Investment and Monetary Policy: Learning and Determinacy of Equilibrium

John Duffy
University of Pittsburgh
230 S. Bouquet Street
Pittsburgh, PA 15260 USA
E–mail: jduffy@pitt.edu

Wei Xiao
SUNY at Binghamton
P.O. Box 6000
Binghamton, NY 13902 USA
E–mail: wxiao@binghamton.edu

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Abstract

We examine determinancy and expectational stability (learnability) of rational expectations equilibrium (REE) in sticky price "New Keynesian" (NK) models of the monetary transmission mechanism. We consider three different New Keynesian models: a labor-only model and two models that add capital – one where capital is allocated in an economy-wide rental market and another where demand for capital is firm-specific. We find that Bullard and Mitra's (2002, 2007) findings on determinacy and learnability of REE under various interest rate rules in the labor-only NK model do not always extend to models with capital. In particular, the Taylor principle, that the response of interest rates should be more than proportionate to changes in inflation, will not generally suffice to guarantee determinate and/or learnable equilibria in NK models with capital.

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1 Introduction

Taylor's principle, that stabilizing rule-based monetary policy requires a more-than-proportional rise in the central bank's target interest rate in response to higher inflation, has attracted considerable attention in the monetary policy literature. Taylor himself (1999) has shown how violations of this principle appear to explain historical episodes of high inflation and low capacity utilization. Woodford (2001, 2003a) has shown that Taylor's principle ensures determinacy of rational expectations equilibria (REE) in New Keynesian models, i.e., dynamic, stochastic general equilibrium models with imperfect competition and Calvo-style staggered price setting. Bullard and Mitra (2002, 2007) show that Taylor's principle further implies that the REE of New Keynesian models are learnable by agents who do not initially possess rational expectations.

A REE is *determinate* if the solution path is locally unique, thereby allowing application of standard comparative static exercises and preventing non-fundamental "sunspot" variables from playing any role. Determinacy of equilibrium is desirable in that it enables policymakers to correctly anticipate the impact of policy changes (e.g., to the interest rate target) on inflation and output. A REE is *learnable* (or "expectationally stable" in the sense of Evans and Honkapohja (2001)) if agents who do not initially possess rational expectations are able to learn that REE using an adaptive real-time updating process such as recursive least squares. The conditions under which a REE is determinate will generally differ from the conditions under which that same REE is learnable. Satisfaction of both conditions is a desideratum of good monetary policy.

Most of the existing literature assessing the determinacy and learnability of rational expectations equilibrium under Taylor-type monetary policy rules has done so in versions of the New Keynesian model (Woodford, 2003, Gali 2008) *where there is no capital or investment.*[1] It is important to add capital to forward-looking, New Keynesian models as such models are more general than the more widely studied "labor–only" version. In the latter framework, both inflation and output are non-predetermined "jump" variables (in the language of Blanchard and Kahn (1980)); in the absence of serially correlated shocks, the forward–looking system would not display any dynamic persistence at all! The addition of capital–a pre-determined, non-jump variable–to the New Keynesian model thus provides for endogenous dynamics and greater persistence. It also allows for an analysis of investment decisions, an important and volatile component of aggregate demand. As Woodford (2003a, p. 352) notes, "one may doubt the accuracy of the conclusions obtained [using

[1] See Bullard (2006) or Evans and Honkapohja (2008) for surveys of the literature on determinacy and learnability of REE in New Keynesian models.

the simple labor–only model], given the obvious importance of variations in investment spending both in business fluctuations generally and in the transmission mechanism for monetary policy in particular." Indeed, in models with capital, central banks' influence on real interest rates may be greatly reduced as the arbitrage-induced equivalence between the real interest rate and the marginal product of capital places an additional constraint on interest rate movements that is not present in the labor-only model.

Two approaches have been taken to adding capital to New Keynesian models. The first, and perhaps most straightforward approach, involves adding an economy-wide rental market for the capital stock. The capital good is demanded by firms for use in combination with labor to produce output and is free to flow to any firm in the economy in response to firms' demands for the capital good. A second approach, advocated by Woodford (2003a, Chp. 5; 2005) and developed further by Sveen and Weinke (2005, 2007), imagines that capital is firm-specific; once the capital good has been purchased for use by a specific firm, that capital cannot be reallocated for use by other firms unlike in the economy-wide rental market model. Woodford's purpose in proposing this firm-specific model of capital was to make price adjustment by those firms who are free to adjust prices (under the standard Calvo pricing assumption) less rapid – more sticky – by comparison with the rental market model of capital. Indeed, an advantage of the firm-specific model of capital is that it does not require an unrealistically high degree of price stickiness to match empirical facts, as is the case in the rental market formulation.

Initial analyses of the impact of Taylor-type monetary policy rules in New Keynesian models with either a rental market for capital or with firm-specific capital have yielded pessimistic findings with regard to the usefulness of Taylor's principle for insuring a determinate equilibrium outcome. For instance, Carlstrom and Fuerst (2005) show that in a New Keynesian model with a rental market for capital, Taylor's principle does not suffice to ensure equilibrium determinacy under a "forward-expectations" Taylor rule, (except under extreme values for the Calvo price adjustment parameter). Similarly, Sveen and Weinke (2005) show that in a New Keynesian model with firm-specific capital, the Taylor principle does not suffice to ensure equilibrium determinacy even under a "current data" version of Taylor's rule.

By contrast, in this paper we make the case for greater optimism with regard to the use of Taylor-type monetary policy rules and the Taylor principle as a rough, though imperfect guide for ensuring equilibrium determinacy in New Keynesian models with capital. We also address the issue of the learnability (E-stability) of equilibrium in New Keynesian models with capital–a largely neglected topic. To address these issues most clearly, we examine a wide variety of Taylor-type monetary policy rules, (e.g., current data, forward-expectations, inertial, etc.), –all those which have appeared in the recent literature on determinacy and learnability of equilibrium–together with three different New Keynesian models - the labor–only model, the model with a rental market for capital and the model with firm–specific capital. Importantly, we compare all three models and policy rules under the *same* calibration of structural parameters which serves to clearly highlight differences among the policy rules and models. We show that for the firm-specific model of capital, the Taylor principle does not suffice for determinacy and learnability of REE under any of the policy rules we consider. However, we also show that under our calibration, there exist empirically plausible calibrations of the policy rule weights (for instance, Taylor's original (1993) calibration) for which all of the Taylor-type monetary policy rules induce both determinacy and learnability of equilibrium in the New Keynesian model with firm-specific capital. Similarly, in the New Keynesian model with an economy-wide rental market for capital, we show that the Taylor principle suffices for determinacy and learnability of REE under just one of the five policy rules we consider. Nevertheless, as in the model with firm-specific capital, we show that there exist empirically plausible calibrations of the policy rule weights (again, Taylor's original calibration will do) for which all monetary policy rules induce both determinacy and learnability of equilibrium in the New Keynesian model with an economy-wide rental market for capital.

Further, as our findings make clear, the differences in the determinacy/learnability conditions between the labor-only New Keynesian model and the two versions of that model that add capital are not as great as the impression given by Carlstrom and Fuerst (2005) or Sveen and Weinke (2005) (who mainly focused on Taylor-type rules that targeted inflation only). As we show, rather small (and not unreasonable) changes, e.g., in capital adjustment costs or in other model parameters can make the differences in determinacy/learnability conditions between the labor-only model and the two models with capital negligible. Indeed, many of the policy implications that emerge from our analysis of determinacy and learnability in the two models with capital reinforce some of the more general conclusions of Bullard and Mitra (2002, 2007) regarding the type of policy rules that are most likely to insure determinacy and learnability of REE in the labor-only model, specifically, the important role of policy-smoothing and of giving appropriate weight to the output gap.

2 Related Literature

As noted in the introduction, the determinacy and/or learnability of REE in New Keynesian models with capital has been explored in several recent papers. Dupor (2001) finds that the Taylor principle can induce an indeterminate REE in a continuous-time sticky price model with money and capital. Carlstrom and Fuerst (2005) show that Dupor's finding is sensitive to the continuous time framework he uses; in a discrete-time variant of Dupor's model, Carlstrom and Fuerst (2005) show that the Taylor principle can suffice to induce a determinate REE, in contrast to Dupor's finding, provided that the interest rate rule depends on current inflation. On the other hand, as noted earlier, if the interest rate rule depends on expected future inflation (forward-looking policy), Carlstrom and Fuerst show that, consistent with Dupor's finding, the Taylor principle does not suffice to implement a determinate REE and will almost always implement an indeterminate equilibrium. Carlstrom and Fuerst consider interest rate rules that respond only to inflation and not to output. Kurozumi and Van Zandweghe (2008) show that Carlstrom and Fuerst's findings can be sensitive to the specification of the interest rate rule, in particular whether some weight is also given to output. They show that, if the interest rate rule gives weight to expected future inflation as well as to *current* (and *not* expected future) output, then the Taylor principle will suffice to implement a determinate REE. Kurozumi and Van Zandweghe (2008) go further and show that the conditions for determinacy of REE generally coincide with the conditions for E-stability or learnability of REE for the interest rate rules they consider. Xiao (2005) adds a small, empirically plausible amount of increasing returns to scale to a New Keynesian model with capital. With this addition, he shows that the Taylor principle no longer suffices to guarantee either determinacy or learnability of REE.

The discrete-time models of Carlstrom and Fuerst, Kurozumi and Van Zandeweghe and Xiao all suppose there is *an economy-wide rental market for capital.* As noted in the introduction, Sveen and Weinke (2005) consider a discrete time version of the New Keynesian model without money but with firm-specific capital and convex capital adjustment costs. They show that in this setting, the Taylor principle does not suffice to ensure determinacy of REE under a interest rate policy rule that responds to *current* inflation only. This finding stands in sharp contrast to Carlstrom and Fuerst's findings for the New Keynesian model with a rental market for capital. Sveen and Weinke show that interest rate rules that respond to both current inflation and current output or which involve some policy smoothing are better able to induce determinate REE than is an interest rate rule that responds only to current inflation. They do not consider the learnability (E-stability) of REE in the firm-specific capital case, and one novelty of our paper is to explore this issue.

A general impression of this literature is that in New Keynesian models with capital, the Taylor principle does not suffice to insure determinacy of REE. However, much less is known about E-stability of these REE; with the exception of Xiao (2006) and Kurozumi and Van Zandweghe (2008), no authors have explored the learnability of REE in New Keynesian models with capital, and the case of firm-specific capital has not been previously considered. The firm-specific case is typically modeled together with the assumption of convex capital adjustment costs which are not present in the rental-market case; we therefore add such adjustment costs to the rental market model to facilitate a more careful comparison between that model and the firm-specific model of capital. More generally, comparisons of determinacy and learnability results between models with and without capital (labor-only) have not been made and there is not much consistency in the choice of interest rate rules and model calibrations used across the various studies. In this paper, we provide a thorough and consistent analysis of determinacy and learnability of REE in three versions of the New Keynesian model - the first generation "labor only" models and the second generation models with either a "rental market" for capital or firm specific capital. In addition, we consider the five main interest rate rules that have appeared in the literature: 1) a current data rule, 2) a lagged data rule, 3) a forward expectations rule, 4) a contemporaneous expectations rule and finally, 5) two versions of a policy smoothing rule. Many of our findings, e.g., the E-stability of REE in New Keynesian Models with firm-specific capital under the five policy rules we consider, *are new*. Some other findings, e.g., for the labor-only model, are previously known, but in the latter case the value added of our paper lies in considering a consistent calibration and set of policy rules across all model specifications (labor-only and the two models of capital).[2] Our approach provides the reader with the clearest available picture to date of the conditions under which Taylor-type interest rate rules work to implement determinate and learnable REE in the most commonly studied versions of the New Keynesian model of the monetary transmission mechanism (with or without capital).

