Chem 1410 HW #2 answers

1. (a) $\langle x^2 \rangle = \frac{2}{a}\int_0^a x^2 \left(\sin\left(\frac{\pi x}{a}\right)\right)^2 dx = a^2\left[\frac{1}{3} - \frac{1}{2\pi^2}\right]$

$\langle p_x^2 \rangle = \frac{2}{a}\hbar^2\left(\frac{\pi}{a}\right)^2 \int_0^a \left(\sin\left(\frac{\pi x}{a}\right)\right)^2 dx = \frac{\pi^2\hbar^2}{a^2}$

(b) $x(x-a) = b_1\psi_1 + b_2\psi_2 + b_3\psi_3 + \cdots$

$b_2 = 0$ by symmetry

$b_1 = \sqrt{\frac{2}{a}}\int_0^a x(x-a)\sin\left(\frac{\pi x}{a}\right)dx = -\sqrt{\frac{2}{a}}\left(\frac{4a^3}{\pi^3}\right)$

$b_2 = \sqrt{\frac{2}{a}}\int_0^a x(x-a)\sin\left(\frac{3\pi x}{a}\right)dx = -\frac{1}{27}\sqrt{\frac{2}{a}}\left(\frac{4a^3}{\pi^3}\right)$

(c) what is the energy associated with the $x(x-a)$ wave function.

$$\langle E \rangle = \frac{\int_0^a x(x-a)\left(-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\right)x(x-a)\,dx}{\int_0^a x^2(x-a)^2 dx}$$

$$= \frac{\hbar^2 a^3/6m}{a^5/30} = \frac{5\hbar^2}{a^2 m}$$

2. In regions I and V, $\psi = 0$

region II, $\psi = A\sin(kx)$, $k = \sqrt{\frac{2mE}{\hbar^2}}$

region III, $\psi = Be^{-Kx} + Ce^{Kx}$, $K = \sqrt{\frac{2m}{\hbar^2}(V_0 - E)}$

region IV, $\psi = D\sin(kx) + E\cos(kx)$

note that this can also be expressed as $\psi = D\sin(kx + \delta)$ where $\delta = n\pi - k(a+b)$.