The Value of Interest Rate Stabilization Polices When Agents are Learning*

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Abstract

We examine expectational stability (E–stability) of rational expectations equilibrium under optimal interest rate rules in the context of the standard, "New Keynesian" model of the monetary transmission mechanism. We focus on the case where the monetary authority adds interest rate stabilization to its traditional objectives of inflation and output stabilization. We consider both the case where the monetary authority lacks a commitment technology and the case of full commitment. We show that for both cases, optimal interest rate rules yield rational expectations equilibria that are E-stable for a wide range of empirically plausible parameter values. This finding stands in contrast to the findings of Evans and Honkapohja (2002, 2003ab) for optimal monetary policy rules in environments where interest rate stabilization is not part of the central bank's objective function.

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1 Introduction

Evans and Honkapohja (2002, 2003ab) examine the stability, under adaptive learning dynamics, of rational expectations equilibrium (REE) in the standard New Keynesian model of the monetary transmission mechanism[1] when the policy rule of the central bank is optimally derived.[2] Specifically, they suppose that the central bank minimizes a quadratic loss function that penalizes deviations of inflation and output from certain exogenous target values. The result of this minimization problem is an optimal interest rate rule which is used together with equations describing private sector behavior to characterize the equilibrium of the economy.

Evans and Honkapohja report that, regardless of whether the central bank operates under commitment or discretion, the REE of the system is always expectationally *un*stable when the policy rule is derived under the incorrect assumption that the private sector has rational expectations – Evans and Honkapohja call the policy rule in this case the "fundamentals–based" policy rule. While the private sector is assumed to use a correctly specified model to form expectations, it does not initially possess knowledge of the REE parameterization of the model; instead it updates the parameters of its model in real time using all relevant data. However, the central bank's fundamentals–based interest rate policy causes this adaptive learning process to diverge away from the REE, and for this reason, the fundamentals–based policy rule is considered undesirable– it is *expectationally unstable*.[3] Evans and Honkapoja (2003) found this instability of optimal policy "deeply worrying," and suggest that the central bank might do well to assume that the private sector does not (initially) possess rational expectations. Indeed, Evans and Honkapohja show that if the central bank does not assume rational expectations on the part of the private sector, the resulting, optimally derived, "expectations–based" interest rate rule, which conditions on the private sector's expectations of inflation and output, results in a REE that is always expectationally stable.

In this paper, we consider an alternative approach to optimal monetary policy under learning using the same New Keynesian framework and maintaining the assumption that the private sector does not have rational expectations. We show that it is possible for the central bank to use an optimally derived policy rule that does not condition on private sector expectations and which results in a REE that is stable under adaptive learning dynamics in contrast to the findings of

[1] See Clarida et al. (1999) for a presentation of this model.

[2] Other authors, e.g., Bullard and Mitra (2002, 2003), Carlstrom and Fuerst (2004), have studied the stability of REE under learning for non-optimal, Taylor-type instrument rules. Honkapohja and Mitra (2004) study the stability of REE under learning using both optimal and non-optimal policy rules.

[3] See Evans and Honkapohja (2001) for a complete treatment of the notion of expectational (in)stability.

Evans and Honkapohja (2002, 2003ab). What is needed to obtain our result is for the central bank to expand its loss function to include interest rate stabilization as a third objective, in addition to the traditional twin objectives of inflation and output stabilization. Our result concerning the stability of REE under adaptive learning holds for a wide range of emprically plausible parameter values across all calibrations found in the literature and regardless of whether the central bank operates under discretion or commitment.

There are several advantages to studying optimal policy using a loss function that gives weight to interest rate stabilization. First, as Woodford (2003) shows, if there are transactions frictions of the type that would give rise to a demand for money then an appropriate welfare-theoretic loss function for the central bank is one that includes interest rate stabilization as a third objective in addition to the standard two objectives of inflation and output stabilization. Second, there is substantial empirical evidence that central banks adjust their interest rate targets only gradually over time – consistent with having interest rate stabilization as a goal (see, e.g., Goodhart (1997)). Finally, as Giannoni and Woodford (2003) note, the optimal policy rules derived under the three-element loss function resemble the much-studied Taylor instrument rule, while rules derived under the more typical two–element (inflation and output stabilization) objective function do not.

Evans and Honkapohja's proposed resolution to the monetary policy instability problem – conditioning policy on private sector expectations – strikes us as problematic for several reasons. First, operationally speaking, the private sector's expectations may not be observable, or may be heterogeneous; figuring out which expectations to use is a complicated task. Second, as Honkapohja and Mitra (2005) point out, if the central bank is known to be conditioning policy on private sector expectations, the private sector might choose to be strategic about its expectations. Third, in certain environments, conditioning on private sector expectations may increase the likelihood that the REE becomes indeterminate, as shown by Bernanke and Woodford (1997).[4] Our approach, which does not require the use of private sector expectations, avoids these problems.