3 A New Keynesian Model with Capital

3.1 The environment

We consider two different environments that differ in their treatment of capital. Our benchmark model is one involving an economy-wide rental market for capital. The alternative model has

[2] Our analysis thus extends and encompasses that of Bullard and Mitra (2002, 2007).

firm-specific capital. The labor-only model is shown to be a special case of these two models.

3.1.1 Rental market for capital

The economy is composed of a large number of infinitely-lived consumers. Each consumes a final consumption good C_t, and supplies labor N_t. Savings can be held in the form of real money balances $\frac{M_t}{P_t}$, bonds B_t, or capital K_t. Consumers seek to maximize expected, discounted life-time utility:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\sigma}}{1-\sigma} + \gamma \frac{(M_t/P_t)^{1-b}}{1-b} - v \frac{N_t^{1+\chi}}{1+\chi} \right],$$

where σ, γ, b, v, $\chi > 0$ and $0 < \beta < 1$. The budget constraint is given by

$$C_t + \frac{M_t}{P_t} + \frac{B_t}{P_t} + I_t = \frac{W_t}{P_t} N_t + \frac{M_{t-1}}{P_t} + \frac{R_t}{P_t} K_t + (1 + i_{t-1}) \frac{B_{t-1}}{P_t} + D_t, \tag{1}$$

and investment is defined as

$$I_t = I\left(\frac{K_{t+1}}{K_t}\right) K_t. \tag{2}$$

The consumer's sources of income are its real labor income $(W_t/P_t)N_t$, real money holdings M_{t-1}/P_t carried over from period $t-1$, real capital rental income $(R_t/P_t)K_t$, its real return on one-period bonds B_{t-1} purchased in period $t-1$ and earning a gross nominal return of $1+i_{t-1}$, and its dividends from ownership of firms, D_t. The consumers allocate this income among consumption C_t, new real money holdings M_t/P_t, new bond purchases B_t/P_t, and new investment I_t. To allow comparisons between this environment and the one with firm specific capital (described below), we follow Woodford (2003) and suppose that each firm faces *capital adjustment costs*. Denote these costs by $I(\frac{K_{t+1}}{K_t})$, where the function $I(\bullet)$ is assumed to satisfy the steady state conditions: $I(1) = \delta$, $I'(1) = 1$, and $I''(1) = \varepsilon_\psi$. Here, $0 < \delta < 1$ denotes the depreciation rate and $\varepsilon_\psi > 0$ characterizes the curvature of the adjustment cost function.[3]

The first order conditions for the consumer's problem can be written as:

$$vN_t^{\chi} = C_t^{-\sigma} \frac{W_t}{P_t}, \tag{3}$$

$$C_t^{-\sigma} = \gamma \left(\frac{M_t}{P_t}\right)^{-b} + \beta E_t C_{t+1}^{-\sigma} \frac{P_t}{P_{t+1}}, \tag{4}$$

$$1 = \beta E_t \left(\frac{C_{t+1}}{C_t}\right)^{-\sigma} \frac{P_t}{P_{t+1}} (1 + i_t), \tag{5}$$

$$\frac{dI_t}{dK_{t+1}} = \beta E_t \left(\frac{C_{t+1}}{C_t}\right)^{-\sigma} \left(\frac{R_{t+1}}{P_{t+1}} - \frac{dI_{t+1}}{dK_{t+1}}\right). \tag{6}$$

[3] The parameter ε_ψ has been interepreted as the elasticity of the investment/capital ratio with respect to Tobin's q, in the steady state.

There exists a continuum of monopolistically competitive firms producing differentiated intermediate goods. The latter are used as inputs by perfectly competitive firms producing the single final good.

The final good is produced by a representative, perfectly competitive firm with a constant returns to scale technology

$$Y_t = (\int_0^1 Y_{jt}^{\frac{\varepsilon-1}{\varepsilon}} dj)^{\frac{\varepsilon}{\varepsilon-1}}, \tag{7}$$

where Y_{jt} is the quantity of intermediate good j used as an input, and $\varepsilon > 1$ governs the price elasticity of individual goods. Profit maximization yields the demand schedule

$$Y_{jt} = (\frac{P_{jt}}{P_t})^{-\varepsilon} Y_t, \tag{8}$$

which, when substituted back into (7), yields

$$P_t = \left(\int_0^1 P_{jt}^{1-\varepsilon} dj \right)^{\frac{1}{1-\varepsilon}}. \tag{9}$$

The intermediate goods market features a large number of monopolistically competitive firms. The production function of a typical intermediate goods firm is:

$$Y_{jt} = K_{jt}^{\alpha} N_{jt}^{1-\alpha}, \tag{10}$$

where K_{jt} and N_{jt} represent the capital and labor services hired by firm j.

These firms' real marginal cost φ_{jt} is derived by minimizing costs:

$$\varphi_{jt} = \frac{1}{(1-\alpha)} \frac{W_t}{P_t} \frac{N_{jt}}{Y_{jt}} = \frac{1}{\alpha} \frac{R_t}{P_t} \frac{K_{jt}}{Y_{jt}}. \tag{11}$$

From this we can derive the expression

$$\frac{K_{jt}}{N_{jt}} = \frac{\alpha}{1-\alpha} \frac{W_t}{R_t}, \tag{12}$$

which implies that the capital-labor ratio is equalized across firms, as is marginal cost itself.

Intermediate firms set nominal prices in a staggered fashion, according to the stochastic time dependent rule proposed by Calvo (1983). Each firm resets its price with probability $1 - \omega$ each period, independent of the time elapsed since the last price adjustment and does not reset its price with probability ω. A firm resetting its price in period t seeks to maximize:

$$E_t \sum_{i=0}^{\infty} \omega^i \beta^i \left(\frac{C_{t+i}}{C_t} \right)^{-\sigma} \left(\frac{P_t^*}{P_{t+i}} Y_{jt+i} - \varphi_{jt+i} Y_{jt+i} \right), \tag{13}$$

where P_t^* represents the (common) optimal price chosen by all firms resetting their prices in period t. This maximization problem yields the first order condition

$$E_t \sum_{i=0}^{\infty} \omega^i \beta^i \left(\frac{C_{t+i}}{C_t}\right)^{-\sigma} Y_{jt+i} \left(\frac{P_t^*}{P_{t+i}} - \frac{\varepsilon}{\varepsilon - 1}\varphi_{t+i}\right) = 0. \tag{14}$$

The equation describing the dynamics for the aggregate price level is

$$P_t = [\omega P_t^{1-\varepsilon} + (1-\omega) P_t^{*1-\varepsilon}]^{\frac{1}{1-\varepsilon}}. \tag{15}$$

Finally, market clearing in the factor and goods markets implies that: $N_t = \int_0^1 N_{jt} dj$, $K_t = \int_0^1 K_{jt} dj$, $Y_t = \int_0^1 Y_{jt} dj$ and $C_t + I_t = Y_t$.

3.1.2 Firm-specific capital

Woodford (2003a, 2005) proposes a different version of the New Keynesian model in which an economy-wide rental market for capital does not exist. Instead, firms are assumed to accumulate capital for their own use only. This assumption implies that a firm's price-setting decision is no longer separate from its capital accumulation decision, (as it is in the rental market case), and this change leads to important changes in the dynamics of the New Keynesian model with capital. The main advantage of the firm-specific approach to capital accumulation is that it does not require an unrealistically high degree of price stickiness to match empirical facts relative to the New Keynesian model with economy-wide rental markets that was examined in the previous section.

With firm-specific capital, the model needs to be modified as follows. First, the consumer's budget constraint (1) is restated as:

$$C_t + \frac{M_t}{P_t} + \frac{B_t}{P_t} = \frac{W_t}{P_t} N_t + \frac{M_{t-1}}{P_t} + (1 + i_{t-1}) \frac{B_{t-1}}{P_t} + D_t. \tag{16}$$

That is, consumers no longer make investment decisions given the absence of any economy-wide capital market.

Second, the firm's problem is now defined as

$$\max \sum_{i=0}^{\infty} \beta^i \left(\frac{C_{t+i}}{C_t}\right)^{-\sigma} \left(\frac{P_{jt+i}}{P_{t+i}} Y_{jt+i} - \frac{W_{t+i}}{P_{t+i}} N_{jt+i} - I_{jt+i}\right) \tag{17}$$

subject to constraint (8), (10), where firm-specific investment is given by:

$$I_{jt} = I\left(\frac{K_{jt+1}}{K_{jt}}\right) K_{jt}, \tag{18}$$

Notice that investment demand (18) is in the same form as (2), (i.e., it involves the same convex adjustment function $I(\bullet)$) but here it is *firm-specific*. Note also that $P_{jt+i+1} = P_{jt+i}$ with probability ω.

Most first order conditions, such as (3), (4), and (5), continue to hold in the New Keynesian model with firm-specific capital. However, three differences between this setup and the rental-market setup will eventually lead to differences in the dynamics of the model.

First, the first order condition associated with capital is different in the firm-specific capital model than in the rental-market for capital model (*c.f.* 6). Maximizing (17) with respect to capital yields:

$$\frac{dI_{jt}}{dK_{jt+1}} = \beta E_t \left(\frac{C_{t+1}}{C_t}\right)^{-\sigma} \left(\frac{MS_{jt+1}}{P_{t+1}} - \frac{dI_{t+1}}{dK_{t+1}}\right), \qquad (19)$$

where MS_{jt+1} denotes the nominal reduction in firm i's labor cost associated with having an additional unit of capital in period $t+1$, and is derived from the firm's maximization problem as

$$MS_{jt} = W_t \frac{MPK_{jt}}{MPL_{jt}}, \qquad (20)$$

where MPK and MPL represent the marginal product of capital and of labor, respectively, of firm j.

Second, marginal cost is now derived from the firm's maximization problem as:

$$\varphi_{jt} = \frac{W_t/P_t}{MPL_{jt}}.$$

The critical feature here is that marginal costs are no longer equalized across firms. They depend on each firm's specific level of capital and labor.

Third, the first order condition associated with P_{jt+i} looks identical to (14), but after substituting in the expression for marginal cost, pricing decisions become a function of firm-specific capital. Since a firm's marginal cost is affected by its current and future capital levels, its pricing decisions must also depend on its current and future capital levels. Future capital levels, on the other hand, depend in turn on today's price and the future prices set by the firm. This complicated mechanism is absent in the rental-market case. Woodford (2005) shows that a linearized inflation equation can be computed by applying the method of undetermined coefficients.

