SOLUTION MANUAL FOR HW # 7

9.2

$$E_n = -\frac{e^2}{8\pi\varepsilon_0 a_0 n^2}$$

$$a.E = -\frac{e^2}{32\pi\varepsilon_0 a_0} \Rightarrow n = 2 \Rightarrow l = 0,1 \Rightarrow m_l = 0,\pm1 \Rightarrow \psi_{200}, \psi_{210}, \psi_{21\pm1}$$

$$b.E = -\frac{e^2}{72\pi\varepsilon_0 a_0} \Rightarrow n = 3 \Rightarrow l = 0,1,2 \Rightarrow m_l = 0,\pm1,\pm2 \Rightarrow \psi_{300}, \psi_{310}, \psi_{31\pm1}, \psi_{320}, \psi_{32\pm1}, \psi_{32\pm2}$$

$$c.E = -\frac{e^2}{128\pi\varepsilon_0 a_0} \Rightarrow n = 4 \Rightarrow l = 0,1,2,3 \Rightarrow m_l = 0,\pm1,\pm2,\pm3 \Rightarrow \psi_{400}, \psi_{410}, \psi_{41\pm1}, \psi_{420}, \psi_{42\pm1},$$

$$\psi_{42\pm2}, \psi_{430}, \psi_{43\pm1}, \psi_{43\pm1}, \psi_{43\pm2}, \psi_{43\pm3}$$

Degenereacy :

a.4 *b*.9 *c*.16

9.4

$$\psi_{210} = \frac{1}{2\sqrt{6}} a_0^{-3/2} (a_0^{-1} r) e^{-a_0^{-1} r/2} \cos\theta$$

$$\psi_{211} = \frac{1}{2\sqrt{6}} a_0^{-3/2} (a_0^{-1} r) e^{-a_0^{-1} r/2} \sin\theta e^{i\varphi}$$

$$\int_0^{\infty}\int_0^{\pi}\int_0^{2\pi} \psi_{210}\psi_{211} d\tau = \frac{1}{24}\int_0^{r} a_0^{-5} r^4 e^{-a_0^{-1} r} dr \int_0^{\pi} \cos\theta \sin^2\theta d\theta \int_0^{2\pi} e^{i\varphi} d\varphi = 0$$

$$\therefore \sin\theta = u \quad \& \cos\theta d\theta = du$$

$$\Rightarrow \int_0^{\pi} \cos\theta \sin^2\theta d\theta = \int_{\theta=0}^{\theta=\pi} u^2 du = \left.\frac{u^3}{3}\right|_{\theta=0}^{\theta=\pi} = \left.\frac{\sin^3\theta}{3}\right|_0^{\pi} = 0$$

$$\therefore e^{i\varphi} = \cos\varphi + i\sin\varphi$$

$$\int_0^{2\pi} e^{i\varphi} d\varphi = \int_0^{2\pi} (\cos\varphi + i\sin\varphi) d\varphi = (\sin\varphi - i\cos\varphi)\Big|_0^{2\pi} = 0$$

9.6

a. n=2 & l=1 → Angular node=1 & 2-1-1=0 Radial node
b. n=2 & l=0 → Angular node=0 & 2-0-1=1 Radial node
c. n=3 & l=2 → Angular node=2 & 3-2-1=0 Radial node
d. n=3& l=2 → Angular node=2 & 3-2-1=0 Radial node

9.11

$$<r>=\int_0^\infty\int_0^\pi\int_0^{2\pi}\psi_{100}^* r\psi_{100} r^2\sin\theta drd\theta d\varphi$$

$$<r>=\int_0^\infty\int_0^\pi\int_0^{2\pi}\frac{1}{\sqrt{\pi}}a_0^{-3/2}e^{-a_0^{-1}r}r^3\frac{1}{\sqrt{\pi}}a_0^{-3/2}e^{-a_0^{-1}r}\sin\theta drd\theta d\varphi$$

$$<r>=\frac{a_0^{-3}}{\pi}(\int_0^\infty r^3e^{-2a_0^{-1}r}dr)(\int_0^\pi\sin\theta d\theta\int_0^{2\pi}d\varphi)=4a_0{}^{-3}\int_0^\infty r^3e^{-2a_0^{-1}r}dr=4a_0^{-3}\frac{3!}{(2a_0^{-1})^{3+1}}=\frac{3}{2}a_0$$

9.17

$$\psi_{100} = \frac{1}{\sqrt{\pi}} a_0^{-3/2} e^{-a_0^{-1} r}$$

$$< z >= \int_0^\infty \int_0^\pi \int_0^{2\pi} \psi_{100}^* z \psi_{100} r^2 \sin\theta dr d\theta d\varphi = \int_0^\infty \int_0^\pi \int_0^{2\pi} \psi_{100}^* r \cos\theta \psi_{100} r^2 \sin\theta dr d\theta d\varphi$$

$$< z >= \frac{3}{4a_0^{-1}} \int_0^\pi \cos\theta \sin\theta d\theta = \frac{3}{8a_0} \sin 2\theta \Big|_0^\pi = 0$$