2 The model

The model of the private sector is the standard, "cashless" New Keynesian model used in analyses of the monetary policy transmission mechanism (as set forth, e.g., in Clarida et al. (1999) or

[4]Evans and Honkapohja (2003ab) are careful to show that indeterminacy of REE is *not* a problem when the central bank uses the optimally derived, expectations-based interest rate rules that condition on private sector expectations. Berardi (2004) reconciles Evans and Honkapohja's (2003ab) findings with those of Bernanke and Woodford (1997).

Woodford (2003)) and consists of the following equations:

$$x_t = -\varphi(i_t - \hat{E}_t\pi_{t+1}) + \hat{E}_t x_{t+1} + g_t \quad (1)$$

$$\pi_t = \lambda x_t + \beta \hat{E}_t \pi_{t+1} + u_t \quad (2)$$

$$v_t = (g_t, u_t)' = F v_{t-1} + e_t, \quad F = \begin{bmatrix} \mu & 0 \\ 0 & \rho \end{bmatrix}, \quad (3)$$

where $|\mu|$, $|\rho| \in (0, 1)$, $e_t = (e_{gt}, e_{ut})$ and $e_{it} \sim$ i.i.d.$(0, \sigma_i^2)$, $i = g, u$. The parameters φ and λ are assumed to be positive, as is the discount factor, $0 < \beta < 1$. The intertemporal IS equation (1) relates the output gap x_t, to its expected future value $\hat{E}_t x_{t+1}$, and to the real interest rate; i_t is the short-term (one-period) nominal interest rate and $\hat{E}_t \pi_{t+1}$ is the expected inflation rate between t and $t+1$. The aggregate supply equation (2) relates the current inflation rate π_t to expected future inflation and the current output gap. Both equations can be derived from explicit microfounded models. Note that $\hat{E}_t$ here refers to expectations of future endogenous variables that are not necessarily rational.[5] The last equation (3) characterizes how the demand and supply shock processes, g_t and u_t, evolve over time.

This model is closed by specifying how the central bank determines the short-term nominal interest rate, i_t. Suppose the central bank's objective is to minimize:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[(\pi_t - \pi)^2 + \alpha_x (x_t - x)^2 + \alpha_i (i_t - i)^2 \right], \quad (4)$$

where π, x and i represent target values for inflation, the output gap and the interest rate. For simplicity, we henceforth set $\pi = x = 0$, though our results would not change if we assumed nonzero values for these targets. The relative weights assigned to output and interest rate stabilization are $\alpha_x > 0$ and $\alpha_i > 0$.

The loss function differs from the one considered by Evans and Honkapohja (2003ab, 2002) by the inclusion of the third, interest rate stabilization element; Evans and Honkapohja have $\alpha_i = 0$. The three-element loss function we use has been given a microfounded welfare-economic justification by Woodford (1999). He shows that when there are non-negligible transaction frictions, as would rationalize a demand for money, the three-element version of the loss function objective represents a quadratic approximation to the optimal, expected utility realized by the representative household

[5] There is some dispute in the literature as to whether equations (1)–(2) represent the correct, microfounded structural equations of the model under this assumption of non-rational expectations (there is no dispute in the case where agents do have rational expectations)– see the debate between Preston (2004) and Honkapohja Mitra and Evans (2003) for the details. As our aim is to contrast our findings with those of Evans and Honkapohja (2002, 2003ab), we chose to work with the same system of structural equations (1)–(2) that they used.

in the same optimizing model that gives rise to the structural equations (1)–(2); in the absence of such transaction costs, the two-element version ($\alpha_i = 0$) is the relevant approximation. Woodford (2003) further rationalizes the inclusion of the third term as a quadratic approximation to the implicit penalty on interest rate variability the central bank faces due to the existence of a zero lower bound on nominal interest rates; the two-element version of the loss function does not take this constraint into account. For these reasons, the three-element loss function can be viewed as the more general form of the central bank's objective function.

3 Discretionary Policy

We first consider the case where the central bank cannot commit to future policies. Optimal monetary policy in this discretionary case amounts to minimization of (4) subject to versions of equations (1)–(2) modified to take account of the central bank's lack of commitment:

$$x_t = -\varphi i_t, \tag{5}$$

$$\pi_t = \lambda x_t. \tag{6}$$

The three first order conditions can be combined to yield the optimality condition:

$$\lambda \pi_t + \alpha_x x_t - \alpha_i \varphi^{-1}(i_t - i) = 0. \tag{7}$$

Equation (7) can be rearranged to yield the optimal interest rate rule:

$$i_t = i + \frac{\varphi\lambda}{\alpha_i}\pi_t + \frac{\varphi\alpha_x}{\alpha_i}x_t. \tag{8}$$

The rule (8) is of the same general form as Taylor's instrument rule, though in this case it has been *optimally derived*.[6] The optimal rule (8) requires knowledge of the contemporaneous values of inflation and output, and for this reason we will refer to it as a "data-based" rule.

The system under discretionary policy thus consists of equations (1), (2) and (8). Letting $y_t = (x_t,\ \pi_t)'$, this system can be further reduced and written as:

$$y_t = \delta_0 + \delta_y \hat{E}_t y_{t+1} + \delta_v v_t, \tag{9}$$

where δ_0, δ_y and δ_v represent comformable vectors or matrices with elements that are combinations of structural model parameters and objective function weights.