3.1.3 Labor Only Model

For comparison purposes, we also study a version of the model in which labor is the only input in production. Setting $I = K = 0$ in our benchmark case will reduce the model to a generic, labor-only New Keynesian model. We assume production has constant returns to scale in labor:

$$Y_{jt} = N_{jt}.$$

The consumer's budget constraint is the same as (16), and the economy wide resource constraint is simply $Y_t = C_t$. The key first order conditions are (3), (4), (5) and (14).

3.2 Reduced linear systems

In the next three subsections we describe the system of linearized equations we use in our analysis of the determinacy and E-stability of REE in each of the three models that we consider. We use lower case letters to denote percentage deviations of a variable from its steady state value.

3.2.1 Benchmark Model: Rental market for capital

In the benchmark model with a rental market for capital, there are six non-dynamic equations and four dynamic equations. The first equation is the linearized version of the labor supply schedule (3):

$$\chi n_t + \sigma c_t = w_t - p_t. \tag{21}$$

The second and third equations are the linearized versions of (11). We are interested in the average level of marginal costs, which are given by

$$\begin{aligned} \varphi_t &= n_t + (w_t - p_t) - y_t, & (22) \\ &= k_t + (r_t - p_t) - y_t. & (23) \end{aligned}$$

The fourth equation is the linearized production function

$$y_t = \alpha k_t + (1 - \alpha) n_t. \tag{24}$$

The first dynamic equation is New Keynesian Phillips curve, which is derived by solving the firm's dynamic price-setting problem and combining it with (15). This equation is given by

$$\pi_t = \beta E_t \pi_{t+1} + \kappa \varphi_t, \tag{25}$$

where $\kappa = \frac{(1-\omega)(1-\beta\omega)}{\omega}$.

The second dynamic equation is the linearized version of (6), which describes the evolution of capital:

$$\Delta k_{+1} = \beta E_t \Delta k_{t+2} + \frac{1}{\varepsilon_\psi} \{[1 - \beta(1 - \delta)] E_t (r_{t+1} - p_{t+1}) - (i_t - E_t \pi_{t+1})\}. \tag{26}$$

The third dynamic equation is the Euler equation (5), which can be linearized as

$$c_t = E_t c_{t+1} - \frac{1}{\sigma}(i_t - E_t \pi_{t+1}). \tag{27}$$

The last dynamic equation is the market clearing condition

$$y_t = \frac{C}{Y} c_t + \frac{K}{Y}[k_{t+1} - (1-\delta)k_t], \tag{28}$$

where C, I and Y represent steady state levels of consumption, investment and output.

Finally, we add the interest rate rule and use the non-dynamic equations to substitute out seven variables $k_t^* = \Delta k_{t+1}$, $w_t - p_t$, $r_t - p_t$, x_t, i_t, φ_t, and y_t. The system becomes a four dimensional linear difference equation system consisting of $s_t = (c_t, n_t, k_t, \pi_t)'$:

$$E_t s_{t+1} = J s_t. \tag{29}$$

3.2.2 Firm-specific capital

With firm-specific capital, the New Keynesian Phillips curve becomes

$$\pi_t = \beta E_t \pi_{t+1} + \kappa^* \varphi_t, \tag{30}$$

which looks quite similar to (25), but the parameter κ^* is different from the parameter κ in (25). Woodford (2005) develops an algorithm that utilizes the method of undetermined coefficients to compute κ^*. Sveen and Weinke (2004) show that κ^* can be approximated by $\frac{1-\alpha}{1-\alpha+\alpha\varepsilon}\kappa$. In our analysis we make use of this approximation. Recall that $0 < \alpha < 1$ is capital's share of output and $\varepsilon > 1$ governs the price elasticity of individual goods. Thus using the approximation, we have that $\kappa > \kappa^*$, so that inflation is *less* responsive to changes in marginal costs in the firm specific model of capital as compared with the rental market model of capital. That is, as Sveen and Weinke (2005) point out, for any given value of the Calvo sticky price parameter ω, prices will be *stickier* in the firm-specific model of capital than they will be in the rental market for capital model.

The marginal return to capital can be derived from (20) as

$$ms_t = w_t - p_t + n_t - k_t,$$

and the aggregate capital accumulation equation is a linearized version of (19):

$$\Delta k_{+1} = \beta E_t \Delta k_{t+2} + \frac{1}{\varepsilon_\psi}\{[1 - \beta(1-\delta)]E_t ms_{t+1} - (i_t - E_t \pi_{t+1})\}.$$

As in the rental-market for capital case, the model with firm-specific capital can be reduced to a four-dimensional linear system of expectational difference equations with the same variables as in (29).

3.2.3 Labor-only model

The labor-only New Keynesian model can be reduced to the New Keynesian Phillips curve and the expectational IS curve:

$$\begin{aligned} \pi_t &= \beta E_t \pi_{t+1} + (\sigma + \chi)\kappa y_t, \\ y_t &= E_t y_{t+1} - \frac{1}{\sigma}(i_t - E_t \pi_{t+1}), \end{aligned}$$

plus the first order condition (4).

3.3 Monetary Authority

The central bank sets the nominal interest rate i_t every period according to a simple linear rule contingent on information about output and inflation. Following Bullard and Mitra (2002, 2007), we consider five variants of the interest rate rule. The first variant is the "contemporaneous data" rule:

$$i_t = \tau_\pi \pi_t + \tau_y y_t, \tag{31}$$

where $\tau_\pi \geq 0$ and $\tau_y \geq 0$, and i_t, π_t and y_t denote percentage deviations of the interest rate, the inflation rate, and output from their steady state values. This is a version of Taylor's original (1993) rule that conditions the interest rate on current output and inflation.[4] The "Taylor principle" is that $\tau_\pi > 1$, or that interest rate changes should be more than proportional to changes in inflation.

A second rule that is commonly considered (e.g., by Clarida et al. 1999), is the "forward expectations rule":

$$i_t = \tau_\pi E_t \pi_{t+1} + \tau_y E_t y_{t+1}, \tag{32}$$

where policy makers use expectations of *future* inflation and output using information available at time t to determine the current interest rate target.

Since current data for output and inflation may not be available at time t, some have suggested restricting attention to the use of time $t-1$ data on output and inflation in the determination of the interest rate target. This consideration gives rise to the next two rules we consider. The third rule is the "lagged data" rule, which may be seen as an alternative to the current data rule (31), is:

$$i_t = \tau_\pi \pi_{t-1} + \tau_y y_{t-1}. \tag{33}$$

[4] Taylor–type interest rate rules typically condition on inflation and output *gaps*, i.e., deviations of inflation from a target level and of output from potential output, rather than on the *levels* of these variables by themselves. As the determinacy/learnability conditions of the systems we consider depend only on the magnitudes of the coefficients impacting on inflation and output *levels*, we choose to work with interest rate rules such as (31) (as well as the four other types of rules that follow) which condition on these levels only; of course, all of our findings will continue to apply to rules that condition on inflation and output *gaps*.

Similarly, the fourth rule we consider, the "contemporaneous expectations" rule may be seen as an alternative to the forward expectations rule (32) and is given by:

$$i_t = \tau_\pi E_{t-1}\pi_t + \tau_y E_{t-1} y_t, \tag{34}$$

where policy depends on forecasts of output and inflation that are formed using data available through time $t-1$.

In addition to the above four rules, we also consider an interest rate smoothing rule, where the policy maker gives some weight ρ to past interest rates and remaining weight $1-\rho$ to the predictions of an interest rate rule such as rules 1-4 given above. Policy smoothing rules have been considered by Bullard and Mitra (2007) for the labor-only model; results for the two models with capital have not been previously examined.

4 Methodology and Calibration

4.1 General Methodology

We now turn to our analysis of the determinacy and E-stability of REE under the three models and five different interest rate rules. When we study E-stability properties, we focus only on REE that are determinate.[5] We use the benchmark model to explain our general methodology and leave the computational details for each different rule to the Appendix.

The determinacy of REE is assessed by computing the eigenvalues of the system (29). Since there is only one predetermined variable k_t and the system is of dimension four, the REE will be determinate in this case if the number of explosive roots is three and the number of stable roots is one (see Blanchard and Kahn (1980)). If the number of stable roots exceeds one, we have an indeterminate REE. If there is no stable root, the system is explosive.

To study adaptive learning, we re-write the system as

$$\begin{aligned} b_z z_t + b_k k_t &= d_k E_t k_{t+1} + d_z E_t z_{t+1}, & (35) \\ k_{t+1} &= e_z z_t + e_k k_t, & (36) \end{aligned}$$

where the second equation is derived from the capital accumulation equation, which does not involve any expectations and so does not need to be learned. We assume that agents use the perceived law

[5] For an analysis of the E-stability properties of indeterminate rational expectations equilibria, see, e.g., Honkapohja and Mitra (2004) and Evans and McGough (2005).

of motion (PLM)

$$\begin{aligned} z_t &= a_1 + \psi k_t, \\ k_t &= a_2 + m k_t, \end{aligned}$$

which is in the same form as the minimal state variable (MSV) RE solution. By contrast with RE, learning agents do not initially know the parameter vectors a_1, a_2, m and ψ and must learn these over time. Given the PLM, we calculate the forward expectations as

$$\begin{aligned} E_t k_{t+1} &= a_2 + m k_t, \\ E_t z_{t+1} &= a_1 + \psi E_t k_{t+1} = a_1 + \psi a_2 + \psi m k_t. \end{aligned}$$

Substituting these expressions into (35), we obtain a T-mapping from $(a_1, a_2, \psi, m)'$ to the actual law of motion of the model. Following Evans and Honkapohja (2001), we say the REE is E-stable (learnable by adaptive agents) if the differential equation, $\frac{d}{d\tau}(a_1, a_2, \psi, m) = T(a_1, a_2, \psi, m) - (a_1, a_2, \psi, m)$, evaluated at the REE solution, is stable. This condition requires that all eigenvalues of $D[T(a_1, a_2, \psi, m) - (a_1, a_2, \psi, m)]$ evaluated at the REE have real parts that are less than zero. Evans and Honkapohja (2001) provide conditions under which this differential equation approximates the limiting behavior of the recursive algorithms that characterize adaptive agent learning.

It is worth pointing out that assumptions about the agents' information set can be crucial in assessing E-stability results. In the baseline case outlined above, we implicitly assume that both the private sector and the central bank can observe current values of the variable k_t. They use this information to obtain forecasts $E_t z_{t+1}$ and $E_t k_{t+1}$, which in turn determine the current values of z_t. This assumption applies in models using the current data rule or the forward expectation rule. However, this assumption is sometimes criticized as being unrealistic, since current data are usually not available to economic agents.[6] An alternative assumption is to assume that the agents can observe current exogenous variables but only lagged values of the endogenous and state variables at time t. We apply this assumption in models using the lagged data rule or the contemporaneous expectations rule. Both the central bank and the private sector are assumed to have symmetric knowledge of the lagged data. With these assumptions, we derive the specific E-stability conditions for each interest rate rule, and present them in the Appendix.

