SOLUTION MANUAL FOR HW # 4

6.2)

$$\left[(\frac{d}{dx}-x),(\frac{d}{dx}+x)\right]=(\frac{d}{dx}-x)(\frac{d}{dx}+x)f-(\frac{d}{dx}+x)(\frac{d}{dx}-x)f$$

$$=(\frac{d^2}{dx^2}+1-1-x^2)f-(\frac{d^2}{dx^2}-1+1-x^2)f=0\Rightarrow do\quad not\quad commute$$

6.16)

a)

$$\therefore E=p_x^2/2m\quad \&\quad \Delta E=(\frac{dE}{dp_x})\Delta p_x$$

$$E=\frac{v_x^2 m}{2}=\frac{p_x^2}{2m}$$

$$\frac{dE}{dp_x}=\frac{2p_x}{2m}=\frac{p_x}{m}=\frac{v_x m}{m}=v_x$$

$$\Delta E=v_x\Delta p_x$$

b)

$$\therefore v_x=\frac{\Delta x}{\Delta t}$$

$$\Delta E=v_x\Delta p_x$$

$$\Delta E=\frac{\Delta x\Delta p_x}{\Delta t}$$

$$\Delta E\Delta t=\Delta x\Delta p_x\geq \hbar/2$$

c)

$$\therefore \Delta E\Delta t\geq \hbar/2$$

$$\Delta E\geq\frac{\hbar}{2\Delta t}$$

$$\Delta t=9.9*10^{-10}s$$

$$\Delta E\geq 5.33*10^{-26}J$$

7.2)

$$H=\frac{-\hbar^2}{2m}\frac{d^2}{dx^2}+\frac{kx^2}{2}\quad \&\quad \Psi_n=N_nH_n(x)e^{-\alpha x^2}$$

$$H\Psi=E\Psi\quad \&\quad \alpha=\frac{1}{2}(\frac{mk}{\hbar^2})^{1/2}\quad \&\quad H_0(y)=1\Rightarrow \Psi_0=N_0e^{-\alpha x^2}$$

$$\frac{-\hbar^2}{2m}\frac{d^2}{dx^2}\Psi+\frac{kx^2}{2}\Psi=\frac{-h^2}{2m}\frac{d^2}{dx^2}(N_0e^{-\alpha x^2})+\frac{kx^2}{2}(N_0e^{-\alpha x^2})\quad (\therefore \frac{d^2}{dx^2}(e^{-\alpha x^2})=2\alpha e^{-\alpha x^2}-4\alpha^2x^2e^{-\alpha x^2})$$

$$=N_0e^{-\alpha x^2}(\frac{2\alpha\hbar^2}{2m}-\frac{4\alpha^2x^2\hbar^2}{2m}+\frac{kx^2}{2})=N_0e^{-\alpha x^2}\{\frac{1}{2}(\frac{mk}{\hbar^2})^{1/2}\frac{\hbar^2}{m}-4(\frac{1}{4}(\frac{mk}{\hbar^2}))\frac{x^2\hbar^2}{2m}+\frac{kx^2}{2}\}$$

$$=N_0e^{-\alpha x^2}\{\frac{1}{2}\sqrt{\frac{k}{m}}\hbar-\frac{kx^2}{2}+\frac{kx^2}{2}\}=E\Psi\Rightarrow E=\frac{1}{2}\sqrt{\frac{k}{m}}\hbar=\frac{1}{2}\hbar\omega\quad where\quad \omega=\sqrt{\frac{k}{m}}$$

7.13)

$$< x^2 >= (n+1/2)\alpha^2 \Rightarrow \sqrt{< x^2 >} = \alpha\sqrt{n+1/2}$$

$$\alpha = (\hbar^2 / km)^{1/4}$$

$$m = \frac{m_H m_{Cl}}{m_H + m_{Cl}} = 1.61 * 10^{-27} kg$$

$$k = 516 Nm^{-1}$$

$$\alpha = 1.07 * 10^{-11} m$$

$$n = 0 \Rightarrow \sqrt{< x^2 >} = 7.60 * 10^{-12} m \Rightarrow \%6 \quad of \quad HCl \quad bond$$

$$n = 1 \Rightarrow \sqrt{< x^2 >} = 1.32 * 10^{-11} m \Rightarrow \%10 \quad of \quad HCl \quad bond$$

$$n = 2 \Rightarrow \sqrt{< x^2 >} = 1.69 * 10^{-11} m \Rightarrow \%13 \quad of \quad HCl \quad bond$$