SOLUTION MANUAL FOR HW # 8

10.5

$$S_{total}^2 = (S_1 + S_2)^2 = S_1^2 + S_2^2 + 2(S_{1x}S_{2x} + S_{1y}S_{2y} + S_{1z}S_{2z})$$

$$\psi_+ = (\alpha(1)\beta(2) + \beta(1)\alpha(2))/\sqrt{2} \quad \& \quad \psi_- = (\alpha(1)\beta(2) + \beta(1)\alpha(2))/\sqrt{2}$$

$$S_1^2\alpha(1)\beta(2) = \frac{3\hbar^2}{4}\alpha(1)\beta(2) \quad \& \quad S_1^2\beta(1)\alpha(2) = \frac{3\hbar^2}{4}\beta(1)\alpha(2)$$

$$S_2^2\alpha(1)\beta(2) = \frac{3\hbar^2}{4}\alpha(1)\beta(2) \quad \& \quad S_2^2\beta(1)\alpha(2) = \frac{3\hbar^2}{4}\beta(1)\alpha(2)$$

$$S_{1x}S_{2x}\alpha(1)\beta(2) = \frac{\hbar}{2}\beta(1)\frac{\hbar}{2}\alpha(2) \quad \& \quad S_{1x}S_{2x}\,\beta(1)\alpha(2) = \frac{\hbar}{2}\alpha(1)\frac{\hbar}{2}\beta(2)$$

$$S_{1y}S_{2y}\alpha(1)\beta(2) = \frac{i\hbar}{2}\beta(1)\frac{-i\hbar}{2}\alpha(2) \quad \& \quad S_{1y}S_{2y}\,\beta(1)\alpha(2) = \frac{i\hbar}{2}\alpha(1)\frac{-i\hbar}{2}\beta(2)$$

$$S_{1z}S_{2z}\alpha(1)\beta(2) = \frac{\hbar}{2}\alpha(1)\frac{-\hbar}{2}\beta(2) \quad \& \quad S_{1z}S_{2z}\beta(1)\alpha(2) = \frac{\hbar}{2}\beta(1)\frac{-\hbar}{2}\alpha(2)$$

$$S_{total}^2\psi_+ = S_1^2(\alpha(1)\beta(2)/\sqrt{2}) + S_1^2(\beta(1)\alpha(2)/\sqrt{2}) + S_2^2(\alpha(1)\beta(2)/\sqrt{2}) + S_2^2(\beta(1)\alpha(2)/\sqrt{2})$$
$$+ (2/\sqrt{2})S_{1x}S_{2x}\alpha(1)\beta(2) + (2/\sqrt{2})S_{1x}S_{2x}\beta(1)\alpha(2) + (2/\sqrt{2})S_{1y}S_{2y}\alpha(1)\beta(2) + (2/\sqrt{2})S_{1y}S_{2y}\beta(1)\alpha(2)$$
$$+ (2/\sqrt{2})S_{1z}S_{2z}\alpha(1)\beta(2) + (2/\sqrt{2})S_{1z}S_{2z}\beta(1)\alpha(2) = 2\hbar^2\psi_+ \Rightarrow Eigenvalue \quad is \quad "2\hbar^2".$$

$$S_{total}^2\psi_- = S_1^2(\alpha(1)\beta(2)/\sqrt{2}) - S_1^2(\beta(1)\alpha(2)/\sqrt{2}) + S_2^2(\alpha(1)\beta(2)/\sqrt{2}) - S_2^2(\beta(1)\alpha(2)/\sqrt{2})$$
$$+ (2/\sqrt{2})S_{1x}S_{2x}\alpha(1)\beta(2) - (2/\sqrt{2})S_{1x}S_{2x}\beta(1)\alpha(2) + (2/\sqrt{2})S_{1y}S_{2y}\alpha(1)\beta(2) - (2/\sqrt{2})S_{1y}S_{2y}\beta(1)\alpha(2)$$
$$+ (2/\sqrt{2})S_{1z}S_{2z}\alpha(1)\beta(2) - (2/\sqrt{2})S_{1z}S_{2z}\beta(1)\alpha(2) = 0$$

10.9

$$E(\alpha)=\frac{\int \phi H\phi d\tau}{\int \phi\phi d\tau} \quad \& \quad \phi(r)=e^{-\alpha r}$$

$a)$

$$H=\frac{-\hbar^2}{2m_e r^2}\frac{\partial}{\partial r}(r^2\frac{\partial}{\partial r})-\frac{e^2}{4\pi\varepsilon_0 r}\Rightarrow H\phi=\frac{-\hbar^2}{2m_e r^2}\frac{\partial}{\partial r}(r^2\frac{\partial}{\partial r}\phi(r))-\frac{e^2}{4\pi\varepsilon_0 r}\phi(r)$$

$$H\phi=\frac{-\hbar^2}{2m_e r^2}\frac{\partial}{\partial r}(r^2\frac{\partial}{\partial r}e^{-\alpha r})-\frac{e^2}{4\pi\varepsilon_0 r}e^{-\alpha r}=\frac{\hbar^2}{2m_e r^2}\frac{\partial}{\partial r}(r^2(-\alpha)e^{-\alpha r})-\frac{e^2}{4\pi\varepsilon_0 r}e^{-\alpha r}$$

$$=\frac{\alpha\hbar^2}{2m_e r^2}(2r-\alpha r^2)-\frac{e^2}{4\pi\varepsilon_0 r}e^{-\alpha r}$$