[6] The case with the current data rule is especially controversial. As pointed out by Bullard and Mitra (2002), it implies that the central bank has "superior information" in that it reacts to current values of y_t and π_t while the private sector does not possess such information.

Parameter	Description	Base value	BM value
α	Capital's share of output	.36	n/a
β	Period discount factor	.99	.99
σ^{-1}	Intertemporal elasticity of substitution	.5	1/.157
χ^{-1}	Labor supply elasticity	1	1
ε	Elasticity of substitution between varieties of consumption goods	11	n/a
δ	Depreciation rate	.025	n/a
ε_ψ	Curvature of the adjustment cost function	3	n/a
$1-\omega$	Fraction of firms free to adjust prices each period	.25	.14†
κ	Parameter relating to the degree of price stickiness	.086†	.024

Table 1: Calibrations, Base=baseline and BM=Bullard-Mitra, used in our numerical analyses, quarterly frequency. † As implied by the relationship $\kappa = (1-\omega)(1-\beta\omega)/\omega$ – see Walsh (2003) for a derivation. Note that Sveen and Weinke approximate κ^* in (30) (firm-specific capital model) by $\frac{1-\alpha}{1-\alpha+\alpha\varepsilon}\kappa$. Using the baseline calibriation for these parameters, we have that $\kappa^* = .012$.

4.2 Calibration

Table 1 provides the two calibrations of model parameters that we use in all of our analyses. Our baseline calibration (also used by Sveen and Weinke (2005)), is used for all three models: labor-only, rental market for capital, and firm-specific capital. A second calibration, due to Woodford and used by Bullard and Mitra, is also used, but only for the labor-only model; the purpose of reporting results under this second calibration is to establish the consistency of our results with those reported by Bullard and Mitra for the labor-only case.

The reader may have noticed that we have excluded exogenous disturbance processes from all three versions of the New Keynesian model we have considered. This was in the interest of simplicity, as our determinacy and learnability findings do not depend in any way on the calibration of these shock processes. Similarly, as certain model parameters such as b, γ and v do not come into play in our analysis of determinacy and learnability, we do not provide calibrations of those parameters.

4.3 Determinacy and Learnability of REE under various interest rate rules

Ideally, we would like to provide analytic results concerning the determinacy and learnability of REE under various interest rate rules. Unfortunately, except in a few special cases, such as those studied by Bullard and Mitra (2002) and Carlstrom and Fuerst (2005), analytic results are not possible. The reason for this is simple: with the addition of capital, the dimension of the systems we are considering is either four or five and too complicated to reduce to a system that would allow for analytic findings. This situation necessitates that we adopt a numerical approach. Still, to the extent possible, we will try to provide some intuition for our numerical findings.[7]

Our approach is as follows. In all simulation exercises, we vary the weights τ_π and τ_y, in the various interest rate rules. The ranges allowed for these weights cover all empirically relevant cases; in particular we search over a fine grid of values for τ_π between 0 and 5 and for τ_y between 0 and 4. We use an increment stepsize of .02. For each possible pair of weights (τ_π, τ_y) in this grid, we check whether the eigenvalues satisfy the conditions for 1) determinacy and 2) E-stability. If both conditions are satisfied, we indicate this in the figures below using a blue color. If neither condition is satisfied, no color is used – the white regions in the figures below. We use a green color to mark weight pairs for which the REE is determinate but E-unstable. Finally, we use the color yellow to indicate weight pairs for which all roots are explosive (greater than one).

For the labor-only model, we conduct our analysis of determinacy and learnability using both the baseline and the BM calibrations for comparison purposes. Then, using only our baseline calibration, which includes parameter choices relevant to the inclusion of capital in the model, we conduct our analysis of determinacy and learnability for the two models of capital, the rental and the firm-specific models. Thus, the figures that follow typically involve four panels, two panels for the labor-only model under our baseline and the BM calibrations and two panels for the two models of capital under our baseline calibration only.

4.3.1 Current data rule

Determinacy and E-stability findings using the current data rule (31) are shown in the four panels of Figure 1. The two top panels show the labor-only model under the BM calibration and our baseline calibration, while the two bottom panels show the rental- and firm-specific models of capital under the baseline calibration only. Values of the monetary policy rule weight τ_π are indicated on the horizontal axis and values of the monetary policy rule weight τ_y are indicated on the vertical axis in these (and all subsequent) figures. Notice that under the current data rule, all four figures show only the color blue or no color (white). The reason for this finding is that in the case of the current data rule (31), if a REE is determinate, it is also E-stable in all three models. The coincidence of determinacy and learnability conditions in the labor-only and rental market for capital models under a current data rule is known from the work of Bullard and Mitra (2002) and Kurozumi and

[7] The Matlab code we used in our numerical analysis is available on request.

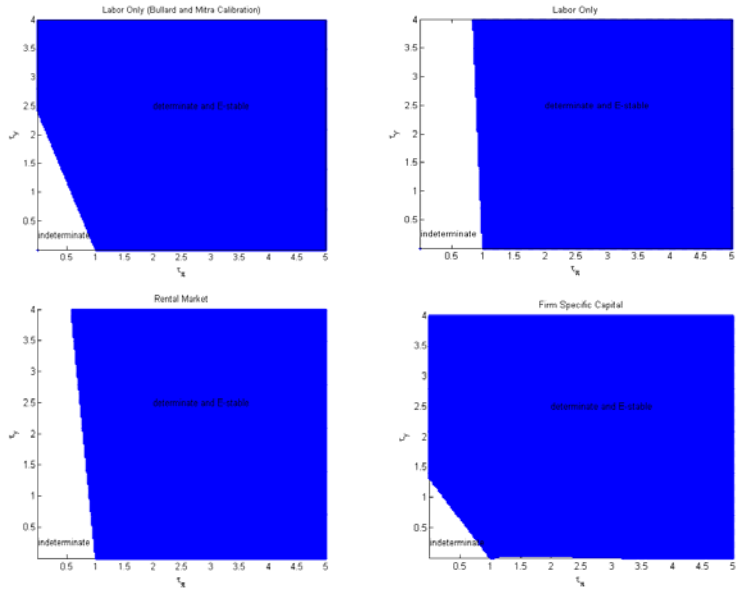

Figure 1: Determinacy and E-Stability Results Under the Current Data Rule

Van Zandweghe (2008), respectively.[8] The difference in the regions of *determinate* REE in the firm-specific and rental market models of capital under a current data rule (as discussed in further detail below) has been pointed out by Sveen and Weinke (2005). However, the finding for the firm-specific model of capital, that determinate REE are also learnable, is a new finding of this paper.

In addition to the important observation that determinacy implies learnability in all three models with a current data policy rule (31), we further observe that the Taylor principle, $\tau_\pi > 1$, suffices to insure both determinacy and learnability of REE in the labor-only and in the rental market for capital models, *but does not suffice in the firm-specific model of capital.* Notice that in the firm-specific capital model (bottom right panel of Figure 1) there is a very small "sliver" where $\tau_y \approx 0$ and $\tau_\pi > 1$, where the REE is both indeterminate and unlearnable. Figure 2 provides a blown-up view of this region. The presence of this region of indeterminacy is consistent with Sveen and Weinke's (2005) findings under a current data rule that gives zero weight to output.

[8] Bullard and Mitra (2002) show that under the current data rule in the labor only model, REE will be both determinate and learnable provided that (in our notation) $\tau_\pi + \frac{1-\beta}{(\sigma+\chi)\kappa}\tau_y > 1$, which they refer to as the "long-run" Taylor principle following Woodford (2003a, Chapter 4). Using our model calibration, it can be shown that this same inequality precisely characterizes the border between determinacy/E-stability and indeterminacy/E-instability in our Figure 1 for the labor-only model. Kurozumi and Van Zandweghe (2008) have similar analytic conditions for the rental-market model of capital, but their conditions are derived under an interest rate rule that gives weight to current output (or its components consumption, investment) and to *future expected inflation*, $E_t\pi_{t+1}$, – a rule that differs from the current data rule (31) that we consider here.

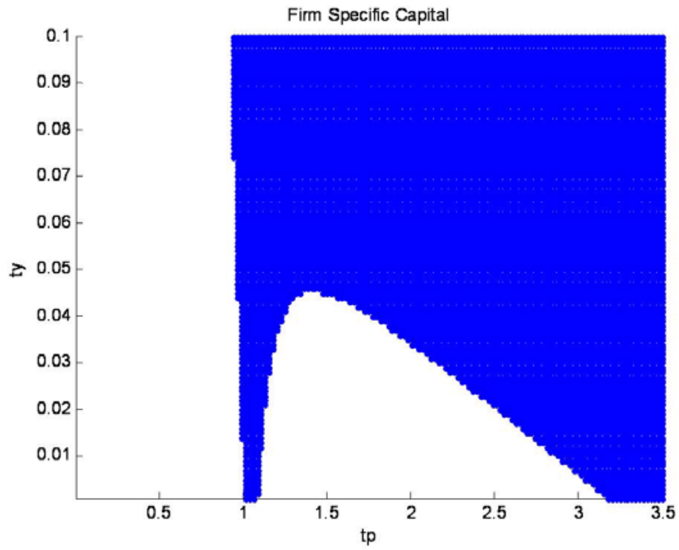

Figure 2: Blown-up View of the Indeterminate and E-unstable Region of the Firm-specific Model of Capital Under the Current Data Rule.

Indeed, they report that for various values of ω, measuring the stickiness of prices (which include our calibrated value $\omega = .75$), the Taylor principle may not suffice to guarantee determinacy of the REE unless sufficient weight is given either to real activity or to lagged interest rates (as in a policy smoothing rule).

The intuition for the difference between the rental market and firm specific capital cases must lie with the different specifications for the New Keynesian Phillips curve (NKPC) (25) and (30) since, as Sveen and Weinke (2005) have noted, this is the only difference between the two linearized versions of the models with capital. Recall that the difference between these two NKPC equations lies in the coefficient attached to marginal costs φ_t, i.e., κ in the rental market case and κ^*, in the firm-specific case, with $\kappa > \kappa^*$. Why is this the case? In the case of firm-specific capital, an increase in the demand for a firm's output will raise the firm's marginal costs, but as these are specific to the firm it will not affect the marginal costs of other firms. Thus in the firm-specific model, a firm that is experiencing increased demand (and is free to adjust prices) will take into account the impact of price changes on its future relative demand and adjust prices *less* than it would in the rental market model where changes in marginal costs are homogeneous across all firms. The resulting decline in the firm's future relative demand leads to a fall in its future relative marginal cost as well, which reinforces the incentive to avoid a large price increase today. Consequently, price setting is more forward–looking (and will appear to be much more sluggish) in the firm specific case relative to the economy wide rental market case, where capital is perfectly mobile across firms making the marginal costs firms face independent of the demand for their output.