[6] It is in this sense that our analysis differs from that of Bullard and Mitra (2002).

To study the stability of REE under adaptive learning, we follow Evans and Honkapohja (2001, section 10.3) and suppose that agents have a *perceived* law of motion for y_t that corresponds to the minimal state variable (MSV) representation of the REE solution to the system (9). This perceived law of motion may be written as:

$$y_t = d_0 + d_v v_t. \tag{10}$$

Using this perceived law of motion (PLM), agents form expectations of y_{t+1}:

$$\hat{E}_t y_{t+1} = d_0 + d_v F v_t.$$

Substituting these expectations into (9) (in lieu of rational expectations) yields a T–mapping from the PLM to the *actual* law of motion (ALM):

$$y_t = T_{d_0}(d_0) + T_{d_v}(d_v) v_t.$$

The rational expectations solution consists of values $\overline{d}_0 = T_{d_0}(\overline{d}_0)$ and $\overline{d}_v = T_{d_v}(\overline{d}_v)$.

Expectational (E)–stability of $(\overline{d}_0, \overline{d}_v)$ is governed by local asymptotic stability of the matrix differential equation:

$$\frac{d}{d\tau}(d_0, d_v) = T(d_0, d_v) - (d_0, d_v),$$

, evaluated at the REE solution values. Specifically, the REE solution to the system (9) is E-stable if the eigenvalues of

$$\begin{aligned} DT_{d_0} &= \delta_y, \\ DT_{d_v} &= \delta_y F, \end{aligned}$$

have real parts less than unity. As Evans and Honkapohja (2003) point out, these conditions correspond closely to whether or not the rational expectations equilibrium of the system (9) is determinate; the condition for determinacy is that the eigenvalues of δ_y are all less than unity. Indeed, given the restrictions imposed on the matrix F it is clear that in this case of discretionary policy, the determinacy and the E–stability conditions exactly coincide. As Duffy (2003) shows, in the case of discretionary policy we have:

$$\delta_y = \frac{1}{\xi} \begin{bmatrix} \alpha_i & \varphi(\alpha_i - \lambda\varphi\beta) \\ \lambda\alpha_i & \varphi(\lambda\alpha_i + \beta\varphi\alpha_x) + \beta\alpha_i \end{bmatrix}, \tag{11}$$

where $\xi = \alpha_i + \varphi^2(\alpha_x + \lambda^2)$. Whether or not the eigenvalues of this matrix are all less than unity depends on the calibration of the structural parameters of the model and the weights chosen for the objective function, and so we turn to a numerical analysis later on in section 5. Duffy (2003) investigated the eigenvalues of this matrix for one calibration of the model, due to Woodford (1999), but in this paper, we provide a more general analysis, considering several different calibrations that have appeared in the literature and allowing the values of the two weights, α_x and α_i in the central bank's objective function to vary over a grid of plausible values. In addition, Duffy (2003) did not consider the more interesting case where the central bank operates under commitment, which we address in the next section.

The difference between the optimal "data-based" interest rate rule (8) we derive under discretionary policy and the optimal "fundamentals-based" rule derived by Evans and Honkapohja (2003a) under discretionary policy is the key to understanding our different stability findings under adaptive learning. In Evans and Honkapohja's model, $\alpha_i = 0$, so the optimality condition we derived above (7) reduces to:[7]

$$\lambda\pi_t + \alpha_x x_t = 0. \tag{12}$$

Since this optimality condition does not involve the interest rate, i_t, Evans and Honkapohja proceed to derive an optimal interest rate rule *under the assumption that the private sector has rational expectations.* They suppose that the private sector forms expectations of future output and inflation using an MSV solution of the form given by (10), i.e., the same solution class we consider. Using this perceived law of motion to form expectations in (1), (2) and using (12), they solve for the REE coefficient values of the MSV solution and further obtain an optimal interest rate rule of the form:

$$i_t = \psi_i + \psi_g g_t + \psi_u u_t, \tag{13}$$

where ψ_i, ψ_g and ψ_u are precisely defined coefficient values under the maintained assumption that the private sector has rational expectations. They call this a "fundamentals-based" ruled because it is a function only of exogenous fundamentals, i.e., the two shocks. Evans and Honkapohja use (13) to eliminate i_t in (1) and write the system (1)–(2) in the form (9), where, in particular, they have that:

$$\delta_y = \begin{bmatrix} \varphi & 1 \\ \beta + \lambda\varphi & \lambda \end{bmatrix}. \tag{14}$$

Their matrix for δ_y, (14), which is critical to their E-stability analysis, is different from the matrix (11) we derive using the data-based rule, and this difference is the reason for our different stability findings, as detailed later. We note that we could have proceeded in the same manner as Evans

[7] *c.f.* equation (6), p. 809 in Evans and Honkapohja (2003a).

and Honakpohja (2003a), using the optimality condition (7) (our data-based interest rate rule) to substitute out for i_t in equation (1) and then finding the MSV REE solution for π_t and x_t as functions of the shocks g_t and u_t. We could have then substituted the REE solutions for π_t and x_t back into our optimal data-based interest rate rule (8) to get a "fundamentals-based" rule of the same form (13) used by Evans and Honkapoja; the ψ coefficients would differ from those in Evans and Honkapohja (2003a) due to our different objective function, but the functional form of the rule would be the same. However, we already know, from Proposition 2 of Evans and Honkapohja (2003a), that *any* interest rate rule of the form (13) will result in an E-unstable REE, irrespective of the coefficient values of that rule; that finding is unaffected by our use of a different loss function.

