CHEM 1440
SOLUTION MANUAL FOR HW # 2

2.6)

$$\Psi(x,t) = A\sin(kx - wt) + 2A\sin(kx + wt)$$
$$\Psi(x,t) = A[\sin kx\cos wt - \cos kx\sin wt] + 2A[\sin kx\cos wt + \cos kx\sin wt]$$
$$\Psi(x,t) = 3A\sin kx\cos wt + A\cos kx\sin wt$$

Not a standing wave

2.8)

$$e^{ix} = \cos x + i\sin x$$

a) $2e^{i\pi/2} = 2[\cos(\pi/2) + i\sin(\pi/2)] = 2\times[0 + i\times 1] = 2i$

b) $2\sqrt{5}e^{-i\pi/2} = 2\sqrt{5}\times[\cos(-\pi/2) + i\sin(-\pi/2)] = -2\sqrt{5}i$

c) $e^{i\pi} = [\cos(\pi) + i\sin(\pi)] = -1$

d)

$$\frac{3\sqrt{2}}{5+\sqrt{3}}e^{i\pi/4} = \frac{3\sqrt{2}}{5+\sqrt{3}}[\cos(\pi/4) + i\sin(\pi/4)]$$
$$= \frac{3\sqrt{2}}{5+\sqrt{3}}\left(\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}\right) = \frac{3}{5+\sqrt{3}}(1+i)$$

2.11)

$\frac{d^3}{dx^3}x^3 = 6 \Rightarrow$ "x^3" not eigenfunction of the operator, "d^3/dx^3".

$x(\frac{\partial}{\partial x}) + y(\frac{\partial}{\partial y})(xy) = x(\frac{\partial}{\partial x}xy) + y(\frac{\partial}{\partial y}xy) = xy + yx = 2xy \Rightarrow$ "xy" eigenfunction of "$x(\partial/\partial x)+y(\partial/\partial y)$" with an eigenvalue of "2".

$\frac{\partial^2}{\partial\theta^2}\sin\theta\cos\phi = \cos\phi\frac{\partial^2}{\partial\theta^2}\sin\theta = -\cos\phi\sin\theta \Rightarrow$ "$\sin\theta\cos\Phi$" eigenfunction of "$\partial^2/\partial\theta^2$" with an eigenvalue of "-1".

2.19)

$$\{\frac{1}{r^2}\frac{d}{dr}[r^2\frac{d}{dr}]+\frac{2}{r}\}(Ae^{-br})=\frac{1}{r^2}\frac{d}{dr}[r^2\frac{d}{dr}Ae^{-br}]+\frac{2Ae^{-br}}{r}$$

$$=\frac{1}{r^2}\frac{d}{dr}[A(-b)r^2e^{-br}]+\frac{2Ae^{-br}}{r}=\frac{1}{r^2}[-2Abre^{-br}+Ab^2r^2e^{-br}]+\frac{2Ae^{-br}}{r}$$

$$=Ae^{-br}[\frac{-2b}{r}+b^2+\frac{2}{r}]=Ae^{-br}[\frac{2-2b}{r}+b^2]$$

$$[\frac{2-2b}{r}+b^2]=Cons\tan t \quad to \quad be \quad eigenvalue \Rightarrow \frac{2-2b}{r}=0\Rightarrow b=1$$

$A \quad any \quad number$

2.21)

$$\int_0^{2\pi}\phi_m^*(\theta)\phi_n(\theta)d\theta=\int_0^{2\pi}e^{-im\theta}e^{in\theta}d\theta=\int_0^{2\pi}e^{i(n-m)\theta}d\theta$$

$$=\int_0^{2\pi}\left(\cos[(n-m)\theta]+i\sin[(n-m)\theta]\right)d\theta$$

$$=\frac{1}{n-m}\sin[(n-m)\theta]\Big|_0^{2\pi}-\frac{i}{n-m}\cos[(n-m)\theta]\Big|_0^{2\pi}$$

$n\neq m\Rightarrow(n-m)\quad$ int$eger$

$$=\frac{1}{n-m}\{\sin[(n-m)2\pi]-\sin[(n-m)0]-\frac{i}{n-m}\{\cos[(n-m)2\pi]-\cos[(n-m)0]\}=0$$

$$\sin[(n-m)2\pi]=0 \quad \& \quad \sin[(n-m)0]=0$$

$$\cos[(n-m)2\pi]=1 \quad \& \quad \cos[(n-m)0]=1$$

$$=\int_0^{2\pi}\phi_m^*(\theta)\phi_n(\theta)d\theta=\int_0^{2\pi}e^{-im\theta}e^{in\theta}d\theta=\int_0^{2\pi}e^{i(m-n)\theta}d\theta=0$$

2.27)

$$f(x) = d_0 + \sum_{n=1}^{5} c_n \sin(\frac{n\pi x}{b}) + d_n \cos(\frac{n\pi x}{b})$$

$$c_n = \frac{1}{b}\int_{-b}^{b} x \sin(\frac{n\pi x}{b}) dx$$

$$u = \frac{n\pi x}{b} \Rightarrow x = (\frac{b}{n\pi})u$$

$$dx = (\frac{b}{n\pi})du$$

$$c_n = \frac{1}{b}\int_{-n\pi}^{n\pi} (\frac{b}{n\pi})u \sin(u)(\frac{b}{n\pi})du$$

$$c_n = (\frac{b}{n^2\pi^2})\int_{-n\pi}^{n\pi} u \sin(u) du$$

$$\therefore \int u \sin(u) du = -u\cos u + \sin u \qquad (by \quad \text{int}\,egration \quad by \quad parts)$$

$$c_n = (\frac{b}{n^2\pi^2})\left[-u\cos u\Big|_{-n\pi}^{n\pi} + \sin u\Big|_{-n\pi}^{n\pi}\right]$$

$$\therefore For \quad n \quad even$$

$$\left[-u\cos u\Big|_{-n\pi}^{n\pi} + \sin u\Big|_{-n\pi}^{n\pi}\right] = -2n\pi$$

$$c_n = \frac{-2b}{n\pi}$$

$$\therefore For \quad n \quad odd$$

$$\left[-u\cos u\Big|_{-n\pi}^{n\pi} + \sin u\Big|_{-n\pi}^{n\pi}\right] = 2n\pi$$

$$c_n = \frac{2b}{n\pi}$$

$$d_0 = \frac{1}{b}\int_{-b}^{b} x dx = \frac{1}{b}\frac{x^2}{2}\Bigg|_{-b}^{b} = 0$$

$$d_n = \frac{1}{b}\int_{-b}^{b} x \cos(\frac{n\pi x}{b}) dx$$

$$d_n = (\frac{b}{n^2\pi^2})\int_{-n\pi}^{n\pi} u \cos(u) du$$

$$\therefore \int u \sin(u) du = u\sin u + \cos u \qquad (by \quad \text{int}\,egration \quad by \quad parts)$$

$$d_n = (\frac{b}{n^2\pi^2})\left[u\sin u\Big|_{-n\pi}^{n\pi} + \cos u\Big|_{-n\pi}^{n\pi}\right] = 0$$

2.29)

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \Rightarrow not \quad eigenfunction$$

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$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \Rightarrow \quad eigenfunction \quad with \quad eigenvalue \quad "1"$$

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix} \Rightarrow \quad eigenfunction \quad with \quad eigenvalue \quad "-1"$$