

## Chpt. 6 - Commutators

The values of two different observables,  $a$  and  $b$ , can be simultaneously determined (precisely) only if the measurement does not change the state of the system.

$$a \leftrightarrow \hat{A}$$

$$b \leftrightarrow \hat{B}$$

$$\begin{aligned}\hat{B} \hat{A} \psi_n(x) &= \hat{B} \alpha_n \psi_n(x), \text{ if } \psi_n \text{ an e.f. of } \hat{A} \\ &= \beta_n \alpha_n \psi_n(x), \text{ if } \psi_n \text{ also an e.f. of } \hat{B}\end{aligned}$$

Note that for this case  $\hat{B} \hat{A} \psi_n = \hat{A} \hat{B} \psi_n$

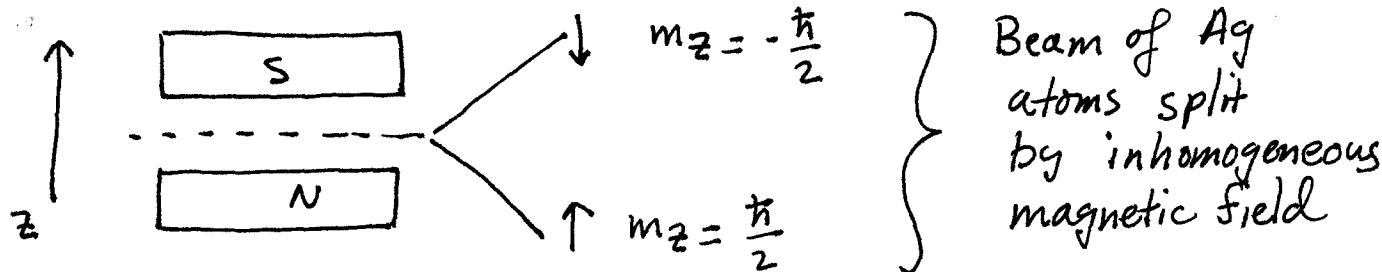
$$(\hat{A} \hat{B} - \hat{B} \hat{A}) f = \underbrace{[A, B] f}_{\text{commutator}}$$

$[A, B] = 0 \Rightarrow$  the corresponding observables can be determined exactly, simultaneously.

$P_x, x$  cannot be known exactly

$P_{x, H}$  cannot be known exactly if  $V \neq 0$  (actually if  $V \neq \text{constant}$ )

Stern-Gerlach expt.