Our contriubtion is to note that the addition of interest-rate stabilization to the central banks' policy objective allows for a *new and completely different* optimal interest rate rule that is "data-based;" this rule cannot be a part of the MSV REE solution under the two-element loss function that Evans and Honkapohja consider. Indeed, it should be emphasized that setting $\alpha_i = 0$ in (11) *does not* result in (14), i.e., there is a discontinuity between our system and the one studied by Evans and Honkapohja (2003a). As noted above, a further important difference is that Evans and Honkapohja (2003) need to assume that the central bank acts as though the private sector has rational expectations in order to derive the central bank's optimal, fundamentals-based interest rate rule. In our approach, we do not need to make such an assumption in order to derive the optimal interest rate rule as this rule (8) follows immediately from combining the first order conditions from the central bank's optimization problem. Finally, we wish to emphasize that data-based rules seem more plausible as candidates for optimal policy rules than rules that condition policy on exogenous shocks; despite the assumption that shock "fundamentals" are known, in practice, such shocks are hard to identify and it seems that central bankers do use endogenously determined variables (π_t, x_t) in formulating policy decisions. The same logic which we have just given for the difference in our findings from those of Evans and Honkapohja (2002, 2003ab) under discretionary policy also extends to the case of policy under commitment. We now turn to an analysis of that case.

4 Policy Under Commitment

If the central bank can credibly commit to future policies, the problem it faces changes to reflect this possibility. In particular, we follow Woodford (2003) in adopting the "timeless perspective" to optimal policy under commitment. This perspective requires that the central bank minimizes (4) subject to the original private sector equations, (1)–(2). The first order conditions can be manipulated to obtain the optimal interest rate rule under commitment:

$$i_t = -\frac{\varphi\lambda i}{\beta} + \frac{\varphi\lambda}{\alpha_i}\pi_t + \frac{\alpha_x\varphi}{\alpha_i}(x_t - x_{t-1}) + \frac{\varphi\lambda+\beta+1}{\beta}i_{t-1} - \frac{1}{\beta}i_{t-2}. \tag{15}$$

As noted by Giannoni and Woodford (2003), the optimal rule (15) closely resembles a "policy-smoothing" version of the Taylor instrument rule, though (15) involves greater history dependence (via the variables x_{t-1}, i_{t-2}) than is typically assumed in policy smoothing versions of Taylor rules. We note further that the rule (15) differs from the optimal "fundamentals-based" rule under commitment studied by Evans and Honkapohja (2002, 2003b).[8] The reason for this difference in the commitment case is analogous to the reason we provided in the discretionary policy case; the use of the two–element objective function does not immediately give rise to an optimal interest rate rule and so Evans and Honkapohja (2002) need to assume rational expectations on the part of the private sector in order to derive the optimal policy rule. With the three-element objective function (4), we do not need to make such an assumption.

Using the optimal rule (15) to substitute out for i_t in (1), we can reduce the system to two equations in x_t and π_t. Defining $y_t = (x_t, \pi_t)'$ and $w_t = (i_t, i_{t-1})'$, the system under commitment can be written as:

$$y_t = \delta_0 + \delta_{y1}\hat{E}_t y_{t+1} + \delta_{y2}y_{t-1} + \delta_w w_{t-1} + \delta_v v_t. \tag{16}$$

The interest rate rule (15) can also be written in matrix notation as

$$w_t = a_0 + a_1 y_t + a_2 y_{t-1} + a_3 w_{t-1}. \tag{17}$$

The perceived law of motion (PLM) consistent with a MSV-REE solution in this case is:

$$y_t = d_0 + d_y y_{t-1} + d_w w_{t-1} + d_v v_t. \tag{18}$$

Given (18), (3) and (17), we obtain the expected value of y_{t+1} as:

$$\hat{E}_t y_{t+1} = d_0 + d_y y_t + d_w(a_0 + a_1 y_t + a_2 y_{t-1} + a_3 w_{t-1}) + d_v F v_t.$$

Since there are two y_t terms in this equation, we need to apply (18) one more time to eliminate them. Doing this yields:

$$\hat{E}_t y_{t+1} = \psi_0 + \psi_y y_{t-1} + \psi_w w_{t-1} + \psi_v v_t.$$

[8] Evans and Honkapohja's (2002) optimal, "fundamentals-based" rule under commitment is $i_t = \psi_x x_{t-1} + \psi_g g_t + \psi_u u_t$,where the ψ coefficients are precisely defined under the maintained assumption of private sector rational expectations.

