

Chem 1410 Exam #1 Answers

$$1 \quad \frac{a}{3} \leq x \leq \frac{2a}{3}, \quad \frac{b}{4} \leq y \leq \frac{3b}{4}$$

$$\begin{aligned} & 2/a \int_{a/3}^{2a/3} \left(\sin \frac{\pi x}{a} \right)^2 dx \int_{b/4}^{3b/4} \left(\sin \frac{\pi y}{b} \right)^2 dy \\ &= \left(\frac{2}{a} \right) \left(\frac{2}{b} \right) \left[\frac{x}{2} - \frac{a}{4\pi} \sin \left(\frac{2\pi x}{a} \right) \right] \Big|_{a/3}^{2a/3} \left[\frac{y}{2} - \frac{b}{4\pi} \sin \left(\frac{2\pi y}{b} \right) \right] \Big|_{b/4}^{3b/4} \\ &= \left(\frac{1}{3} + \frac{\sqrt{3}}{2\pi} \right) \left(\frac{1}{2} + \frac{1}{\pi} \right) \end{aligned}$$

$$V = \frac{1}{2} k (x^2 + y^2)$$

There are two quantum #'s, n_x and n_y , $= 0, 1, 2, \dots$

$$E = \hbar \omega (1 + n_x + n_y)$$

$$E_{00} = \hbar \omega$$

$$E_{10} = E_{01} = 2\hbar \omega$$

$$E_{20} = E_{11} = E_{02} = 3\hbar \omega$$

$$\begin{aligned} \psi_{00} &= e^{-dx^2/2} e^{-dy^2/2} \quad d = \sqrt{\frac{k \hbar \omega}{T^2}} \\ \psi_{10} &= x e^{-dx^2/2} e^{-dy^2/2} \quad \psi_{01} = y e^{-dx^2/2} e^{-dy^2/2} \\ \psi_{11} &= xy e^{-dx^2/2} e^{-dy^2/2} \\ \psi_{20} &= (2dx-1) e^{-dx^2/2} e^{-dy^2/2} \\ \psi_{02} &= (2dy-1) e^{-dx^2/2} e^{-dy^2/2} \end{aligned}$$

In polar coordinates, the ground state wave function is $e^{-dr^2/2}$. The wave functions for the first two excited states are $r e^{-dr^2/2} e^{i\phi}$, $r e^{-dr^2/2} e^{-i\phi}$.

$$3. \quad \boxed{[\frac{d}{dx}, x \frac{d}{dx}] = \frac{d}{dx}}$$

$$4. \quad \Psi_I = A \sin(kx), \quad \Psi_{II} = B e^{-kx}$$

$$k = \sqrt{2E}, \quad K = \sqrt{2(m_0 - E)}, \quad \text{using atomic units}$$

$$\left\{ \begin{array}{l} A \sin(ka) = B e^{-ka} \\ A k \cos(ka) = -B K e^{-ka} \end{array} \right. \quad \left| \begin{array}{l} \text{: match } \Psi_I(a) \text{ and } \Psi_{II}(a) \\ \text{: match } \Psi'_I(a) \text{ and } \Psi'_{II}(a) \\ \text{can solve for } E \text{ numerically or graphically} \end{array} \right.$$

Also $B = e^{ka} \sin(ka)$

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1. $\frac{a}{3} \leq x \leq \frac{2a}{3}, \frac{b}{4} \leq y \leq \frac{3b}{4}$

$$\begin{aligned} & 2/a \int_{a/3}^{2a/3} \left(\sin \frac{\pi x}{a} \right)^2 dx \int_{b/4}^{3b/4} \left(\sin \frac{\pi y}{b} \right)^2 dy \\ &= \left(\frac{2}{a} \right) \left(\frac{2}{b} \right) \left[\frac{x}{2} - \frac{a}{4\pi} \sin \left(\frac{2\pi x}{a} \right) \right] \Big|_{a/3}^{2a/3} \left[\frac{y}{2} - \frac{b}{4\pi} \sin \left(\frac{2\pi y}{b} \right) \right] \Big|_{b/4}^{3b/4} \\ &= \left(\frac{1}{3} + \frac{\sqrt{3}}{2\pi} \right) \left(\frac{1}{2} + \frac{1}{\pi} \right) \end{aligned}$$

2. $V = \frac{1}{2} k(x^2 + y^2)$

There are two quantum #'s, n_x and n_y , $= 0, 1, 2, \dots$

$$E = \hbar\omega(1 + n_x + n_y)$$

$$E_{00} = \hbar\omega$$

$$E_{10} = E_{01} = 2\hbar\omega$$

$$E_{20} = E_{11} = E_{02} = 3\hbar\omega$$

$$\begin{aligned} \psi_{00} &= e^{-\alpha x^2/2} e^{-\alpha y^2/2} & \alpha = \sqrt{\frac{k\mu}{\hbar^2}} \\ \psi_{10} &= x e^{-\alpha x^2/2} e^{-\alpha y^2/2}, \quad \psi_{01} = y e^{-\alpha x^2/2} e^{-\alpha y^2/2} \\ \psi_{11} &= xy e^{-\alpha x^2/2} e^{-\alpha y^2/2} \\ \psi_{20} &= (2x^2 - 1) e^{-\alpha x^2/2} e^{-\alpha y^2/2} \\ \psi_{02} &= (2y^2 - 1) e^{-\alpha x^2/2} e^{-\alpha y^2/2} \end{aligned}$$

In polar coordinates, the ground state wave function is $e^{-\alpha r^2/2}$. The wave functions for the first two excited states are $r e^{-\alpha r^2/2} e^{i\phi}$, $r e^{-\alpha r^2/2} e^{-i\phi}$.

3. $\boxed{[\frac{d}{dx}, x \frac{d}{dx}] = \frac{d}{dx}}$

4. $\Psi_I = A \sin(kx), \Psi_{II} = B e^{-Kx}$

$$k = \sqrt{2E}, K = \sqrt{2(V_0 - E)}, \text{ using atomic units}$$

$$\left\{ \begin{array}{l} A \sin(ka) = B e^{-Ka} \\ Ak \cos(ka) = -B K e^{-Ka} \end{array} \right. \quad \left| \begin{array}{l} \text{match } \Psi_I(a) \text{ and } \Psi_{II}(a) \\ \text{match } \Psi'_I(a) \text{ and } \Psi'_{II}(a) \\ \text{can solve for } E \text{ numerically or graphically} \end{array} \right.$$

Also $B = e^{Ka} \sin(ka)$

$$5. E = \frac{\hbar^2 \pi^2}{2m a^2} + \frac{2b}{a} \int_0^a x \sin^2\left(\frac{\pi x}{a}\right) dx$$
$$= \frac{\hbar^2 \pi^2}{2ma^2} + \frac{ab}{2}$$