Chem. 1410 Exam #1 Answers

1 $a/3 \le x \le \frac{2a}{3}, \quad \frac{b}{4} \le y \le \frac{3b}{4}$

$$2/a \int_{a/3}^{2a/3} \left(\sin\frac{\pi x}{a}\right)^2 dx \; \frac{2}{b}\int_{b/4}^{3b/4} \left(\sin\frac{\pi y}{b}\right)^2 dy$$

$$= \left(\frac{2}{a}\right)\left(\frac{2}{b}\right)\left[\frac{x}{2} - \frac{a}{4\pi}\sin\left(\frac{2\pi x}{a}\right)\right]\Big|_{a/3}^{2a/3} \left[\frac{y}{2} - \frac{b}{4\pi}\sin\left(\frac{2\pi y}{b}\right)\right]\Big|_{b/4}^{3b/4}$$

$$= \left(\frac{1}{3} + \frac{\sqrt{3}}{2\pi}\right)\left(\frac{1}{2} + \frac{1}{\pi}\right)$$

$V = \frac{1}{2} k (x^2 + y^2)$

There are two quantum #'s, n_x and n_y, $= 0, 1, 2, \ldots$

$E = \hbar\omega(1 + n_x + n_y)$

$E_{00} = \hbar\omega$

$E_{10} = E_{01} = 2\hbar\omega$

$E_{20} = E_{11} = E_{02} = 3\hbar\omega$.

$\psi_{00} = e^{-\alpha x^2/2} e^{-\alpha y^2/2}$, $\alpha = \sqrt{\frac{k\mu}{\hbar^2}}$

$\psi_{10} = x e^{-\alpha/2(x^2+y^2)}$, $\psi_{01} = y e^{-\alpha/2(x^2+y^2)}$

$\psi_{11} = xy e^{-\alpha/2(x^2+y^2)}$

$\psi_{20} = (2\alpha x^2 - 1) e^{-\frac{\alpha}{2}(x^2+y^2)}$

$\psi_{02} = (2\alpha y^2 - 1) e^{-\frac{\alpha}{2}(x^2+y^2)}$

In polar coodinates, the ground state wave function is $e^{-\alpha/2 r^2}$. The wave functions for the first two excited states are $r e^{-\alpha r^2/2} e^{i\phi}$, $r e^{-\alpha r^2/2} e^{-i\phi}$.

3. $\left[\frac{d}{dx}, x\frac{d}{dx}\right] = \frac{d}{dx}$

4. $\psi_I = A \sin(kx)$, $\psi_{II} = B e^{-Kx}$

$k = \sqrt{2E}$, $K = \sqrt{2(V_0 - E)}$, using atomic units

$A \sin(ka) = B e^{-Ka}$: match $\psi_I(a)$ and $\psi_{II}(a)$

$A k \cos(ka) = -BK e^{-Ka}$: match $\psi_I'(a)$ and $\psi_{II}'(a)$

$\rightarrow k \cot(ka) = -K$ | can solve for E numerically or graphically

Also $B = e^{Ka} \sin(ka)$

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1. $a/3 \le x \le \frac{2a}{3}$, $\frac{b}{4} \le y \le \frac{3b}{4}$

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$$= \left(\frac{2}{a}\right)\left(\frac{2}{b}\right)\left[\frac{x}{2} - \frac{a}{4\pi}\sin\left(\frac{2\pi x}{a}\right)\right]\Bigg|_{a/3}^{2a/3} \left[\frac{y}{2} - \frac{b}{4\pi}\sin\left(\frac{2\pi y}{b}\right)\right]\Bigg|_{b/4}^{3b/4}$$

$$= \left(\frac{1}{3} + \frac{\sqrt{3}}{2\pi}\right)\left(\frac{1}{2} + \frac{1}{\pi}\right)$$

2. $V = \frac{1}{2}k(x^2 + y^2)$

There are two quantum #'s, n_x and n_y, $= 0, 1, 2, \ldots$

$E = \hbar\omega(1 + n_x + n_y)$

$E_{00} = \hbar\omega$ $\qquad \psi_{00} = e^{-\alpha x^2/2} e^{-\alpha y^2/2}$, $\alpha = \sqrt{\frac{k\mu}{\hbar^2}}$

$E_{10} = E_{01} = 2\hbar\omega$ $\qquad \psi_{10} = x e^{-\alpha/2(x^2+y^2)}$, $\psi_{01} = y e^{-\alpha/2(x^2+y^2)}$

$E_{20} = E_{11} = E_{02} = 3\hbar\omega$. $\qquad \psi_{11} = xy e^{-\alpha/2(x^2+y^2)}$

$\psi_{20} = (2\alpha x^2 - 1) e^{-\frac{\alpha}{2}(x^2+y^2)}$

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In polar coodinates, the ground state wave function is $e^{-\alpha/2 r^2}$. The wave functions for the first two excited states are $r e^{-\alpha r^2/2} e^{i\phi}$, $r e^{-\alpha r^2/2} e^{-i\phi}$.

3. $\left[\frac{d}{dx}, x\frac{d}{dx}\right] = \frac{d}{dx}$

4. $\psi_I = A\sin(kx)$, $\psi_{II} = Be^{-Kx}$

$k = \sqrt{2E}$, $K = \sqrt{2(V_0 - E)}$, using atomic units

$A\sin(ka) = Be^{-Ka}$: match $\psi_I(a)$ and $\psi_{II}(a)$

$Ak\cos(ka) = -BKe^{-Ka}$: match $\psi'_I(a)$ and $\psi'_{II}(a)$

$\rightarrow k\cot(ka) = -K$ | can solve for E numerically or graphically

Also $B = e^{Ka}\sin(ka)$

5. $E = \frac{\hbar^2\pi^2}{2ma^2} + \frac{2b}{a}\int_0^a x \sin^2\left(\frac{\pi x}{a}\right)dx$

$= \frac{\hbar^2\pi^2}{2ma^2} + \frac{ab}{2}$