Substituting these expectations into (16), we get a T-mapping from the PLM to the ALM:

$$y_t = T(d_0) + T(d_y)y_{t-1} + T(d_w)w_{t-1} + T(d_v)v_t.$$

Rather than calculate all possible REE solutions–there can be several fixed points to the T-mapping in this case–we focus on the unique, saddle path stable solution found using the Blanchard-Kahn technique (see e.g., Evans and Honkapohja (2001, Section 10.8)). This unique determinate equilibrium is the one relevant to policy discussions and, as Giannoni and Woodford (2003) show, it is the optimal equilibrium from the timeless perspective.

The conditions for E-stability of the REE solution to the system (16) are given in Evans and Honkapohja (2001, section 10.3) The conditions are that the eigenvalues of the matrices DT_{d_j}, $j = 0, y, w, v$, all have real parts less than unity. The relevant matrices are:

$$\begin{aligned} DT_{d_0} &= \delta_{y1}(I + \overline{d_y} + \overline{d_w}a_1), \\ DT_{d_y} &= \overline{d_y'} \otimes \delta_{y1} + I \otimes (\delta_{y1}\overline{d_y} + \delta_{y1}\overline{d_w}a_1), \\ DT_{d_w} &= \overline{d_w'} \otimes \delta_{y1}a_1 + I' \otimes \delta_{y1}(\overline{d_y} + a_1\overline{d_w} + a_3), \\ DT_{d_v} &= \delta_{y1}\overline{d_y} + \delta_{y1}\overline{d_w}a_1 + \delta_{y1}F. \end{aligned}$$

In the case of optimal policy under commitment, it is no longer the case that determinacy and stability of equilibrium under adaptive learning are inextricably linked; while we focus on the unique determinate REE, this equilibria may or may not satisfy the E-stability conditions given above. Again, it is not possible to obtain analytic results, so we must resort to numerical methods to assess whether the REE in the commitment case are stable under adaptive learning. We now turn to this numerical exercise.

5 Numerical Analysis

The calibrated values of the structural model parameters we consider in our numerical exercise are due to Woodford (W) (1999), Clarida, Gali, and Gertler (CGG) (1999), and McCallum and Nelson (MN) (2000) and are given in Table 1. We further assume that $\rho = \mu = .35$ in (3) for all three model calibrations. We consider whether the REE is E-stable or not for each of the three structural model calibrations and for both discretionary and commitment regimes – a total of 6 numerical exercises. For each exercise, we vary each of the loss function weights α_i and α_x, over a fine grid of values, ranging from .001 to 2, with a step size of .04.[9]

[9] We conducted a further numerical analysis (not illustrated here), where we searched over all combinations of the two weights in the interval $[.001, .01]$ using a step size of .01; the results are equivalent to those illustrated below

Author	φ	λ
W	1/0.157	0.024
CGG	1	0.3
MN	0.164	0.3

Table 1: Three values of the structural parameters of the model

5.1 Numerical Findings Under Discretionary Policy

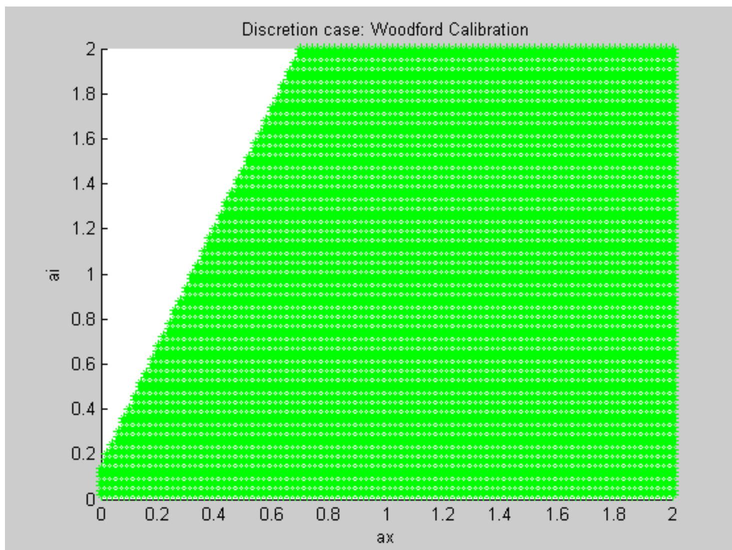

Figure 1: Discretionary policy, Woodford calibration.

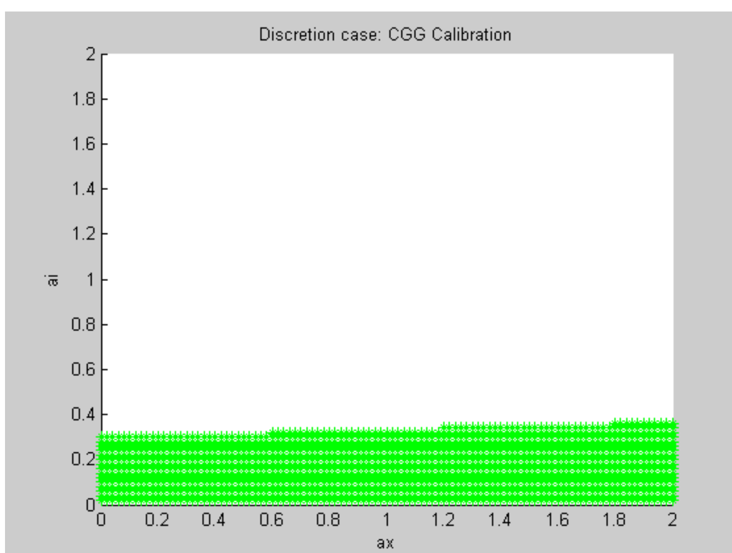

Figure 2: Discretionary policy, Clarida et al. calibration.

when each weight is set equal to .01, i.e., in all of these cases, the REE is E–stable.

