Chem 1410 HW #1 Answers

1. $\psi = c_1 \sin(ax) + c_2 \cos(bx)$

$\frac{d\psi}{dx} = c_1 a \cos(ax) - c_2 b \sin(bx)$

$\psi$ is not an eigenfunction of $\frac{d}{dx}$ regardless of the choice of $a, b, c_1, c_2$.

$\frac{d^2\psi}{dx^2} = -c_1 a^2 \sin(ax) - c_2 b^2 \cos(bx)$

If $a = b$, $\psi$ is an eigenfunction of $d^2/dx^2$ regardless of the choice of $c_1$ and $c_2$

2. $\int_0^{\pi/2} \sin(x)\cos(x)\,dx = \frac{\sin^2(x)}{2}\Big|_0^{\pi/2} = 1/2$

The area under the curve is positive for $0 \le x \le \pi/2$

3. $\psi = 1 - x^2$

$\int_{-1}^{1} (1-x^2)^2 dx = \int_{-1}^{1} (1 - 2x^2 + x^4)\,dx =$

$= x - \frac{2x^3}{3} + \frac{x^5}{5}\Big|_{-1}^{1} = 2 - \frac{4}{3} + \frac{2}{5} = \frac{16}{15}$

So the normalized wavefunction is $\psi = \sqrt{\frac{15}{16}}(1-x^2)$

4. Is $A\sin(kx - wt) + 2A\sin(kx + wt)$ a standing wave

Use $\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$
$\sin(x-y) = \sin(x)\cos(y) + \cos(x)\sin(y)$.

Thus we rewrite the starting function as

$A[\sin(kx)\cos(wt) - \cos(kx)\sin(wt)]$
$+ 2A[\sin(kx)\cos(wt) + \cos(kx)\sin(wt)]$

$= 3A\sin(kx)\cos(wt) + A\cos(kx)\sin(wt)$

This is not a standing wave.