{2} \qquad (if\ \kappa a \gg 1\ e.g. when\ the\ barrier\ is\ very\ wide\ or\ E \ll V_0)$$

$$\left|\frac{F}{A}\right|^2 = \frac{1}{1 + \frac{(k^2+\kappa^2)^2}{16(\kappa k)^2}e^{2\kappa a}} = \left(1 + \frac{(k^2+\kappa^2)^2}{16(\kappa k)^2}e^{2\kappa a}\right)^{-1}$$

$$\Rightarrow \left|\frac{F}{A}\right|^2 \propto e^{-2\kappa a}$$

5-5

$$C = C - C = C - C = C - C = C$$

$$length\ of\ the\ box = 3 \times 154 + 4 \times 135 = 1002\ pm$$

Because there are 8 pi electrons, they occupy the first four energy levels. So exciting one pi electron means going from 4th to 5th level.

$$E = \frac{n^2 h^2}{8ma^2} \Rightarrow E_5 - E_4 = \frac{(5^2 - 4^2) \times h^2}{8 \times 9.11 \times 10^{-31} \times (1002 \times 10^{-12})^2} = 1.23 \times 10^{48} \times h^2$$

$$\lambda = \frac{c}{\nu} = \frac{c}{\frac{E}{h}} = \frac{2.998 \times 10^{8}}{1.23 \times 10^{48} \times 6.626 \times 10^{-34}} = 368 \times 10^{-9} m$$

6-5

a) $\Delta E = \left(\frac{dE}{dp_x}\right)\Delta p_x = \frac{2p_x}{2m}\Delta p_x = \frac{p_x}{m}\Delta p_x = \mathrm{v}_x \Delta p_x$

b) $\Delta E \Delta t = \mathrm{v}_x \Delta p_x \Delta t = \frac{\Delta x}{\Delta t}\Delta p_x \Delta t = \Delta x \Delta p_x \;\Rightarrow \Delta E \Delta t \geq \frac{\hbar}{2}$

c) $\Delta E \Delta t \geq \frac{\hbar}{2} \Rightarrow \Delta E \geq \frac{\hbar}{2\Delta t} = \frac{h}{4\pi \Delta t} = h\Delta\nu$

$$\Delta\nu = \frac{1}{4\pi\Delta t} = \frac{1}{4\pi(1.0 \times 10^{-9})} = 8.0 \times 10^{7} s^{-1}$$

$$\Delta\nu(cm^{-1}) = \frac{\Delta\nu(s^{-1})}{c} = \frac{8.0 \times 10^{7} s^{-1}}{2.998 \times 10^{10} \frac{cm}{s}} = 0.00265\ cm^{-1}$$

$$\Delta\nu = \frac{1}{4\pi\Delta t} = \frac{1}{4\pi(1.0 \times 10^{-11})} = 8.0 \times 10^{9} s^{-1}$$

$$\Delta\nu(cm^{-1}) = \frac{\Delta\nu(s^{-1})}{c} = \frac{8.0 \times 10^{9} s^{-1}}{2.998 \times 10^{10} \frac{cm}{s}} = 0.265\ cm^{-1}$$

6-7

$$\left[x^2 - \frac{d^2}{dx^2}, x - \frac{d}{dx}\right] f(x) = \left(x^2 - \frac{d^2}{dx^2}\right)\left(x - \frac{d}{dx}\right) f(x) - \left(x - \frac{d}{dx}\right)\left(x^2 - \frac{d^2}{dx^2}\right) f(x)$$

$$= \left(x^2 - \frac{d^2}{dx^2}\right)\left(xf(x) - \frac{df(x)}{dx}\right) - \left(x - \frac{d}{dx}\right)\left(x^2 f(x) - \frac{d^2 f(x)}{dx^2}\right)$$

$$= \left(x^3 f(x) - x^2 \frac{df(x)}{dx} - \frac{d^2}{dx^2}\left(xf(x)\right) + \frac{d^3 f(x)}{dx^3}\right)$$

$$- \left(x^3 f(x) - x\frac{d^2 f(x)}{dx^2} - \frac{d}{dx}\left(x^2 f(x)\right) + \frac{d^3 f(x)}{dx^3}\right)$$

$$= -x^2 \frac{df(x)}{dx} - \frac{d^2}{dx^2}\left(xf(x)\right) + x\frac{d^2 f(x)}{dx^2} + \frac{d}{dx}\left(x^2 f(x)\right)$$

For clarity, we evaluate the second and forth terms below:

$$-\frac{d^2}{dx^2}\left(xf(x)\right) = -\frac{d}{dx}\left(f(x) + x\frac{df(x)}{dx}\right) = -\frac{df(x)}{dx} - \frac{df(x)}{dx} - x\frac{d^2 f(x)}{dx} = -2\frac{df(x)}{dx} - x\frac{d^2 f(x)}{dx}$$

$$\frac{d}{dx}\left(x^2 f(x)\right) = 2xf(x) + x^2 \frac{df(x)}{dx}$$

So:

$$= -x^2 \frac{df(x)}{dx} - 2\frac{df(x)}{dx} - x\frac{d^2 f(x)}{dx} + x\frac{d^2 f(x)}{dx^2} + 2xf(x) + x^2 \frac{df(x)}{dx} = -2\frac{df(x)}{dx} + 2xf(x)$$