Figures 1-3 show our numerical findings under discretionary policy for the three calibrations given in Table 1 for various (α_x, α_i) weight pairs. For each weight pair, our numerical routine checks the eigenvalues of the matrix δ_y, (11), to determine whether either eigenvalue has real part less than unity. Regions where all eigenvalues have real parts less than unity are shaded in gray; in this case, the REE is both E–stable and determinate. Regions where at least one eigenvalue has a real part greater than unity are left blank; these are the regions where the REE is both E–unstable and indeterminate.

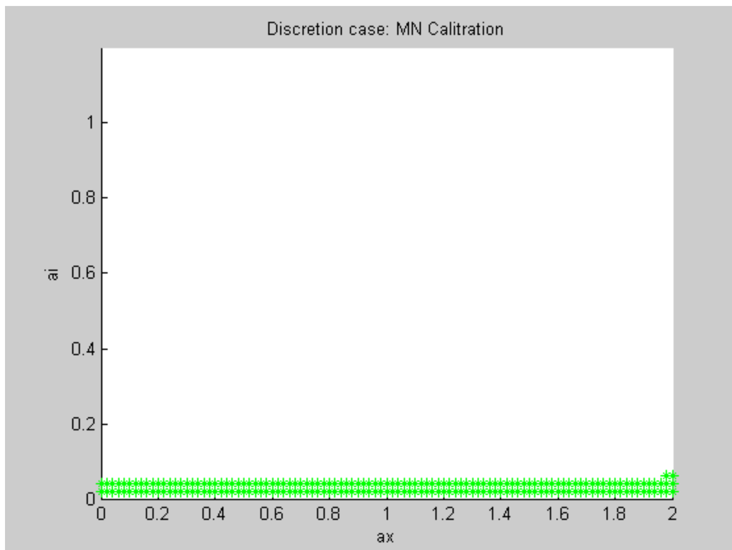

Figure 3: Discretionary policy, McCallum-Nelson calibration.

The figures reveal that for all three calibrations there always exist (α_x, α_i) combinations for which the REE solution is E-stable and determinate. This finding stands in contrast to Evans and Honkapohja's finding of E–instability for the fundamentals-based rule under discretion derived under the assumption that $\alpha_i = 0$. Of course, Evans and Honkapohja's result holds for any choice of weights/calibration of the model and our finding applies only for certain weight choices that are not independent of the model calibration.

Woodford (1999) proposes the weights $\alpha_x = .047$ and $\alpha_i = .233$. With these choices, and the Woodford (W) calibration of the structural parameters, Figure 1 reveals that the REE is both E–stable and determinate. Woodford (2003, Table 6.1) proposes somewhat different weights of $\alpha_x = .048$ and $\alpha_i = .077$ under the same W calibration of the structural parameters. In that case the REE is again found to be both E–stable and determinate. As Figure 1 reveals, determinacy and E-stability obtain for all values of α_x so long as α_i is not too great. This same conclusion holds for the other two calibrations (CGG, MN) as seen in Figures 2–3. The range of (α_x, α_i) pairs for which E-stability obtains is greatest in the W calibration, and smallest in the MN calibration. However, there is always some (α_x, α_i) pair for which the REE is E–stable and determinate.

5.2 Numerical Findings Under Commitment

Figures 4-6 show comparable results for the case of optimal policy under commitment.[10] To assess E–stability, we again used a fine grid of values for the weight pairs ($\alpha_x,\ \alpha_i$), allowing each weight to vary over the range $[0, 2]$.

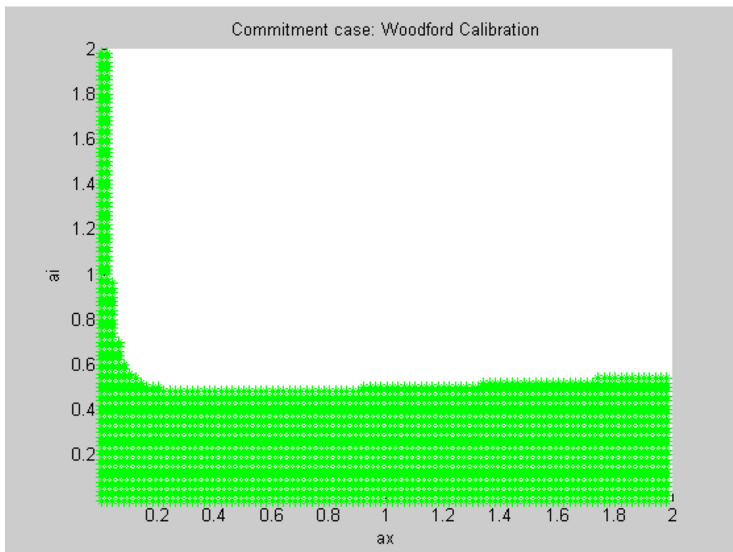

Figure 4: Commitment policy, Woodford calibration.

