Additional notes chapters 2 and 3

$\psi^*(x)\psi(x) \equiv$ probability of finding particle at *x*

$\int_{-\infty}^{\infty} \psi^* \psi \, dx = 1 \longleftarrow$ probability of finding particle somewhere

(assumes ψ normalized)

ψ continuous, single valued
ψ' continuous

Single measurement of an observable $\hat{A}$ can only give an eigenvalue of $\hat{A}$.
If ψ not an eigenfunction of $\hat{A}$, one can still calculate an average

$$\langle \hat{A} \rangle = \frac{\int \psi \hat{A} \psi \, d\tau}{\int \psi \psi \, d\tau}$$

here τ denotes the variable(s) being integrated over.

Consider the case where ϕ_1 and ϕ_2 are eigenfunctions of $\hat{A}$

$$\hat{A}\phi_1 = a_1\phi_1$$

$$\hat{A}\phi_2 = a_2\phi_2$$

and suppose you have the wavefunction

$$\psi = \frac{1}{2}\phi_1 + \frac{\sqrt{3}}{2}\phi_2$$

$$\int \psi^* \psi \, d\tau = \frac{1}{4}\int \left(\phi_1\phi_1 + \underbrace{2\sqrt{3}\phi_1\phi_2}_{\downarrow \; 0} + 3\phi_2\phi_2 \right) d\tau$$

$$= \frac{1}{4}[1 + 3 = 1]$$

So ψ is normalized

It has been assumed that ϕ_1 and ϕ_2 are real.

$$\int \psi^* \hat{A} \psi \, d\tau$$

$$\frac{1}{4}\int \left(\phi_1 + \sqrt{3}\phi_2 \right) \hat{A} \left(\phi_1 + \sqrt{3}\phi_2 \right) d\tau$$

$$= \frac{1}{4}\int \left(\phi_1 + \sqrt{3}\phi_2 \right) \left[a_1\phi_1 + \sqrt{3}a_2\phi_2 \right] d\tau$$

$$= \frac{1}{4}\left[a_1 + 3a_2 \right]$$

¼ of measurements, get a_1

¾ of measurements, get a_2

Now consider the example from the text of three superposition states

$$\psi_1(x) = \frac{\sqrt{11}}{4}\phi_1(x) + \frac{1}{4}\phi_2(x) + \frac{1}{2}\phi_3(x)$$

$$\psi_2(x) = \frac{1}{2}\phi_1(x) + \frac{1}{4}\phi_2(x) + \frac{\sqrt{11}}{4}\phi_3(x)$$

$$\psi_3(x) = \frac{1}{2}\phi_1(x) + \frac{\sqrt{11}}{4}\phi_2(x) + \frac{1}{4}\phi_3(x)$$

$$\left.\begin{array}{l} \phi_1 \to a_1 \\ \phi_2 \to 4a_1 \\ \phi_3 \to 9a_1 \end{array}\right\} \text{ eigenvalues}$$

All measurements give one of these three values. But probabilities differ, depending on whether one has ψ_1, ψ_2, or ψ_3

To determine average, need to do measurements on many identical systems.

Cannot get the answer by doing multiple measurements on one system. Why?

Suppose you do a measurement and get a_1 for the value of $\hat{A}$. Then all successive measurements give a_1.

But if you do measurements on a set of identically prepared systems, you will get the distribution.

In classical universe – measurement does not change system.

In QM systems, measurements change the system (if it is not in an eigenstate of the observable being measured).

Consider three wavefunctions that differ only in signs of contributions of eigenstates

$$\psi_1 = \frac{\sqrt{11}}{4}\phi_1 + \frac{1}{4}\phi_2 + \frac{1}{2}\phi_3(x)$$

$$\psi_4 = \frac{\sqrt{11}}{4}\phi_1 - \frac{1}{4}\phi_2 + \frac{1}{2}\phi_3(x)$$

$$\psi_5 = \frac{\sqrt{11}}{4}\phi_1 - \frac{1}{4}\phi_2 - \frac{1}{2}\phi_3(x)$$

multiple measurements on each give same probabilities of $\phi_1,\ \phi_2,\ \phi_3$

Cannot fully determine a quantum mechanical wavefunction by measurements

Due to the fact it is $\psi^*\psi$ that we can measure not ψ.

The order of operators can matter

$$\hat{x}\hat{p}_x \sin x = x\left(\frac{\hbar}{i}\frac{\partial}{\partial x}\right)\sin x$$

$$= \frac{\hbar}{i} x \cos x$$

$$\hat{p}_x \hat{x} \sin x = \frac{\hbar}{i}\left[\sin x + x\cos x\right]$$

This is the origin of the uncertainty principle between $\hat{x}$ and $\hat{p}_x$.

So the order matters

In classical world can know x and p_x precisely, and order of measurements does not matter.

What about $\hat{p}_y \hat{x}$ and $\hat{x}\hat{p}_y$?

Link to a java applet for two coupled pendula

http://www.walter-fendt.de/ph14e/cpendula.htm

Note that this system has two natural frequencies and can be prepared in either of these or in a superposition of the two natural motions.