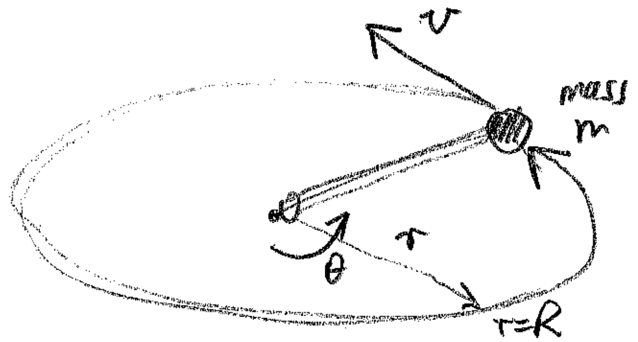
Bohr-Sommerfeld Quantization of a rotor



In familiar coordinates

Kinetic Energy $T = \frac{1}{2} m v^2 = \frac{1}{2} m r^2 \dot{\theta}^2$

Potential energy $= U = 0$

Lagrangian formulation

$\theta \longrightarrow q$

$L = T - U = \frac{1}{2} m R^2 \dot{q}^2 \qquad \bullet = d/dt$

Euler Lagrange equation

$$\frac{\partial L}{\partial q} - \frac{d}{dt}\frac{\partial L}{\partial \dot{q}} = 0 \Rightarrow 0 = mR^2 \ddot{q}$$

$\dot{q} = \text{constant} = \omega$

$q = q_0 + \omega t$

Canonical momentum

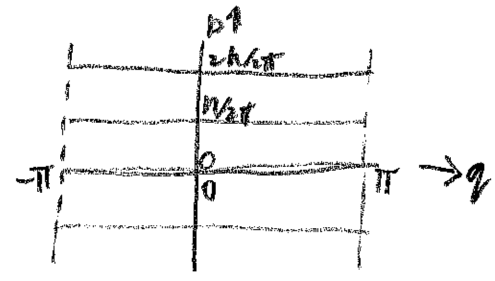
$$p = \frac{\partial L}{\partial \dot{q}} = mR^2 \dot{q} = mR^2 \omega = \text{angular momentum}$$

Quantization

$$nh = \oint p\, dq = \oint mR^2 \omega \, d\theta = mR^2 \omega [2\pi - 0]$$

Angular momentum $= mR^2 \omega = n h/2\pi$

Phase space diagram:



P.9 Silberstein's estimate of mass H nucleus from spectrum Helium

For Hydrogen

$$\frac{1}{\lambda} = N\left[\frac{1}{n'^2} - \frac{1}{n^2}\right]$$

$$N = \frac{2\pi^2 \overset{\text{electron}}{e^2}\, \overset{\text{nucleus}}{e^2}\, \overset{\text{mass electron}}{m}}{ch^3}$$

charge nucleus $e \to 2e$

For Helium

$$\frac{1}{\lambda} = \bar{N}\left[\frac{1}{n'^2} - \frac{1}{n^2}\right]$$

$$\bar{N} = \left[\frac{2\pi^2 e^2 (2e)^2 m}{ch^3}\right]$$

$$= N\left[\frac{1}{(n'/2)^2} - \frac{1}{(n/2)^2}\right]$$

Factor of 4 goes here

Same N

correct for mass nucleus M

$$N_H = \frac{1}{1 + m/M} N$$

correct for mass nucleus $= 4M$

$$N_{He} = \frac{1}{1 + m/4M} N$$

$$\frac{N_{He}}{N_H} = \frac{1 + m/M}{1 + m/4M} = \underbrace{\frac{48764}{48744.4}}_{\text{observed}}$$

solve for $\dfrac{m}{M} = \dfrac{1}{1865}$ which is close to $\dfrac{1}{1836}$ Silberstein reports