To see why stickier price adjustment in the firm-specific model might lead to indeterminacy, under the current data rule, consider whether an exogenous, (sunspot-driven) investment boom could be self-fulfilling. The answer depends on how it affects current and future marginal costs and inflation and on how capital is modeled. Under a rental market for capital, the increase in investment demand will immediately drive up the marginal costs that all firms face and via the NKPC, will increase current inflation. An activist monetary policy focused on current inflation only ($\tau_\pi > 1$, $\tau_y = 0$) responds to the increase in current inflation by raising interest rates, thereby killing off the speculative investment boom. By contrast, under a firm-specific model of capital because investment is firm specific, price setting is more strategic (forward–looking) with the result that price adjustment (by those firms free to adjust prices) is more sluggish. An increase in investment will raise marginal costs, but the impact on inflation will be reduced relative to the rental market case for the reasons given above. Furthermore, in the firm-specific model, the increase in firm-specific investment will lead to lower, future firm-specific marginal costs, lower future inflation and hence lower future real interest rates, and with the more forward looking view of firms making firm specific investments, this can serve to make the investment boom self-fulfilling. Note that this indeterminacy possibility would be reduced, if not eliminated, if monetary policy also put some weight on current output, as the investment boom would increase y_t and lead to an even higher increase in current interest rates.

Alternatively put, *for the baseline calibration we use*, $\omega = .75$, prices will be sufficiently flexible in the rental market model of capital to avoid the indeterminacy outcome when the Taylor principle holds, but the same will not be true in the firm-specific model of capital.[9] As we shall see, this same "sliver of a region" of indeterminacy/E-instability in the firm-specific capital model can also arise under all four of the interest rate rules we consider that don't involve policy smoothing. Adding some policy inertia may also work to eliminate this region of indeterminacy as will be shown later in the paper.

Of course, a judicious (and empirically plausible) choice of policy rule weights will also ensure that the REE is determinate and learnable in all three models under a current data rule. For

[9] For calibrations other than the one we consider, e.g. higher, but empirically implausible values for ω, the small sliver of indeterminacy we observe for the firm-specific model of capital under the current data rule when $\tau_y \approx 0$ will also appear in the rental market model of capital under the current data rule so that the Taylor principle will not suffice to insure determinacy and learnability of REE *for such calibrations.*

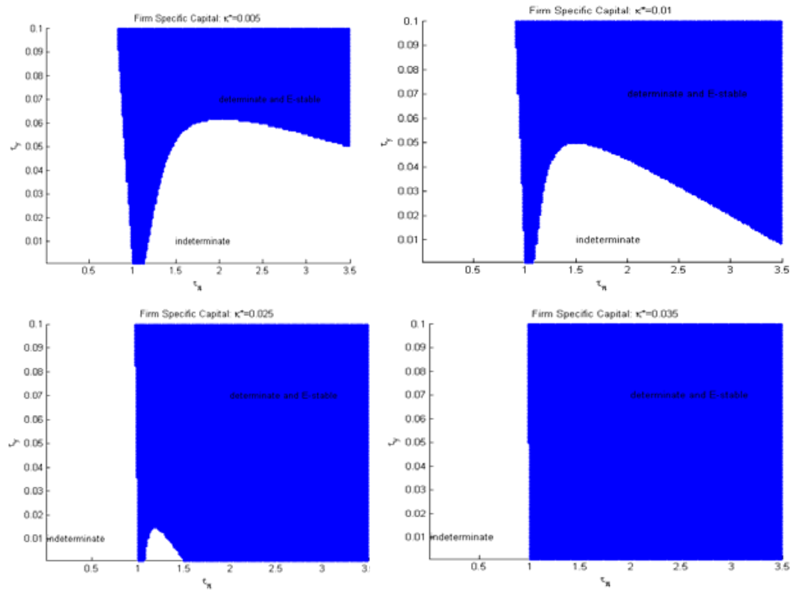

Figure 3: Determinacy and E-stability Results in the Firm Specific model Under the Current Data Rule and Various Values for κ^*.

instance, Taylor's (1993) original calibration of the Taylor Rule (current data rule), adapted for the quarterly frequency of our calibrations, has $\tau_\pi = 1.5$ and $\tau_y = 0.125$.[10] This calibration succeeds in implementing a determinate and learnable REE in all three models as Figures 1-2 confirm. The clear recommendation that follows from our findings using the current data rule is that the Taylor principle, in tandem with some positive weight being given to real activity (or possibly to lagged interest rates) will reliably implement both a determinate and learnable REE in models with capital.

Aside from policy rule changes, we can also eliminate the sliver of indeterminacy in the firm-specific capital model under the current data rule by assuming more flexible prices, e.g., values of ω that are closer to zero, which raises κ and hence κ^*.[11] Alternatively, holding ω fixed, we can reduce α or ϵ or both, which will also increase κ^*. The impact of such changes (higher values for κ^*) on the area of indeterminacy in the firm-specific model under the current data rule are shown in Figure 3. We see that for sufficiently high levels of κ^* –our baseline calibration value is 0.012 – the indeterminacy problem is eliminated.

[10] As noted by Woodford (2003, p. 245) and Gali (2008, p. 83), Taylor's original calibration of the weight on the output gap is 0.5, but Taylor used annualized rates for interest and inflation. Thus, Taylor's calibration of the weight on the output gap under our quarterly model frequency is, appropriately, $0.5/4 = 0.125$.

[11] However, Sveen and Weinke (2006) show that if wages are also modeled as being sticky, much lower values for ω (greater price flexibility) does not eliminate the indeterminacy problem.

4.3.2 The forward expectations rule

Determinacy and E-stability results for the forward expectations rule (32) in the three models are shown in Figure 4. Under this rule, the Taylor principle does not suffice to insure determinacy and learnability of REE in *any* of the three models and there are large differences in the regions giving rise to determinate and learnable REE across the three models. Specifically, the addition of capital either via an economy-wide rental market or via firm-specific demand leads to a big reduction in the region for which REE is determinate and learnable relative to baseline calibration of the labor-only case, though not relative to Bullard and Mitra's calibration of the labor-only case. An important observation from Figure 4 is that in models with capital, the weight assigned to output under a forward expectations rule that obeys the Taylor principle should neither be too aggressive nor too modest.[12] Notice further that the weight regions giving rise to determinate, E-stable REE appear to be empirically plausible ones; Taylor's (1993) calibration will again work to insure determinacy/learnability of the REE in all three models.

The fact that the forward expectations rule (32) makes it more likely (relative to the labor-only case) that REE is indeterminate and unstable under adaptive learning in the rental market for capital model is essentially known from the work of Carlstrom and Fuerst (2005) and Kurozumi and Van Zandweghe (2008). Our findings for the firm-specific model of capital are new, and our findings for the rental market model of capital differ somewhat from those of Carlstrom and Fuerst and Kurozumi and Van Zandweghe. Those authors show that the use of interest rate rules that condition on expectations of future inflation alone or on future output as well *always* result in an indeterminate and unlearnable REE in the rental market for capital model, whereas we find that there is a plausible determinate/learnable region in the rental market case (as well as in the firm-specific capital market case).

What accounts for our different finding? We begin by noting that the addition of investment to the New Keynesian model imposes an arbitrage relationship between the return on bonds and on physical capital. As Carlstrom and Fuerst (2005) show, if the policy rule is purely forward-looking, this arbitrage relationship will hinge entirely on future expected variables, which produces a zero eigenvalue in the system. This forces the only state variable, the capital stock, to become a jump

[12] The finding that the Taylor principle does not suffice for both determinacy and learnability of REE in the labor-only model under the forward expectations rule was previously shown by Bullard and Mitra (2002). They also report that under the forward expectations rule, REE will be determinate and learnable only if the weight assigned to output τ_y is not too great. The lowering of this upper bound for τ_y in models with capital and the addition of a new lower bound for τ_y are new findings of this paper.

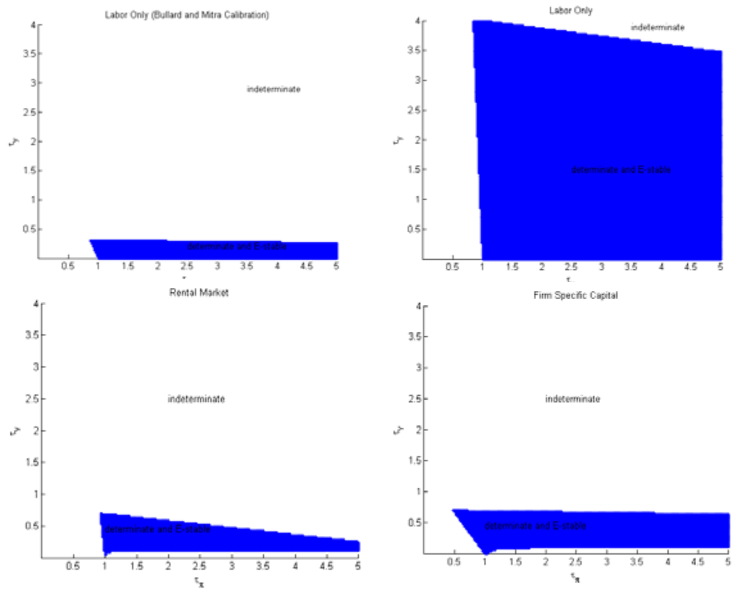

Figure 4: Determinacy and E-Stability Results Under the Forward Expectations Rule

variable, and this insures that the equilibrium will be indeterminate. A critical feature in the model we consider is the inclusion of capital adjustment costs.[13] Capital adjustment costs make capital accumulation dependent on *current* and not just future capital; this makes the arbitrage relationship not entirely forward-looking and eliminates the zero eigenvalue, which, in combination with a purely forward looking interest rate rule will implement a determinate REE in certain cases, i.e., with sufficiently high costs of adjustment.[14]

To establish that capital adjustment costs are responsible for our different determinacy/E-stability findings under the forward expectations rule, Figure 5 shows the consequences of varying these adjustment costs in the *rental* market model of capital (similar results obtain if we vary adjustment costs in the firm-specific model of capital). More precisely we vary the parameter governing the curvature of the capital adjustment cost function, ε_{ψ}, in the three panels of Figure 5 from a value very close to 0–0.1 (i.e., no adjustment costs), to 1 and finally to 15. Recall that

[13] We included capital adjustment costs in the rental market as they are included as part of Woodford's and Sveen and Weinke's model of firm-specific capital, and we wanted to make the comparison between the two models as close as possible. Of course, the inclusion of capital adjustment costs is a standard practice in neoclassical investment theory. Carlstrom and Fuerst (2005) briefly discuss the addition of capital adjustment costs to the rental market for capital model they examine, and note that such adjustment costs may overturn their conclusions for forward-looking policy rules.

[14] An alternative mechanism for achieving the same end, as pursued by Kurozumi and Van Zandweghe (2008), is to have a *hybrid* policy rule that conditions on future expected inflation but on *current* output or its components (consumption, investment).

