For the first stochastic difference equation

$$y_t = 3 + 0.75y_{t-1} - 0.125y_{t-2} + \varepsilon_t,$$

the particular solution for the deterministic process is

$$\begin{aligned} c &= 3/(1 - .75 + .125) \\ &= 8 \end{aligned}$$

To find the particular solution for the stochastic process, posit a linear *challenge solution*

$$y_t = \sum_{i=0}^{\infty} \alpha_i \varepsilon_{t-i},$$

and then substitute the challenge solution into the difference equation

$$\alpha_0\varepsilon_t + \alpha_1\varepsilon_{t-1} + \alpha_2\varepsilon_{t-2} + ... = .75[\alpha_0\varepsilon_{t-1} + \alpha_1\varepsilon_{t-2} + \alpha_2\varepsilon_{t-3} + ...] - .125[\alpha_0\varepsilon_{t-2} + \alpha_1\varepsilon_{t-3} + \alpha_2\varepsilon_{t-4} + ...] + \varepsilon_t$$

Collect like terms

$$(\alpha_0 - 1)\varepsilon_t + (\alpha_1 - .75\alpha_0)\varepsilon_{t-1} + (\alpha_2 - .75\alpha_1 + .125\alpha_0)\varepsilon_{t-2} + ... = 0$$

Verify that there are coefficient values that make the challenge solution a solution for the difference equation.

$$\begin{gathered} (\alpha_0 - 1) = 0 \\ (\alpha_1 - .75\alpha_0) = 0 \\ (\alpha_2 - .75\alpha_1 + .125\alpha_0) = 0 \\ \vdots \end{gathered}$$

Solving for α_i, we have $\alpha_i = .75\alpha_{i-1} - .125\alpha_{i-2}$. To reduce this further, we need to solve for the characteristic roots.

The homogeneous solutions will take the form $y_t^h = A\alpha^t$. Start by substituting for y_t

$$A\alpha^t - .75A\alpha^{t-1} + .125A\alpha^{t-2} = 0.$$

Divide by $A\alpha^{t-2}$

$$\alpha^2 - .75\alpha + .125 = 0.$$

There are two solutions. We solve for α_1 and α_2 using the quadratic formula

$$\alpha_1, \alpha_2 = \frac{.75 \pm \sqrt{.5625 - 4(.125)}}{2} = .5, .25$$

So, now we have

$$y_t = A_1(.5)^t + A_2(.25)^t.$$

We can eliminate the arbitrary constants if we know the outcome in the initial periods y_0 and y_1. This gives us two equations and two unknowns

$$y_0 = 8 + A_1 + A_2$$
$$y_1 = 8 + A_1(.5) + A_2(.25)$$

Solving gives us $A_1 = 4y_1 - y_0 - 24$ and $A_2 = 16 - 4y_1 + 2y_0$.
The particular solution for the stochastic process can be written compactly as

$$\sum_{i=0}^{\infty} \left(2(.5)^i - (.25)^i\right) \varepsilon_{t-i}$$

Putting it all together gives

$$y_t = 8 + (.5)^t \left[4y_1 - y_0 - 24\right] + (.25)^t \left[2y_0 - 4y_1 + 16\right] + \sum_{i=0}^{\infty} \left(2(.5)^i - (.25)^i\right) \varepsilon_{t-i}.$$

If we assume the process starts in equilibrium (i.e., $y_0 = y_1 = 8$), the solution simplifies to

$$y_t = 8 + \sum_{i=0}^{\infty} \left(2(.5)^i - (.25)^i\right) \varepsilon_{t-i}.$$