$$< z^2 >= \int_0^\infty \int_0^\pi \int_0^{2\pi} \psi_{100}^* z^2 \psi_{100} r^2 \sin\theta dr d\theta d\varphi = \int_0^\infty \int_0^\pi \int_0^{2\pi} \psi_{100}^* r^2 \cos^2\theta \psi_{100} r^2 \sin\theta dr d\theta d\varphi$$

$$< z^2 >= \frac{3}{2a_0^{-2}} \int_0^\pi \cos^2\theta \sin\theta d\theta = \frac{3}{2a_0^{-2}} (-\frac{\cos^3\theta}{3}) \Big|_0^\pi = a_0^2$$

$$< x >= \int_0^\infty \int_0^\pi \int_0^{2\pi} \psi_{100}^* x \psi_{100} r^2 \sin\theta dr d\theta d\varphi = \int_0^\infty \int_0^\pi \int_0^{2\pi} \psi_{100}^* r \sin\theta \cos\varphi \psi_{100} r^2 \sin\theta dr d\theta d\varphi$$

$$< x >= \frac{3}{8\pi a_0^{-1}} \int_0^\pi \sin\theta \sin\theta d\theta \int_0^{2\pi} \cos\varphi d\varphi = \frac{3}{8\pi a_0^{-1}} \cdot \frac{\pi}{2} \cdot 0 = 0$$

$$< x^2 >= \int_0^\infty \int_0^\pi \int_0^{2\pi} \psi_{100}^* x^2 \psi_{100} r^2 \sin\theta dr d\theta d\varphi = \int_0^\infty \int_0^\pi \int_0^{2\pi} \psi_{100}^* r^2 \sin^2\theta \cos^2\varphi \psi_{100} r^2 \sin\theta dr d\theta d\varphi$$

$$< x^2 >= \frac{4!}{(2a_0^{-1})^{4+1}\pi} \int_0^\pi \sin^2\theta \sin\theta d\theta \int_0^{2\pi} \cos^2\varphi d\varphi = \frac{4!}{(2a_0^{-1})^{4+1}\pi} \int_0^\pi (1-\cos^2\theta) \sin\theta d\theta \int_0^{2\pi} \cos^2\varphi d\varphi$$

$$< x^2 >= \frac{4!}{(2a_0^{-1})^{4+1}\pi} \int_0^\pi (1-u^2) - du \int_0^{2\pi} \frac{1}{2}(\cos 2\varphi + 1) d\varphi = \frac{4!}{(2a_0^{-1})^{4+1}\pi} (\frac{u^3}{3} - u) \Big|_1^{-1} (\frac{\sin 2\varphi}{4} + \frac{\varphi}{2}) \Big|_0^{2\pi} =$$

$$< x^2 >= \frac{4! a_0^{-3}}{(2a_0^{-1})^{4+1}\pi} \cdot \frac{4}{3} \cdot \pi = a_0^2$$

$$< y >= \int_0^\infty \int_0^\pi \int_0^{2\pi} \psi_{100}^* y \psi_{100} r^2 \sin\theta dr d\theta d\varphi = \int_0^\infty \int_0^\pi \int_0^{2\pi} \psi_{100}^* r \sin\theta \sin\varphi \psi_{100} r^2 \sin\theta dr d\theta d\varphi$$

$$< x >= \frac{3}{8\pi a_0^{-1}} \int_0^\pi \sin\theta \sin\theta d\theta \int_0^{2\pi} \sin\varphi d\varphi = \frac{3}{8\pi a_0^{-1}} \cdot \frac{\pi}{2} \cdot 0 = 0$$

$$< y^2 >= \int_0^\infty \int_0^\pi \int_0^{2\pi} \psi_{100}^* y^2 \psi_{100} r^2 \sin\theta dr d\theta d\varphi = \int_0^\infty \int_0^\pi \int_0^{2\pi} \psi_{100}^* r^2 \sin^2\theta \sin^2\varphi \psi_{100} r^2 \sin\theta dr d\theta d\varphi$$

$$< y^2 >= \frac{4!}{(2a_0^{-1})^{4+1}\pi} \int_0^\pi \sin^2\theta \sin\theta d\theta \int_0^{2\pi} \sin^2\varphi d\varphi = \frac{4!}{(2a_0^{-1})^{4+1}\pi} \int_0^\pi (1-\cos^2\theta) \sin\theta d\theta \int_0^{2\pi} \sin^2\varphi d\varphi$$

$$< y^2 >= \frac{4!}{(2a_0^{-1})^{4+1}\pi} \int_0^\pi (1-u^2) - du \int_0^{2\pi} \frac{1}{2}(1 - \cos 2\varphi 1) d\varphi = \frac{4!}{(2a_0^{-1})^{4+1}\pi} (\frac{u^3}{3} - u) \Big|_1^{-1} (\frac{\varphi}{2} - \frac{\sin 2\varphi}{4}) \Big|_0^{2\pi} =$$

$$< y^2 >= \frac{4! a_0^{-3}}{(2a_0^{-1})^{4+1}\pi} \cdot \frac{4}{3} \cdot \pi = a_0^2$$

$$\langle x\rangle = \langle y\rangle = \langle z\rangle = 0 \;\&\; \langle x^2\rangle = \langle y^2\rangle = \langle z^2\rangle = a_0^2$$