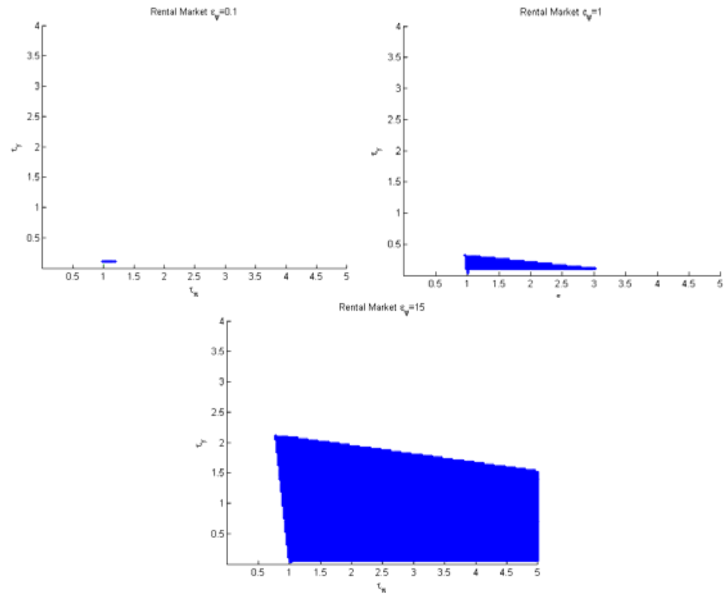

Figure 5: Determinacy and E-Stability Results Under the Forward Expectations Rule In the Rental Market for Capital for Various Values of the Parameter ε_ψ (0.1, 1, and 15) Characterizing the Degree of Adjustment Costs.

our baseline calibration had $\varepsilon_\psi = 3$ (*c.f.* Figure 5 with Figure 4, rental market case). As capital adjustment costs increase, firms increasingly avoid adjustments to their capital stock, and the model increasingly resembles the labor-only model.

Figure 5 shows clearly that when ε_ψ is (essentially) zero, there are essentially no weight pairs for which the REE is both determinate and E-stable, consistent with the findings of Carlstrom and Fuerst (2005) and Kurozumi and Van Zandweghe (2008). However as ε_ψ is increased above zero, the determinacy/E-stability region increases as well which is consistent with the intuition we have provided: the increasing convexity of adjustment costs means investment becomes both more costly and more tied to the current level of the capital stock; as the latter variable is predetermined, it makes the indeterminacy (and E-instability) outcome less likely.

4.3.3 The lagged data rule

Results for the lagged data policy rule (33) in the three models are shown in Figure 6. In these figures, several different colors are now visible. As a reminder to the reader, the blue colored regions indicate weight pairs (τ_π τ_y) for which the REE is both determinate and learnable. The green colored region is determinate but not learnable, the yellow colored region is where all roots are explosive and no color (white) represents regions where REE is indeterminate.

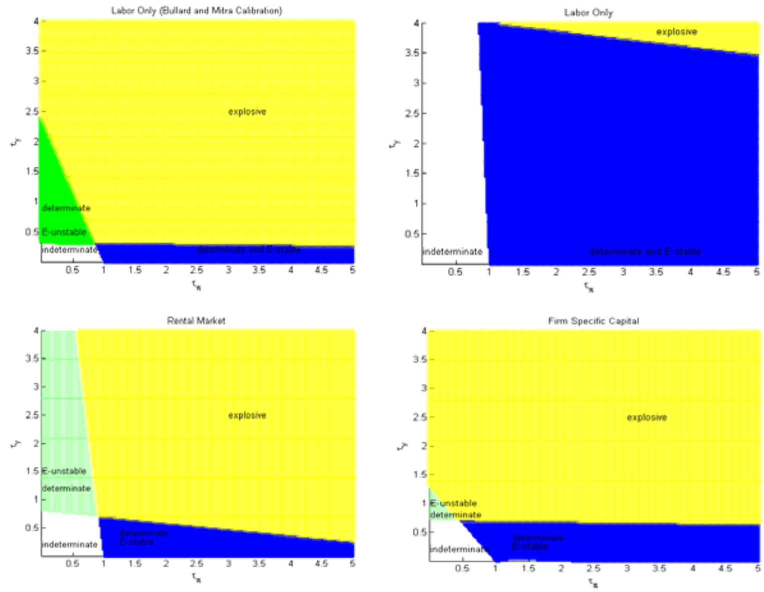

Figure 6: Determinacy and E-Stability Results Under the Lagged Data Rule.

In this case we observe that *in all three models*, the simple Taylor principle does not suffice to insure both a determinate and learnable REE. This finding, for the labor-only model only, was earlier reported by Bullard and Mitra (2002). (Notice, however, that the problem appears much more severe under Bullard and Mitra's calibration for the labor-only model than our own). The novel finding we report here is for the two models *with* capital: the determinate and learnable parameter regions in the models with capital are considerably smaller relative to the labor–only model, suggesting that a much more modest response to output is needed for determinacy and learnability of the REE. In the firm-specific model of capital there is again a small sliver of indeterminate REE for values of τ_y that are close to 0 and values for τ_π between 1 and 3.

However, there is a sense in which the findings for the models with capital follow somewhat continuously from the labor-only model; if we increased the range of values of τ_y we considered above 4 in Figure (6) in the labor-only model, we would also have a green-colored region for the labor-only model where the Taylor principle was not satisfied, i.e. $\tau_\pi < 1$ and $\tau_y > 4$ and where the REE was determinate but E-unstable. That is, qualitatively, the pictures for all 3 models are quite similar.

One explanation for smaller region of determinacy and learnability in and the models with capital is the greater dimension of the system when there is capital and a lagged data policy rule –5 equations – as compared with the labor–only model which has a dimension of just 3 equations under the lagged data policy rule (two lagged values of capital versus one). Thus the models with capital require that 2 out of 5 of the eigenvalues (equal to the number of predetermined variables) should be stable as opposed to just 1 out of 3 in the labor only model and 1 out of 4 in models with capital using current data, both of which are less restrictive requirements than in the models with capital and a lagged data rule.

Comparing the two different approaches to modeling capital, the firm-specific case leads to a slightly larger region of determinate and learnable REE, though the firm-specific case continues to have a sliver of a region where equilibrium is both indeterminate and E-unstable. Nevertheless, for reasonable parameterizations of the Taylor rule, for instance Taylor's original (1993) calibration (adapted to quarterly data) $\tau_\pi = 1.5$ and $\tau_y = 0.125$, determinacy and learnability of the REE are assured in all three models.

4.3.4 The contemporaneous expectations rule

Results for the contemporaneous expectations rule (34) in the three models are shown in Figure 7. This case yields results that at first glance appear to be quite similar to the current data rule (compare Figure 7 with Figure 1). Indeed, under the contemporaneous expectations rule the Taylor principle again suffices to implement a determinate and learnable REE in the labor-only model. However, by contrast with the current data rule, under the contemporaneous expectations rule, the Taylor principle no longer suffices to insure both determinacy and learnability of REE in either model of capital. In both the rental and firm-specific models of capital, the determinacy conditions under the contemporaneous expectations rule are exactly the same as under the current data rule. The Taylor principle suffices to insure determinacy of REE in the rental market case, but in the firm-specific case, there is the same small sliver of a region where $\tau_y \approx 0$ and τ_π is between 1 and 3 for which the REE is indeterminate. However, under the contemporaneous expectations rule there is a further difference relative to the current data rule: in both the rental and firm-specific models of capital there is now a small sliver of a region (colored green) where τ_y is close to 0 and τ_π is between 1 and 1.5 (rental market) or between 1 and 3 (firm-specific) for which the REE *is* determinate but is *not* E-stable. Thus in the rental market model under contemporaneous expectations, the Taylor principle may suffice for determinacy of REE but it no longer suffices for E-stability of REE. In the firm-specific model under the contemporaneous expectations rule, the region of determinate-but-E-unstable REE is a very small green sliver (that is admittedly difficult to see) but which lies along the border between the indeterminate (white) and determinate (blue) regions in Figure (7) As

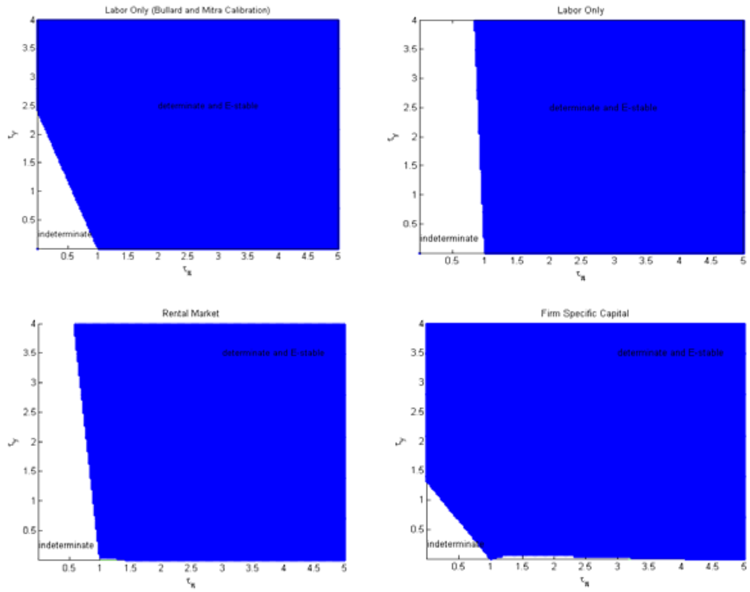

Figure 7: Determinacy and E-Stability Results under the Contemporaneous Expectations Rule.

Figure 7 shows, these regions of indeterminacy or E-instability can be easily avoided by setting τ_y sufficiently high. Indeed we observe that there is again a very wide range of plausible calibrations, (e.g. Taylor's (1993) calibration, $\tau_\pi = 1.5$ and $\tau_y = 0.125$) that result in determinate and learnable REE in all three models under the contemporaneous expectations rule.

4.3.5 Interest rate smoothing rules

We next consider two variants of interest rate smoothing rules studied in the labor-only model by Bullard and Mitra (2007). The first version is a lagged data policy smoothing rule:

$$i_t = \rho i_{t-1} + (1-\rho)\left[\tau_\pi \pi_{t-1} + \tau_y y_{t-1}\right], \tag{37}$$

where $\rho \in (0,1)$ is the weight given to the past interest rate target. The RHS term in square brackets is just the lagged data rule considered earlier.

The second variant is a forward expectations policy smoothing rule:

$$i_t = \rho i_{t-1} + (1-\rho)\left[\tau_\pi E_t \pi_{t+1} + \tau_y E_t y_{t+1}\right], \tag{38}$$

where again the RHS term in square brackets is just the forward expectations rule considered earlier.

We focus on these two versions of policy smoothing rules as the lagged data and forward expectations rules without policy inertia were previously found to be the most troublesome in terms of implementing determinate and learnable REE in New Keynesian models with capital. We know from the findings of Bullard and Mitra (2007) for the labor-only model that the addition of policy inertia (interest rate smoothing) can work to enlarge the region of policy weights for which a policy rule yields determinate and learnable REE; indeed, Bullard and Mitra (2007) show in the labor-only model that for sufficiently large policy inertia, the Taylor principle suffices for determinacy of REE.

