Answers to Homework # 2

1. a) [1/2 point] The first mover, Player 1, is the buyer and the second mover, Player 2 is the seller. In a two round game, Player 1 knows that Player 2, will propose to keep the entire difference, M=$12,000, for herself in the second and final round. Since the period discount factor δ=.2, Player 1 knows he must give Player 2 $\delta\times$$12,000 in round 1 to make Player 2 indifferent between accepting in round 1 or waiting to make a counterproposal in round 2, Thus, Player 1 proposes in round 1 that Player 2 gets (.2)$12,000=$2,400, and Player 1 gets the remainder, $12,000-$2,400=$9,600 for himself. Player 2 agrees since there is no benefit to be gained by waiting. Since Player 1 is assumed to be the buyer, the equilibrium sale price for the home is $188,000+$2,400=$190,400.

   b) [1/2 point] There is no change in the answer to part a. Only the seller's (Player 2's) discount factor matters in the calculation for the two round game, and that value hasn't changed.

   c) [1 point] In the infinitely repeated game, we showed in class that if Player 1 (the buyer) moves first, he will propose to keep $[(1\text{-}d)/(1\text{-}d^2)]$M for himself with the remainder going to Player 2 (the seller). This proposal will be made in the first round and is acceptable to Player 2. With δ=.2 player 1 (the buyer) will propose to keep $[(1\text{-}.2)/(1\text{-}(.2)^2)]$ 12,000 = (5/6)(12,000) = $10,000 for himself with the remainder, $2,000 going to player 2 (the seller). So the equilibrium sale price in this case is $190,000 ($188,000+$2,000). Player 1 would prefer an infinite number of rounds to just a single round since he gets to keep slightly more of M ($400 more) for himself if he is the first mover.

2. a) [1 point] For Prisoner's Dilemma, you can use elimination of dominated strategies or best-response analysis. The only (unique) pure strategy Nash equilibrium is: Defect, Defect, with a payoff of 40,40.

   b) [1 point] In Stag Hunt, there are two pure strategy Nash equilibria which you can find by best response analysis. These are: Cooperate, Cooperate with a payoff of 70,70 and Defect, Defect with a payoff of 40, 40.

   c) [1 point] In Chicken, there are two pure strategy Nash equilibria which you can find by best response analysis. These are: Player 1: Cooperate, Player 2: Defect with a payoff of 50,80 and Player 1: Defect, Player 2: Cooperate with a payoff of 80, 50.

3. a) [1/2 point] Assigning a payoff of 4 to best, 3 to second best, 2 to third best and 1 to worst, the normal form of this game would be written as:

		Sam's Choice	
		Ball Game	Concert
Pat's Choice	Ball Game	4,3	3,2
	Concert	1,1	2,4

b) [1/2 point] The unique Nash equilibrium for this game, found via best response analysis, is for both players to choose the Ball Game. Pat's dominant strategy is to choose Ball Game, while Sam does not have a dominant strategy. But Sam, understanding Pat's strong preference for the game, will know to go to the ball game rather than the concert, even without being able to communicate with Pat.

4. a) [1 point] The payoff table for the Jack and Jill game should look like this:

		Jill's Choice					
		G	K	L	Q	R	W
Jack's Choice	G	**3,6**	0,0	0,0	0,0	0,0	0,0
	K	0,0	**2,5**	0,0	0,0	0,0	0,0
	L	0,0	0,0	**6,4**	0,0	0,0	0,0
	Q	0,0	0,0	0,0	**3,3**	0,0	0,0
	R	0,0	0,0	0,0	0,0	**4,2**	0,0
	W	0,0	0,0	0,0	0,0	0,0	**5,1**

The Nash equilibria in pure strategies are all choices that result in an outcome along the diagonal (6 total): (G,G), (K,K), (L,L), (Q,Q), (R,R), (W,W). These can be found using best response analysis.

b) [1/2 point] Focal is a subjective concept as we have observed in class; any of the six equilibria could be focal depending on the underlying common cultural understandings of the two players, though to achieve a focal equilibrium, their expectations would have to converge. One possible candidate is (Q,Q), because it is the only equilibrium that yields both players the *same* payoff (3), (equity considerations, envy-freeness). Another candidate might be (L,L), as it yields the highest total sum of payoffs (10) (efficiency). Yet another might be (G,G), e.g. in a society that emphasized the importance of being first, etc.

5. a) [1 point] In the Cournot duopoly game, each firm seeks to maximize profits. The profits for the two firms, 1 and 2, are given by:

$$\pi_1 = (p-c)q_1 = [a - b(q_1+q_2) - c]q_1,$$
$$\pi_2 = (p-c)q_2 = [a - b(q_1+q_2) - c]q_2.$$

Profit maximization by each firm involves taking the derivative of the profit function with respect to the quantity supplied by the firm and setting this derivative equal to zero to find the optimal, profit maximizing price. That is, we set

$$\partial\pi_1/\partial q_1 = 0 \Leftrightarrow a - 2bq_1 - bq_2 - c = 0,$$
$$\partial\pi_2/\partial q_2 = 0 \Leftrightarrow a - bq_1 - 2bq_2 - c = 0.$$

Rearranging we have the *best response functions* for each firm:

$$q_1 = \frac{a - bq_2 - c}{2b},$$
$$q_2 = \frac{a - bq_1 - c}{2b}.$$

These are two equations in two unknowns. Solving (by substituting one into the other, we get the desired result: $q_1 = q_2 = \frac{a-c}{3b}$.

Using these values for q, we can get the price, $p = a - b($ 2(a-c)/3b)= a-2/3a+2/3c = 1/3a+2/3c. Profits, were defined above as (p-c)q. Since the price and quantities are the same for both firms, so too will profits. Using the equilibrium price and quantities, we have:

$$\pi_1 = \pi_2 = \frac{(a/3 + 2c/3 - c)(a-c)}{3b} = \frac{(a-c)^2}{9b}.$$

b) [1.5 points] This is similar to part a, except the firms collude and maximize joint output. In this case, you can think of the firm quantities as production quotas for each firm.

i) The maximization problem is given in the homework question. The first order conditions for profit maximization in this case are:

$$\partial\pi/\partial q_1 = 0 \Leftrightarrow a - 2bq_1 - 2bq_2 - c = 0,$$
$$\partial\pi/\partial q_2 = 0 \Leftrightarrow a - 2bq_1 - 2bq_2 - c = 0.$$

These differ slightly from part a, because again, the firms are not acting independently as they were in part a.

ii) Solving these two equations, we find that:

$$q_1 = q_2 = \frac{a-c}{4b}.$$

The cartel price $p = a - b(2(a-c)/4b) = a - a/2 + c/2 = 1/2(a+c).$

The profit earned by each firm is (p-c)q =
$(1/2a + 1/2c - c)(a - c)/4b = (a - c)^2/8b$..

iii) Notice that the cartel quantity is lower than in the duopoly case, prices are higher as are firm profits. The reason is simple: collusion by the two firms allows them to act as a monopolist, and therefore they can restrict quantity and enjoy monopoly pricing and profits.