

Time Series Analysis

Intervention Models and Transfer Functions

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Outline

- 1 Intervention Analysis
- 2 Transfer Function Models

Intervention Analysis

- Intervention models allow analysts to test for a (permanent or temporary) change in the mean of a time series at the point when an intervention occurs.
- An intervention is an event (singular or sustained) that is theorized to have an impact on the time series we are modeling.
- If we are modeling skyjackings, an intervention might be the use of metal detectors in airport security.
- If we are modeling unemployment rates, an intervention might be a switch from a right to left-wing government (or the presence of a left-wing government).
- If we are modeling President Bush's approval ratings, an intervention might be the 9/11 terrorist attacks.

Intervention Analysis

- The ARMA(p,q) version of the model is

$$y_t = a_0 + A(L)y_{t-1} + C(L)I_{t-d} + B(L)\varepsilon_t$$

where $A(L) = (a_1L + a_2L^2 + \dots + a_pL^p)$,
 $B(L) = (1 + \beta_1L + \beta_2L^2 + \dots + \beta_qL^q)$, and
 $C(L) = (c_d + c_{d+1}L + c_{d+2}L^2 + \dots + c_{d+h}L^{d+h})$.

- Consider a simple example. If $p = 1$, $q = 0$, $d = 0$ and $h = 0$, then we have

$$y_t = a_0 + a_1y_{t-1} + c_0I_t + \varepsilon_t, |a_1| < 1$$

- Without an initial condition, the solution to this model is

$$y_t = a_0/(1 - a_1) + c_0 \sum_{i=0}^{\infty} a_1^i I_{t-i} + \sum_{i=0}^{\infty} a_1^i \varepsilon_{t-i}$$

Types of Intervention Functions

- Interventions can take several forms.

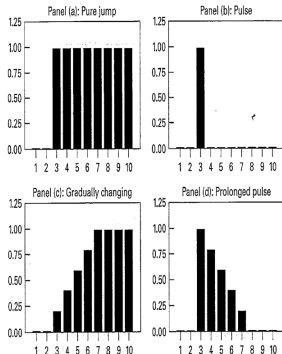


FIGURE 5.3 Typical Intervention Functions

- The step (pure jump) and pulse functions are the most commonly used.

Types of Intervention Functions

- In the case of a step function, which represents a sustained intervention, the indicator variable I_t takes a value of zero prior to the intervention and one at the time of the intervention and afterwards.
- In the case of a pulse function, which represents a singular event, the indicator variable I_t takes a value of zero in all periods except the one in which the intervention occurs.
- Ultimately, we are interested in the effect of the intervention over time. With our simple, AR(1), intervention model, the *impulse response function* for a sustained intervention is

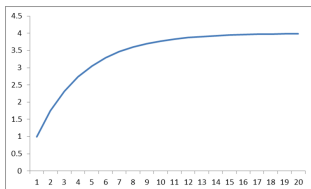
$$dy_{t+j}/dI_t = c_0 [1 + a_1 + \dots + (a_1)^j]$$

The *impulse response function* for a pulse is

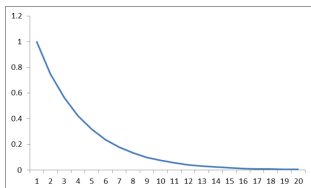
$$dy_{t+j}/dI_t = c_0(a_1)^j$$

Impulse Response Functions

IRF: Sustained Intervention, $a_1 = .75$ & $c_0 = 1$.



IRF: Pulse Intervention, $a_1 = .75$ & $c_0 = 1$.



Estimating Intervention Models

- 1 Use the longest data span, either the pre or post-intervention period, to identify a set of plausible ARMA models.
- 2 Estimate the set of models over the entire sample including the intervention.
- 3 Perform diagnostics on the estimated models.

Selecting an Intervention Model

- 1 The coefficient estimates should be statistically significant and imply a convergent process.
- 2 The residuals should be white noise.
- 3 The model should outperform the alternatives in terms of fit and forecasting.

Transfer Function Models

- The transfer function model is a generalization of the intervention model.
- We replace the deterministic dummy variable $\{I_t\}$ with $\{z_t\}$, which can take other forms such as white-noise and ARMA processes.
- The ARMA(p,q) version of the transfer function model is

$$y_t = a_0 + A(L)y_{t-1} + C(L)z_{t-d} + B(L)\varepsilon_t$$

Identifying Transfer Function Models

- If $\{z_t\}$ is a white-noise process, we rely on the cross-correlations between $\{y_t\}$ and $\{z_{t-i}\}$ to identify the model.
- This cross-correlation is defined as

$$\rho_{yz}(i) \equiv \text{cov}(y_t z_{t-i}) / (\sigma_y \sigma_z)$$

- The standardized cross-covariance is defined as

$$\gamma_{yz}(i) \equiv \text{cov}(y_t z_{t-i}) / (\sigma_z^2)$$

- Plotting the cross-correlations and standardized cross-covariances give the cross-correlation function (CCF) and standardized cross-covariance function (CCVF) respectively.

Identifying Transfer Function Models

The theoretical CCF and CCVF have the following characteristics

- 1 All the $\rho_{yz}(i)$ and $\gamma_{yz}(i)$ will be zero until the first nonzero element of the polynomial $C(L)$.
- 2 The form of $B(L)$ does not affect the theoretical CCF and CCVF.
- 3 Spikes in the CCF and CCVF indicate a nonzero element of $C(L)$. A spike at lag d indicates that z_{t-d} directly affects y_t .
- 4 The nature in which spikes decay reveals information about $A(L)$. We read these patterns in the same way we read the ACF for an ARMA model.

Identifying Transfer Function Models

Some examples of theoretical CCVFs

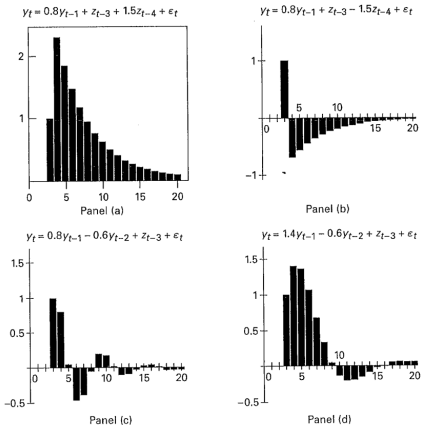


FIGURE 5.4 Standardized Cross-Correlograms

Identifying Models with Higher-Order Input Processes

- If $\{z_t\}$ is an ARMA(p,q), we have $D(L)z_t = E(L)\varepsilon_{zt}$
- Substituting into $y_t = a_0 + A(L)y_{t-1} + C(L)z_{t-d} + B(L)\varepsilon_t$ allows us to trace the effects of a one-unit shock to ε_{zt} on y_t .
- We start by estimating the parameters of the polynomials $D(L)$ and $E(L)$.
- We use the cross-correlations between $\{y_t\}$ filtered by $D(L)/E(L)$ and $\hat{\varepsilon}_{zt}$ to identify the forms of $A(L)$ and $C(L)$.
- Estimate $[1 - A(L)L]y_t = C(L)z_t + e_t$, and then use e_t to find the best model for $B(L)\varepsilon_t$
- Estimate $A(L)$, $B(L)$ and $C(L)$ simultaneously.