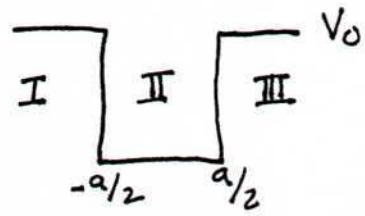


(18)

Particle in box - real world

$$V(x) = 0, -a/2 < x < a/2 \\ = V_0, \text{ for } x \geq \frac{a}{2}, x \leq -\frac{a}{2}$$



inside box:  $\frac{d^2\psi}{dx^2} = -\frac{2mE}{\hbar^2} \psi$

outside box:  $\frac{d^2\psi}{dx^2} = \frac{2m(V_0-E)}{\hbar^2} \psi$

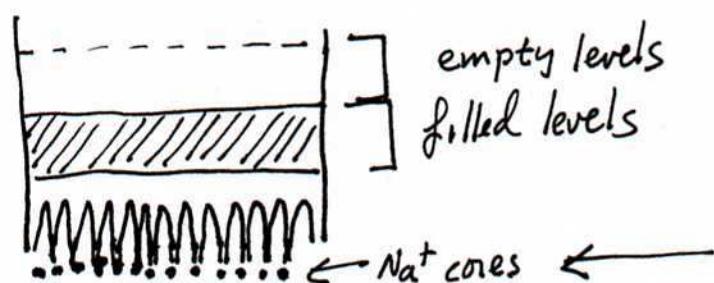
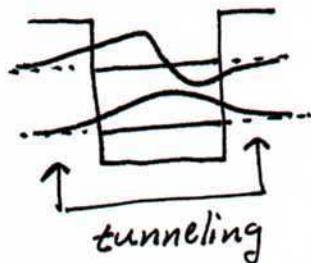
$$\begin{aligned} \text{I } \psi(x) &= A'e^{kx} + B'e^{-kx} \\ \text{III } \psi(x) &= A'e^{-kx} + B'e^{kx} \end{aligned}$$

$$k = \sqrt{\frac{2m(V_0-E)}{\hbar^2}}$$

$$\begin{aligned} \text{I } \psi &= B'e^{kx} \\ \text{III } \psi &= A'e^{-kx} \end{aligned} \quad ] \text{ exponentially decaying}$$

$$\text{II } \psi = C \sin kx + D \cosh kx, \quad k = \sqrt{\frac{2mE}{\hbar^2}}$$

require that  $\psi$  and  $\frac{d\psi}{dx}$  for I and II and for II and III  
match at the boundaries



can model with a series  
of particle-in-box potentials

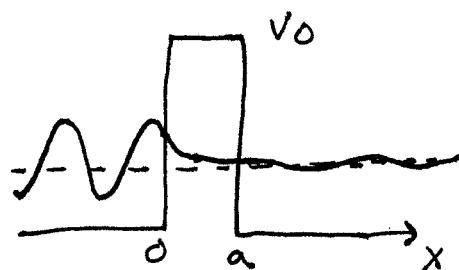
zero energy gap  $\Rightarrow$  metal  
large energy gap  $\Rightarrow$  insulator

small gap  $\rightarrow$  semiconductor

Apply electric potential  $\rightarrow$  charge will flow. (for metal)

Tunneling through barrier.

tunneling probability related  
to  $e^{-2a\sqrt{2m(V_0-E)/\hbar^2}}$



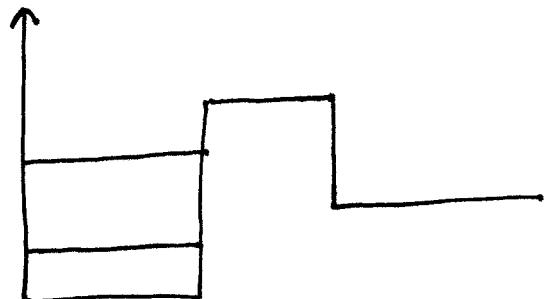
most important when: particle light  
energy near top of barrier  
 $a$  is small

Note if  $E > V_0$ , particle can be reflected by barrier.

resonances:

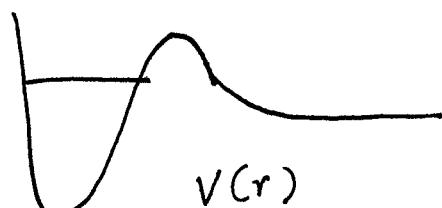
particle can  
escape by  
tunneling.

resonance →  
bound →



examples: radioactive decay.

$\text{Be}^-$ ,  $\text{N}_2^-$ , benzene  
electron falls off  
in  $10^{-14}$  sec.



How can one measure something with such a short lifetime?