

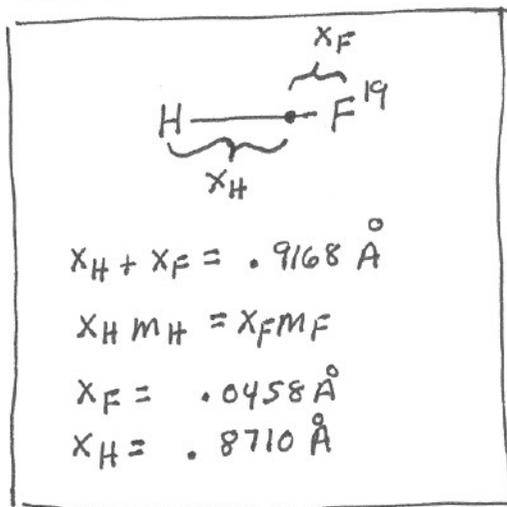
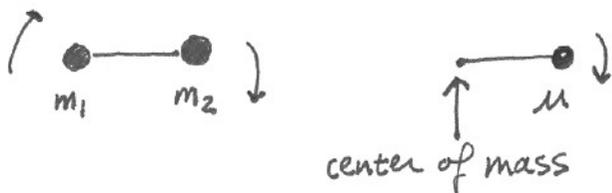
Rotation in 2 dimensions

$$H_{total} = H_{trans}(r_{cm}) + H_{vib}(r_{internal}) + H_{rot}(\theta, \phi)$$

$$E_{total} = E_{trans} + E_{vib} + E_{rot}$$

$$\psi_{tot} = \psi_{trans} \psi_{vib} \psi_{rot}$$

separation of variables



$V(x,y) = 0$ everywhere

$$-\frac{\hbar^2}{2\mu} \left(\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} \right)_{r=r_0} = E\psi$$

↑ fixed radius

Switch to polar coordinates

$$-\frac{\hbar^2}{2\mu r_0^2} \frac{d^2 \Phi}{d\phi^2} = E \Phi \rightarrow \Phi = e^{im\phi}, \quad m = 0, \pm 1, \pm 2, \dots$$

$0 \leq \phi \leq 2\pi$

$$e^{im(\phi+2\pi)} = e^{im\phi} \Rightarrow e^{im2\pi} = 1$$

$$e^{im2\pi} = \cos 2\pi m + i \sin 2\pi m = 1 \Rightarrow m = 0, \pm 1, \pm 2, \dots$$

$$E = \frac{\hbar^2 m^2}{2\mu r_0^2} = \frac{\hbar^2 m^2}{2I}$$

$I = \mu r_0^2 = \text{moment of inertia}$

Quantization due to boundary condition $\Psi(0) = \Psi(2\pi)$

Note: there is no zero-point energy. Why?

Classically $E = \frac{|\vec{L}|^2}{2I} = \frac{1}{2} I \omega^2$ | All energies possible

\vec{L} = angular momentum

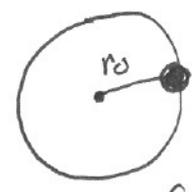
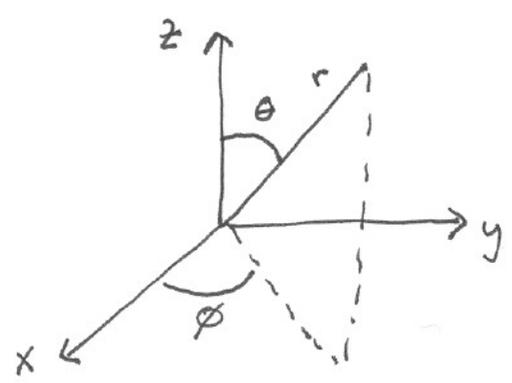
angular momentum in z direction: $\hat{L}_z = \frac{\hbar}{i} \frac{\partial}{\partial \phi}$

$$\hat{L}_z \Psi = \frac{\hbar}{i} \frac{1}{\sqrt{2\pi}} \frac{d}{d\phi} e^{im\phi} = m\hbar \Psi$$

$P(\phi)d\phi = \frac{d\phi}{2\pi}$, all ϕ values equally probable }
 angular momentum in z direction }
 precisely defined

$\hat{L}_z, \hat{\phi}$ do not commute

On to 3 dimensions.



motion of particle on the surface of a sphere
 \equiv Rigid rotor

$0 \leq r \leq \infty$
 $0 \leq \theta \leq \pi$
 $0 \leq \phi \leq 2\pi$

} volume element $r^2 \sin\theta dr d\theta d\phi$

$x = r \sin\theta \cos\phi$
 $y = r \sin\theta \sin\phi$
 $z = r \cos\theta$

$$-\frac{\hbar^2}{2\mu r_0^2} \left[\frac{1}{\sin\theta} \frac{\partial}{\partial\theta} \left(\sin\theta \frac{\partial Y}{\partial\theta} \right) + \frac{1}{\sin^2\theta} \frac{\partial^2 Y}{\partial\phi^2} \right] = E Y$$

$$Y = Y(\theta, \phi)$$

$$\beta = \frac{2\mu r_0^2}{\hbar^2} E$$

$$\underbrace{\sin\theta \frac{\partial}{\partial\theta} \left(\sin\theta \frac{\partial Y}{\partial\theta} \right) + \beta \sin^2\theta Y}_{\text{depends only on } \theta} = - \underbrace{\frac{\partial^2 Y}{\partial\phi^2}}_{\text{depends only on } \phi}$$

$$\Rightarrow Y(\theta, \phi) = \Theta(\theta) \Phi(\phi)$$

$$\underbrace{\frac{1}{\Theta} \sin\theta \frac{d}{d\theta} \left(\sin\theta \frac{d\Theta}{d\theta} \right) + \beta \sin^2\theta}_{\text{depends only on } \theta}$$

depends only on θ

spherical harmonics

$$= \underbrace{-\frac{1}{\Phi} \frac{d^2\Phi}{d\phi^2}}_{\text{depends only on } \phi}$$

depends only on ϕ