$b)$

$$\int \phi H\phi d\tau=\int_0^\infty\int_0^\pi\int_0^{2\pi}\left\{\frac{\alpha\hbar^2}{2m_e r^2}(2r-\alpha r^2)e^{-2\alpha r}-\frac{e^2}{4\pi\varepsilon_0 r}e^{-2\alpha r}\right\}r^2 dr\sin\theta d\theta d\varphi$$

$$=\int_0^\infty\left\{\frac{\alpha\hbar^2}{2m_e r^2}(2r-\alpha r^2)e^{-2\alpha r}-\frac{e^2}{4\pi\varepsilon_0 r}e^{-2\alpha r}\right\}r^2 dr(\int_0^\pi\sin\theta d\theta\int_0^{2\pi}d\varphi)$$

$$=4\pi\int_0^\infty\left\{\frac{\alpha\hbar^2}{2m_e}(2r-\alpha r^2)e^{-2\alpha r}-\frac{e^2}{4\pi\varepsilon_0}re^{-2\alpha r}\right\}dr$$

$$\therefore\int_0^\infty re^{-2\alpha r}dr=\frac{1}{(2\alpha)^2} \quad \& \quad \int_0^\infty r^2e^{-2\alpha r}dr=\frac{2!}{(2\alpha)^3}$$

$$=4\pi\left\{\frac{\alpha\hbar^2}{m_e}\frac{1}{(2\alpha)^2}-\frac{\alpha^2\hbar^2}{2m_e}\frac{2!}{(2\alpha)^3}-\frac{e^2}{4\pi\varepsilon_0}\frac{1}{(2\alpha)^2}\right\}=\frac{\pi\hbar^2}{2m_e\alpha}-\frac{e^2}{4\varepsilon_0\alpha^2}$$

$c)$

$$\int \phi\phi d\tau=\int_0^\infty\int_0^\pi\int_0^{2\pi}\left\{e^{-2\alpha r}\right\}r^2 dr\sin\theta d\theta d\varphi=4\pi\frac{2!}{(2\alpha)^3}=\pi/\alpha^3$$

$d)$

$$E(\alpha)=\frac{\int \phi H\phi d\tau}{\int \phi\phi d\tau}=\frac{\frac{\pi\hbar^2}{2m_e\alpha}-\frac{e^2}{4\varepsilon_0\alpha^2}}{\pi/\alpha^3}=\frac{\hbar^2\alpha^2}{2m_e}-\frac{e^2\alpha}{4\pi\varepsilon_0}$$

$$\frac{dE(\alpha)}{d\alpha}=\frac{\hbar^2\alpha}{m_e}-\frac{e^2}{4\pi\varepsilon_0}=0\Rightarrow\alpha=\frac{e^2m_e}{4\pi\varepsilon_0\hbar^2}\Rightarrow E(\alpha)=\frac{e^4m_e}{32\pi^2\varepsilon_0^2\hbar^2}-\frac{e^4m_e}{16\pi^2\varepsilon_0^2\hbar^2}=-\frac{e^4m_e}{32\pi^2\varepsilon_0^2\hbar^2}$$

$e)$

Energy is geater than true energy.

10.11

$a)$

$Y_1^0(\theta,\phi) = (3/4\pi)^{1/2}\cos\theta$

$Y_1^1(\theta,\phi) = (3/8\pi)^{1/2}\sin\theta e^{i\phi}$

$Y_1^{-1}(\theta,\phi) = (3/8\pi)^{1/2}\sin\theta e^{-i\phi}$

$\left|Y_1^0(\theta,\phi)\right|^2 = \left|\{(3/4\pi)^{1/2}\cos\theta\}\{(3/4\pi)^{1/2}\cos\theta\}\right| = (3/4\pi)\cos^2\theta$

$\left|Y_1^1(\theta,\phi)\right|^2 = \left|\{(3/8\pi)^{1/2}\sin\theta e^{i\phi}\}\{(3/8\pi)^{1/2}\sin\theta e^{-i\phi}\}\right| = (3/8\pi)\sin^2\theta$

$\left|Y_1^1(\theta,\phi)\right|^2 = \left|\{(3/8\pi)^{1/2}\sin\theta e^{-i\phi}\}\{(3/8\pi)^{1/2}\sin\theta e^{i\phi}\}\right| = (3/8\pi)\sin^2\theta$

$b)$

$\left|Y_1^0(\theta,\phi)\right|^2 + \left|Y_1^1(\theta,\phi)\right|^2 + \left|Y_1^1(\theta,\phi)\right|^2 = (3/4\pi)\cos^2\theta + (3/8\pi)\sin^2\theta + (3/8\pi)\sin^2\theta$

$= (3/4\pi)\cos^2\theta + (3/4\pi)\sin^2\theta = (3/4\pi)(\cos^2\theta + \sin^2\theta) = (3/4\pi)$

c) Because charge density does not depend on angular part only radial part.

10.21

a) s^1d^5: ^7S **b)** f^3: L = 6, S = 3/2, ^4I **c)** g^2: L = 7, S = 1, ^3J

10.26

a) ^2S **b)** ^2P **c)** ^1S **d)** ^2S **e)** ^1S **f)** ^4S **g)** ^2D