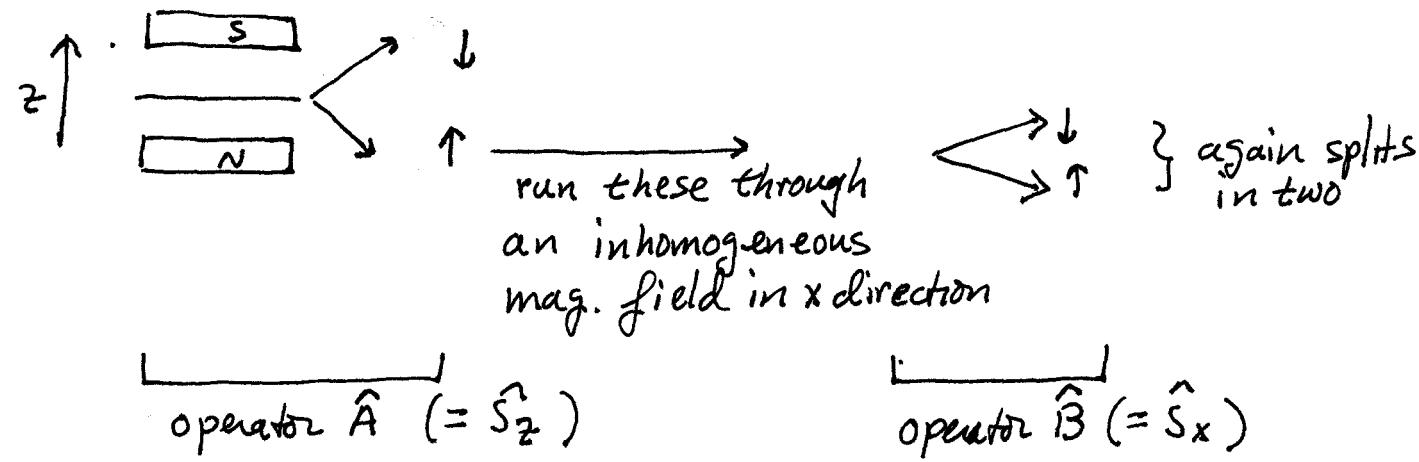


Ag atoms - unpaired  $e^-$  has spin  
 $\rightarrow$  magnetic moment in  $z$  direction  
 $\rightarrow$  deflected by external magnetic field

Expt.  $\rightarrow$  only 2 values of spin possible in the  $z$  direction

$\rightarrow$  eigenfunctions  $\alpha$  (up spin),  $\beta$  (down spin)

Initial w.f.  $\psi = \frac{1}{\sqrt{2}}(c_1\alpha + c_2\beta)$ ,  $|c_1|^2 + |c_2|^2 = 1$



Now take beam of the downward deflected atoms and pass through magnetic field in  $z$  direction.

The beam is split in two ( $\alpha, \beta$ )  
 $\Rightarrow \hat{A}$  and  $\hat{B}$  do not commute.

$\mu_z$  and  $\mu_x$  cannot be simultaneously well defined

Stern Gerlach expt (1921) was carried out to confirm the Bohr model.

Electron spin was discovered several years later

uncertainty principle (Heisenberg)

$$\Delta p \cdot \Delta x \geq \frac{\hbar}{2} \quad \neq 0 \text{ because } \hat{p}_x \text{ and } \hat{x} \text{ do not commute.}$$

$$\sigma_p \sigma_x \geq \frac{\hbar}{2}$$

$\uparrow \uparrow$  standard deviations

$$\sigma_x^2 = \langle x^2 \rangle - \langle x \rangle^2 \quad \sigma_p^2 = \langle p^2 \rangle - \langle p \rangle^2$$

$$\sigma_x = \sqrt{\frac{1}{N} \sum_{i=1}^N (x_i - \langle x \rangle)^2} = \sqrt{\langle x^2 \rangle - \langle x \rangle^2}$$

particle-in-box example

$$\langle x \rangle = \frac{a}{2}$$

$$\langle x^2 \rangle = a^2 \left( \frac{1}{3} - \frac{1}{2\pi^2 n^2} \right)$$

$$\langle p \rangle = 0$$

$$\langle p^2 \rangle = \frac{n^2 \pi^2 \hbar^2}{a^2}$$

$$\sigma_p = \frac{n\pi\hbar}{a}$$

$$\sigma_x = a \sqrt{\left( \frac{1}{12} - \frac{1}{2\pi^2 n^2} \right)}$$

$$\sigma_p \sigma_x = 0.57 \hbar > \frac{\hbar}{2} \text{ for } n=1$$

Supplemental material:

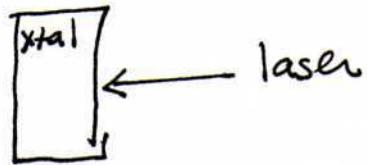


$$\psi = a\psi_{\text{left}} + b\psi_{\text{right}}$$

Implications of looking into left-hand box

(26)

$\psi = a\psi_{\text{left}} + b\psi_{\text{right}}$  : superposition of a single particle



- photons exit xtal with  $1/2$  original freq
- one photon in  $\rightarrow$  2 photons out

polarization can be horizontal (H) or vertical (V)

whatever polarization is measured for one photon,  
the opposite is found for the other

$$\text{entanglement } \psi = \frac{1}{\sqrt{2}} (\psi_1(H)\psi_2(V) + \psi_1(V)\psi_2(H))$$

Action at a distance.

implications for teleportation and quantum computing