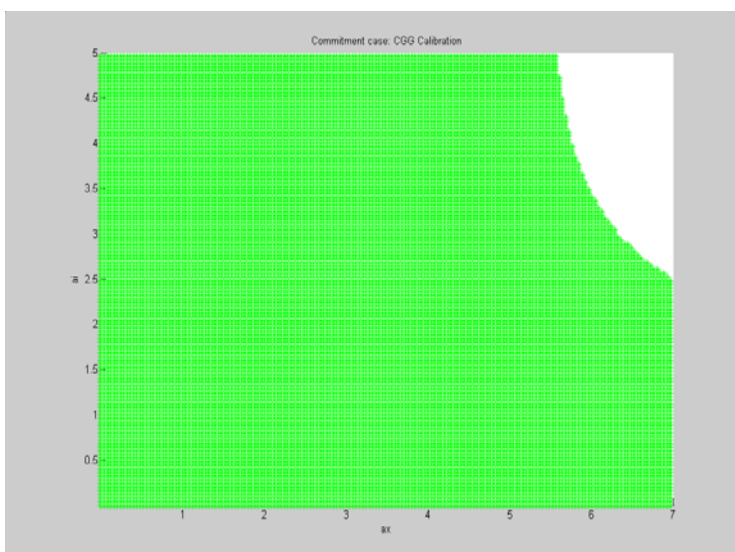

Figure 5: Commitment Policy, Clarida et al. calibration.

[10]Recall that for this case, we restricted attention to the unique, determinate, saddlepath stable REE since, in the commitment case, the conditions for E–stability and determinacy of equilibrium need not coincide.

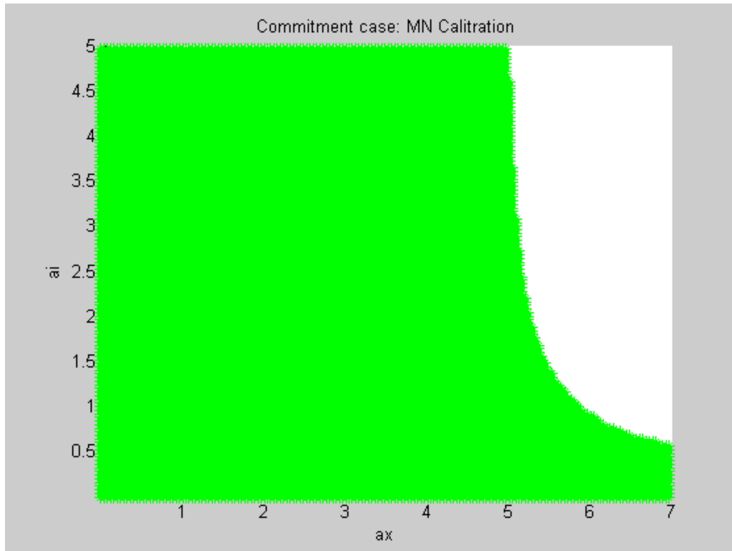

Figure 6: Commitment policy, McCallum-Nelson calibration.

However, for the Clarida et al. and McCallum-Nelson parameterizations, we had to extend the range of admissible weights so we can illustrate regions where E-stability fails to hold. In particular, for those two calibrations, we search for values of α_x over $[0, 7]$ and we search for values of α_i over $[0, 5]$, where again, the step size is .04. For each weight pair, we assess whether all the eigenvalues of the matrices DT_{d_j}, $j = 0$, y, w, v, have real parts less than unity so that the REE is E–stable; such regions are indicated by gray shading. Regions where one or more eigenvalue has a real part greater than unity are left blank, indicating regions where the REE is E-unstable.

Figures 4–6 confirm that in the commitment case, there always exist (α_x, α_i) pairs such that the REE is E–stable. Moreover, for the Clarida et al. and McCallum-Nelson calibrations, the set of policy weight pairs for which E–stability holds is much less restrictive under commitment than under discretion. For the Woodford (W) calibration, Figure 4 reveals that the weights proposed by Woodford (1999) $\alpha_x = .047$ and $\alpha_i = .233$, are consistent with an E–stable REE. This finding also holds for Woodford's (2003) alternative weights, $\alpha_x = .048$ and $\alpha_i = .077$.