Here, in contrast to Bullard and Mitra (2007), we follow the convention in much of the literature on monetary policy rules (e.g. Rudebusch (2002)) and imagine that the weight assigned to the lagged interest rate, i_{t-1}, and to the prescription of the policy rule [in square brackets] add up to unity; in this case the interest rate rule without smoothing can be regarded as the special limiting case where $\rho \to 0$. Woodford (2003b) has shown how such a "partial adjustment" model of monetary policy inertia may result from optimizing behavior on the part of the central bank.[15] Thus we add the choice of $\rho = .5$ to our baseline calibration (Table 1) for both policy smoothing rules, but we later explore the impact of changes in this persistence parameter. As we consider a different class of policy smoothing rules than Bullard and Mitra (2007), we do not report results using their calibration in this section. Instead we use only our baseline calibration.

Determinacy and learnability results for the three models under the lagged data rule with policy smoothing (37) and our baseline calibration are shown in Figure 8. We see that in this case, the Taylor principle suffices to guarantee both determinacy and learnability of REE in the labor-only model but not in the two models that include capital. Comparing Figure 8 with Figure 6 which showed results for the lagged data rule without inertia ($\rho = 0$), we observe that the addition of policy inertia (specifically, $\rho = 0.5$) greatly enlarges the range of policy weights for which REE are determinate and E-stable in both models with capital. Thus policy inertia, like a positive weight attached to output, helps policymakers avoid indeterminacy and E-instability. As in the case of the other rules, one can find a large range of empirically plausible values for the policy weights (τ_π, τ_y), for which the REE is both determinate and learnable, e.g., Taylor's original calibration.

Determinacy and learnability results for the three models under the forward expectations policy

[15] Some authors e.g., Rotemberg and Woodford (1998), Giannoni and Woodford (2003) have derived optimal policy rules where the coefficient on the lagged interest rate is greater than 1. However, such a *super-inertial* policy rule appears to be at odds with estimated interest rate rules. For instance, using U.S. data, Amato and Labauch (1999) estimate the current data rule (31) with the addition of a lagged interest rate (dependent) variable and report that the unrestricted coefficient estimate on the lagged interest rate is always less than one. While we think it would be of interest to consider super-inertial interest rate rules, a virtue of the partial adjustment model we examine is that it requires just one additional parameter, ρ, making it easier to see whether our findings without inertia generalize to the addition of some inertia.

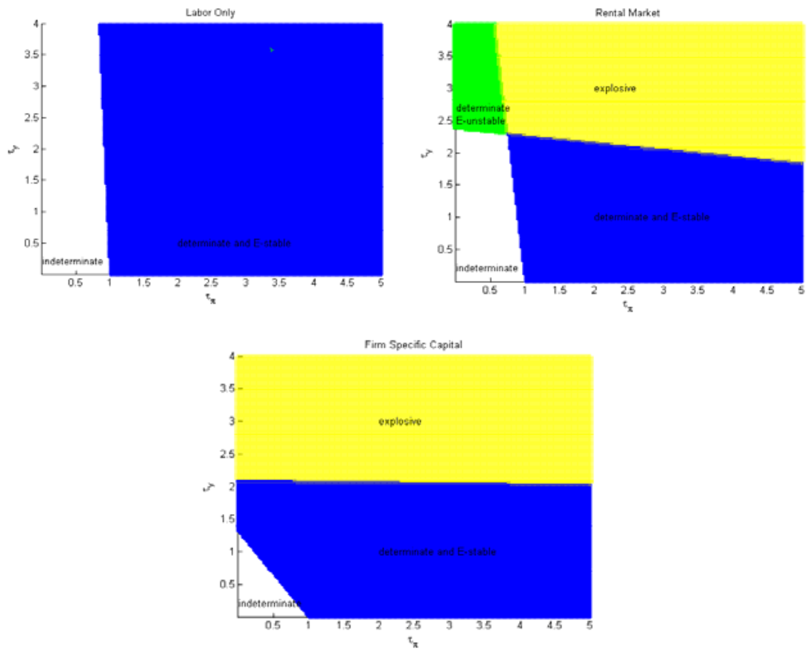

Figure 8: Determinacy and E-Stability Results Under the Lagged Data Policy Smoothing Rule.

smoothing rule (38) are shown in Figure 9. In this case, we see, once again that the Taylor principle suffices to guarantee both determinacy and learnability of REE in the labor-only model but not in the two models that include capital. Comparing Figure 9 with Figure 4 we again observe that the addition of policy inertia ($\rho = 0.5$) greatly enlarges the range of policy weights for which REE are determinate and E-stable in the two models with capital. However it remains the case that in both models with capital, determinacy and E-stability of REE requires both the Taylor principle, together with a choice for τ_y that is neither neither too small nor too large.

Finally, we explore the sensitivity of our findings using policy smoothing rules to changes in the persistence parameter ρ. We focus on 1) the rental market case and 2) the lagged data policy smoothing rule (37), though similar results obtain for the firm-specific capital case and under the forward expectations policy smoothing rule (available on request). Figure 10 below shows that in the case of a rental market for capital, the region of weight pairs for which REE is both determinate and E-stable increases as ρ increases. Specifically, the lines in Figure 10 show how the boundaries of the determinate and E-stable region increase with increases in ρ. For instance, the determinate and E-stable polygon for the baseline $\rho = 0.5$ is the same in Figure 10 as the blue determinate

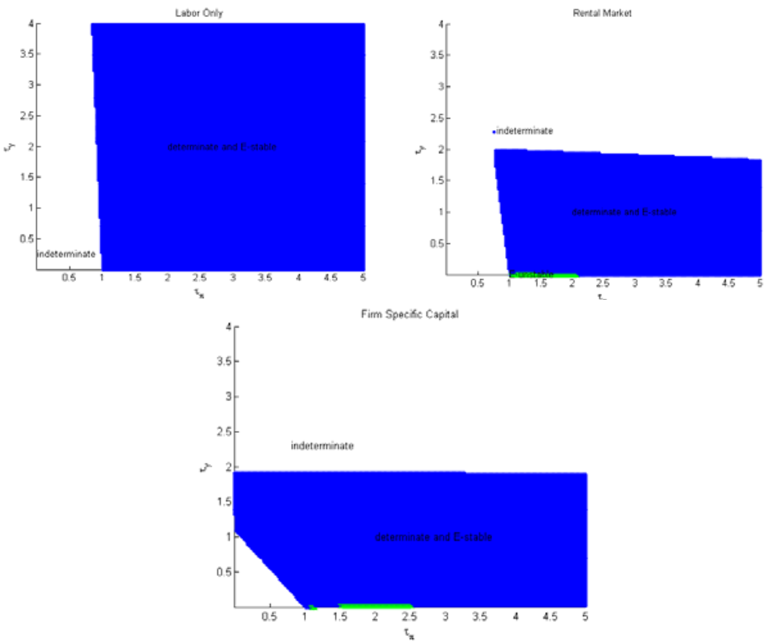

Figure 9: Determinacy and E-Stability Results Under the Forward Expectations Policy Smoothing Rule.

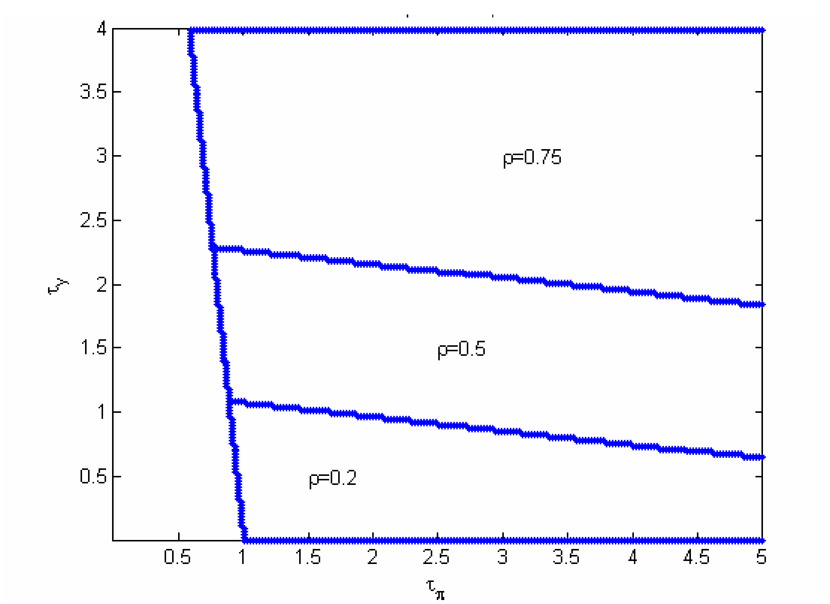

Figure 10: Sensitivity Analysis for the Rental Market Model Under the Lagged Data Policy Smoothing Rule Showing how the Determinate and E-stable Region Varies with Changes in the Value of ρ

and E-stable polygon in Figure 8. As ρ is lowered to 0.2, the upper bound to this determinate and E-stable region falls relative to the baseline case and as ρ is raised to 0.75, the upper bound to the determinate and E-stable region rises relative to the baseline case as Figure 10 illustrates.

As similar finding obtains for the rental market if we use the forward expectations policy smoothing rule or if we use either policy smoothing rule in the firm-specific model of capital. The main finding from this analysis is that increasing persistence in policy (the value of ρ) in models with capital leads to an expansion in the region where equilibrium is both determinate and learnable.

5 Conclusions

We have studied determinacy and learnability of REE in 3 different New Keynesian models. The first model with labor only is a standard, benchmark model. The other two models add productive capital: one via an economy-wide rental market for capital and one via firm-specific demand for capital. The addition of capital to the New Keynesian model allows for the study of investment decisions and may serve to temper the efficacy of central bank policies, as movements in real interest rates are now affected by capital market activity.

Determinacy and learnability are two highly desirable properties for REE and it should be the aim of central banks to adopt interest rate policies that implement equilibria possessing both of these properties. While Bullard and Mitra (2002, 2007) found that the Taylor principle nearly always suffices for both determinacy and learnability of REE in the labor only model, the addition of capital to the New Keynesian model requires some further qualifications to this conclusion. In particular, we find that in the model with a rental market for capital, the Taylor principle continues to suffice to insure both determinacy and learnability of REE if the interest rate rule responds to current data on inflation and output. However the Taylor principle need not suffice for both determinacy and learnability of equilibrium if the interest rate rule responds to future or contemporaneous expectations of inflation and output or to lagged values of these variables or if the central bank uses a policy smoothing rule. Perhaps our most important finding is that in the model with firm-specific capital the Taylor principle *never* suffices to insure both determinacy and learnability of REE for the calibration we consider. There is always at least some small region of policy weights that satisfy the Taylor principle but for which the REE is neither determinate nor learnable. Our findings suggest that this region can be avoided by giving sufficient weight to output or by adopting a sufficiently strong policy smoothing stance or both.[16]

Under the forward-looking policy rule (32) our results underscore the important role played by capital adjustment costs. As we have shown in either model of capital, in the absence of capital adjustment costs the forward-looking policy rule (32) results in REE that are always indeterminate and E-unstable. Introducing capital adjustment costs ties investment decisions to the current level of the current capital stock thus making determinacy and E-stability of the REE a possibility.