However, like Figures 1–3, Figures 4–6 reveal that there exist $(\alpha_x,\ \alpha_i)$ pairs for which the REE is E-unstable; the precise regions depend on the calibration of the structural model parameters. These E–unstable regions do not appear to be empirically relevant, though the literature is not so clear about empirically plausible weight choices. Nevertheless, our finding is not a strong the one found in Evans and Honkapohja (2002); they show that an optimally derived, expectation-based interest rate rule that has the central bank condition on private sector expectations *always* implements an

E–stable REE, regardless of parameter values. Still, our finding that there exist parameter regions under which an optimal interest rate rule that does not condition on private sector expectations can nevertheless implement an E-stable REE is of interest, as Evans and Honkapohja are unable to show such a result using the two–element loss function objective.

6 Conclusions

We have show that if central bankers give some weight to interest rate stabilization, the resulting optimal policy rule implements a REE that *is* E-stable for several calibrations found in the literature under either discretionary or commitment policy regimes. This result obtains *without* the assumption that the private sector has rational expectations or that the central bank conditions its policy on private sector expectations, in contrast to the findings of Evans and Honkapohja (2002, 2003ab). While Evans and Hokapohja called the instability under learning of their optimal, fundamentals-based rule "deeplying worrying," our findings suggest that learnability of REE need not conflict with optimization of an appropriately defined policy objective function. Indeed, Evans and Honkapohja's proposed solution – the conditioning of policy on private sector expectations – is problematic, as such expectations are likely to be unobservable or heterogeneous and such conditioning might lead to gaming on the part of the private sector or indeterminacy of the REE.

Our finding differs from Evans and Honkapohja's because the optimal interest rate rule derived under the assumption that $\alpha_i > 0$ closely resembles Taylor–type, "instrument rules," where, the interest rate responds to contemporaneous and/or lagged values of the endogenous variables (inflation, the output gap, interest rates), and not to exogenous disturbance terms as in Evans and Honkapohja (2002, 2003ab). Bullard and Mitra (2002) have shown that, under certain conditions, Taylor-type instrument rules can implement E-stable REE, and it seems this finding carries over to optimally derived policy rules that resemble Taylor rules. We conclude that the value of interest rate stabilization as a central bank objective is that it may aid private sector learning of the rational expectations equilibrium relative to the case where this objective is absent.

References

Bernanke, Ben S., and Michael Woodford. (1997). "Inflation Forecasts and Monetary Policy." Journal of Money, Credit, and Banking 29, 653–684.

Berardi, Michele. (2004). "Should Monetary Policy Respond to Private Expectations?" working paper.

Bullard, James, and Kaushik Mitra. (2002). "Learning About Monetary Policy Rules." Journal of Monetary Economics 49, 1105-1129.

Bullard, James, and Kaushik Mitra. (2003). "Determinacy, Learnability, and Monetary Policy Inertia." Working paper No. 2000-030, Federal Reserve Bank of St. Louis.

Carlstrom, Charles T., and Timothy S. Fuerst. (2004). "Learning and the Central Bank." Journal of Monetary Economics 51, 327-338.

Clarida, Richard, Jordi Gali, and Mark Gertler. (1999). "The Science of Monetary Policy: A New Keynesian Perspective." Journal of Economic Literature 37, 1661–1707.

Duffy, John. (2003). "Comment on 'Adaptive Learning and Monetary Policy Design'." Journal of Money, Credit and Banking 35, 1073-1079.

Evans, George W., and Seppo Honkapohja. (2001). Learning and Expectations in Macroeconomics. Princeton: Princeton University Press.

Evans, George W., and Seppo Honkapohja. (2002). "Monetary Policy, Expectations and Commitment." working paper.

Evans, George W., and Seppo Honkapohja. (2003a). "Expectations and the Stability Problem for Optimal Monetary Policies." Review of Economic Studies 70, 807–824.

Evans, George W., and Seppo Honkapohja. (2003b). "Adaptive Learning and Monetary Policy Design." Journal of Money, Credit, and Banking 35, 1045-1072.

Giannoni, Marc P., and Michael Woodford. (2003). "How Forward-Looking is Optimal Monetary Policy?" Journal of Money, Credit, and Banking 35, 1425-1469.

Goodhart, Charles A.E. (1997). "Why Do the Monetary Authorities Smooth Interest Rates?" In European Monetary Policy, edited by S. Collignon, pp. 119-174. Washington, DC: Pinter.

Honkapohja, Seppo, and Kaushik Mitra. (2004). "Are Non-Fundamental Equilibria Learnable in Models of Monetary Policy?" Journal of Monetary Economics 51, 1743-1770.

Honkapohja, Seppo, and Kaushik Mitra. (2005). "Performance of Monetary Policy with Internal Central Bank Forecasting." Journal of Economic Dynamics and Control 29, 627–658.

Honkapohja, Seppo, Kaushik Mitra, and George W. Evans. (2003). "Notes on Agents' Behavioral Rules Under Adaptive Learning and Recent Studies of Monetary Policy." working paper.

McCallum, Bennett T., and Edward Nelson. (2000). "Timeless Perspective vs. Discretionary Monetary Policy in Forward–Looking Models." working paper.

Preston, Bruce. (2004). "Adaptive Learning and the Use of Forecasts in Monetary Policy." working paper.

Woodford, Michael. (1999). "Optimal Monetary Policy Inertia." working paper.

Woodford, Michael. (2003). Interest and Prices, Princeton: Princeton University Press.