A difficulty with our analysis is that it is entirely numerical; analytic results for four or five dimensional systems are difficult to obtain, and the lack of analytic results inhibits our understanding of the causal mechanisms.[17] On the other hand, as we have noted, our numerical approach confirms many existing analytical findings (especially for the simpler, labor-only model) thus providing us with a high degree of confidence in these findings; at the same time, our numerical approach has enabled us to provide several new findings, especially in the case of firm-specific capital. Our adoption of a single model calibration greatly facilitates comparisons across the three models and provides the clearest picture yet of the conditions under which various interest rate rules will yield determinate and learnable REE.

While the Taylor principle may not suffice to guarantee determinacy and learnability in all models considered, we can still reach several practical conclusions that should be of interest to central bankers. First, while the specific findings for the labor-only model do not generalize to models with capital and investment decisions, there appears to be considerable continuity in several of the broader policy recommendations. Specifically, two of the rules we consider, the current data rule and the contemporaneous expectations rule, fare the best in all three models in terms of admitting the largest possible regions of determinate and learnable REE. Second, the results for the firm-specific and rental models of capital suggest that there is high value to interest rates that obey both the Taylor principle and give some weight to output and/or to policy smoothing so as to avoid indeterminacy and instability under learning. Under a forward-looking policy rule, the response to output should not be too modest nor too aggressive, and under a policy-smoothing rule, the weight attached to past interest rates should not be too small. Finally, we note that some $(\tau_\pi,\, \tau_y)$ pairs succeed in implementing determinate and learnable REE in all models and for all interest rate rules that we have considered. In particular, the parameterization proposed by Taylor

[16] Alternatively, it may be avoided if prices are sufficiently (but perhaps implausibly) flexible in the firm-specific model.

[17] Such difficulties would carry over to an analysis of optimally derived monetary policy rules in New Keynesian models with capital, a topic we leave to future research.

(1993) adjusted for the quarterly frequency of our model, – $\tau_\pi = 1.5$ and $\tau_y = 0.125$ – belongs to that class. Perhaps the empirical success of Taylor's (1993) rule rests as much with the parameter values he chose as with the principle that bears his name.

Appendix

In this appendix we derive conditions for E-stability for all variants of the Taylor rule used in the text.

Current Data Rule and Forward Expectations Rule

Under the current data and forward expectations rules, the linearized system can be written in matrix form as

$$b_z z_t + b_k k_t = d_k E_t k_{t+1} + d_z z_{t+1}, \tag{39}$$

$$k_{t+1} = e_z z_t + e_k k_t, \tag{40}$$

where $z_t = (c_t, n_t, \pi_t)'$. The three equations involving agents' expectations are summarized by (39), while (40) is the capital accumulation equation that does not involve any expectations.

The perceived law of motion (PLM) is

$$z_t = a_1 + \psi k_t, \tag{41}$$

$$k_t = a_2 + m k_{t-1}, \tag{42}$$

which corresponds to the MSV solution of the model.

Given the PLM, the expectations are given by

$$\begin{aligned} E_t k_{t+1} &= a_2 + m k_t, \\ E_t z_{t+1} &= a_2 + \psi E_t k_{t+1} = a_1 + \psi a_2 + \psi m k_t. \end{aligned}$$

Substituting the these into (1) and (2), we have

$$\begin{aligned} k_{t+1} &= e_z a_1 + (e_z \psi + e_k) k_t, \\ z_t &= b_z^{-1}(d_k a_2 + d_z a_1 + d_z \psi a_2) + b_z^{-1}(d_k m + d_z \psi m - b_k) k_t. \end{aligned}$$

Therefore the T-mappings are

$$\begin{aligned}
T(a_1) &= b_z^{-1}(d_k a_2 + d_z a_1 + d_z \psi a_2), \\
T(a_2) &= e_z a_1, \\
T(m) &= e_z \psi + e_k, \\
T(\psi) &= b_z^{-1}(d_k m + d_z \psi m - b_k).
\end{aligned}$$

In principle, these T-mappings can be solved to obtain the REE equilibrium. However, solving the T-mappings directly may sometimes yield multiple solutions, and we need to find among these solutions the unique one that is consistent with equilibrium determinacy. To avoid this complication, we solve the REE equilibrium by applying the Blanchard and Kahn (1980) algorithm, which always yields a unique REE.

The T-mappings for ψ and m form an independent system, and we start by computing the derivatives

$$\begin{aligned}
dT_m(m, \psi) &= 0, \\
dT_\psi(m, \psi) &= e_z, \\
dT_m(\psi, m) &= b_z^{-1}(d_k + d_z \psi), \\
dT_\psi(\psi, m) &= b_z^{-1} d_z m,
\end{aligned}$$

For E-stability, we require that the matrix

$$\begin{pmatrix} 0 & e_z \\ b_z^{-1}(d_k + d_z \psi) & b_z^{-1} d_z m \end{pmatrix}$$

has eigenvalues less than 1. Since $a_1 = (0, 0, 0)'$ and $a_2 = 0$, the corresponding derivatives are easy to compute:

$$\begin{aligned}
dT_{a_1}(a_1) &= b_z^{-1} d_z \\
dT_{a_2}(a_1) &= b_z^{-1}(d_k + d_z \psi) \\
dT_{a_1}(a_2) &= e_z \\
dT_{a_2}(a_2) &= 0
\end{aligned}$$

For E-stability, we require the matrix

$$\begin{pmatrix} b_z^{-1} d_z & b_z^{-1}(d_k + d_z \psi) \\ e_z & 0 \end{pmatrix},$$

to have eigenvalues less than 1.

Lagged Data Rule

Under the lagged data rule, it is assumed that agents can only observe lagged variables (otherwise they could have used current data in the rule). As a result, the MSV solution must have a different form. There are now 4 state variables instead of one: $c_{t-1}, n_{t-1}, \pi_{t-1}$ and k_{t-1}.

To obtain the MSV solution, we first substitute

$$i_t = \tau_\pi \pi_{t-1} + \tau_y y_{t-1}$$

into (39), then we rewrite (40) as

$$k_t = e_z z_{t-1} + e_k k_{t-1}.$$

Combining (39) and (40), the whole system can then be rewritten as

$$X_t = F E_t X_{t+1} + L X_{t-1}, \tag{43}$$

where $X_t = (z_t, k_t)'$.

The PLM is

$$X_t = a + \gamma X_{t-1}.$$

Using this PLM, we obtain the expectation

$$E_t X_{t+1} = a + \gamma a + \gamma^2 X_{t-1}.$$

Substituting this expectation into (43), we have

$$X_t = F(a + \gamma a) + (F\gamma^2 + L) X_{t-1}.$$

The T-mappings are

$$\begin{aligned} T(a) &= F(a + \gamma a), \\ T(\gamma) &= F\gamma^2 + L. \end{aligned}$$

We need to evaluate the eigenvalues of

$$\begin{aligned} DT(a) &= F(I + \gamma), \\ DT(\gamma) &= \gamma' \otimes F + I \otimes F\gamma \end{aligned}$$

to determine if the REE is E-stable.

Contemporaneous expectations

Under the contemporaneous expectations rule, the informational assumption is that agents can only observe lagged variables, and the expectations are made at time $t-1$ rather than at time t. As a result, the MSV solution must have a different form. To obtain it, we first substitute

$$i_t = \tau_\pi E_{t-1}\pi_t + \tau_y E_{t-1} y_t$$

into the system, and re-write the the resulting system as

$$\begin{aligned} g_k E_{t-1}k_t + g_z E_{t-1}z_t + b_z z_t + b_k k_t &= d_k E_t k_{t+1} + d_z E_t z_{t+1}, & (44)\\ k_{t+1} &= e_k k_t + e_z z_t. & (45) \end{aligned}$$

The PLM is

$$\begin{aligned} z_t &= a + \gamma k_t, \\ k_t &= b + m k_{t-1}. \end{aligned}$$

Agents' expectations are thus

$$\begin{aligned} E_{t-1}z_t &= a + \gamma k_t, \\ E_{t-1}z_{t+1} &= a + \gamma(b + m k_t), \\ E_{t-1}k_t &= b + m k_{t-1}, \\ E_{t-1}k_{t+1} &= b + m k_t. \end{aligned}$$

Substituting these expectations into (44) and (45), we get the T-mappings

$$\begin{aligned} T(a) &= b_z^{-1}(d_k b + d_z a + d_z \gamma b - g_z a, \\ T(b) &= e_z a, \\ T(m) &= e_z \gamma + e_k, \\ T(\gamma) &= b_z^{-1}(d_k m + d_z \gamma m - g_k - b_k - g_z \gamma. \end{aligned}$$

For E-stability, we require the following two Jacobian matrices to have eigenvalues less than 1:

$$\begin{pmatrix} 0 & e_z \\ b_z^{-1}(d_k + d_z) & b_z^{-1}(d_z m - g_z) \end{pmatrix}$$

and

$$\begin{pmatrix} b_z^{-1}(d_z - g_z) & b_z^{-1}(d_k + d_z \gamma) \\ e_z & 0 \end{pmatrix}.$$

Interest rate smoothing

The lagged data rule with policy inertia, is given by:

$$i_t = \rho i_{t-1} + (1-\rho)(\tau_\pi \pi_{t-1} + \tau_y y_{t-1}).$$

Substituting in this rule, the system can be written as:

$$X_t = FE_t X_{t+1} + LX_{t-1}, \tag{46}$$

where $X_t = (z_t, k_t, i_t)'$.

The PLM and the critical matrices are the same as in the lagged data case, except that there is an extra state variable i_{t-1} in the solution.

The forward-looking rule with policy inertia is given by:

$$i_t = \rho i_{t-1} + (1-\rho)(\tau_\pi E_t \pi_{t+1} + \tau_y E_t y_{t+1}),$$

Substituting in this rule, the system becomes:

$$\begin{aligned} b_z z_t + b_s s_t &= d_s E_t s_{t+1} + d_z E_t z_{t+1}, \\ s_{t+1} &= e_s s_t + e_z z_t + f_z E_t z_{t+1} + f_s E_t s_{t+1}, \end{aligned}$$

where z_t is defined as before and $s_t = (k_t, i_{t-1})'$. The PLM is

$$\begin{aligned} z_t &= a_1 + \psi s_t, \\ s_{t+1} &= a_2 + m s_t. \end{aligned}$$

Following the same procedures as explained above, we obtain the T-mappings

$$\begin{aligned} T(a_1) &= b_z^{-1}(d_s + d_z\psi)a_2 + b_z^{-1} d_z a_1, \\ T(a_2) &= (e_z + f_z)a_1 + (f_z\psi + f_s)a_2, \\ T(m) &= e_s + e_z\psi + (f_z\psi + f_s)m, \\ T(\psi) &= b_z^{-1}(d_s m + d_z \psi m - b_s). \end{aligned}$$

For E-stability, the two required Jacobian matrices are

$$\begin{pmatrix} b_z^{-1} d_z & b_z^{-1}(d_s + d_z\psi) \\ e_z + f_z & f_z\psi + f_s \end{pmatrix}$$

and

$$\begin{pmatrix} I \otimes (f_z\psi + f_s) & I \otimes e_z + m' \otimes f_z \\ I \otimes (b_z^{-1} d_s + b_z^{-1} d_z \psi) & m' \otimes b_z^{-1} d_z \end{pmatrix}.$$

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