Physics 0081 Fall 1999 (00-1)
Handout #14

Waves of Probability

Physics theories from the atomic and molecular level and down to ever smaller scales are described in the language of quantum mechanics. This is true as well for the theories describing elementary particles and their interactions. Today great effort is bing made to include gravitation under the quantum umbrella.

We discussed some quantum phenomena, such as the appearance of discrete quantized energy levels in oscillators and atoms. We discussed the deBroglie wavelength of a particle, but we have not fully discussed the quantum description of nature. Here goes a thumbnail sketch.

- With every particle there is associated a deBroglie wave described by a wave function ψ. If the particle has definite momentum, $P = mV$ the deBroglie wave, as we already discussed, has wavelength $\lambda = h/P$, where h is Planck's constant. The development of this subject is called wave mechanics, which is the basis for modern quantum mechanics.

- We already discussed that macroscopic objects have an undetectably small deBroglie wavelength (because the momentum is so large "compared" to Planck's constant).

- The probability that the particle described by ψ will be found at a particular spatial point at a particular time is proportional to the value of $|\psi|^2$ at that particular point in space and time. A visualization of the value of $|\psi|^2$ appears in many chemistry and physics books to show "probability clouds" for electrons in atomic or molecular orbitals. The electron is more likely to be found where the cloud is dense and vice versa. The dependence ψ on position and time is governed by the Schrödinger equation – analogous to the way Maxwell's equations govern the propagation of electromagnetic waves.

- For a matter wave (or particle wave) to have a definite wavelength λ there must be many oscillations of the wave in space, so the uncertainty of the position of the particle, Δx, will be relatively large. This leads to Heisenberg's Uncertainty Principle: the product of the uncertainty in position and the uncertainty in the particle's momentum must be at least h. In other words $\Delta x \Delta P > h$. So if the momentum is certain (i.e., ΔP is very small), the position must be very uncertain, and vice versa. Such limitations arise from the wave description. They have consequence that it is impossible to observe something without disturbing it.

- The wave nature gives rise to phenomena strange to classical (ordinary Newtonian, Galilean) mechanics. In classical physics a particle can't get over a hill unless it has enough energy to get over it. In quantum mechanics, picturesquely speaking, a praticle has some probability to "tunnel" through the hill. The famous α-decays, which provided Rutherford his α particles, arise because the particles "tunnel" their way out of a heavy nucleus.

- Along with Heisenberg's uncertainty principle goes one with uncertainty in energy and time, namely $\Delta E \Delta t > h$. This means that if the energy of an atomic state is very precise (ΔE is very small), it's lifetime Δt will be very long (and vice versa). Same with elementary particles with finite lifetime. Using these sorts of arguments Yukawa was able to predict some properties of the (yet to be discovered) mesons.

- The quantum description of the atomic and subatomic world forces a probabilistic description of physical processes. There are still serious discussions about the interpretation of quantum mechanics, but as a practical tool it has been extraordinarily successful in describing atomic and sub-atomic physics (e.g., atomic processes, chemical bonding, energy levels of nuclei, and so on).

Exercises:

1. Suppose one measures the momentum of an electron (A) with some error and then measures the momentum of another electron (B) with a factor of two smaller error How are the smallest allowed errors in the positions of A and B related? (a) Position error for A is 2 times smaller than position error for B (b) The position errors for A and B are the same. (c) Position error for B is 2 times smaller than position error for A (d) The smallest allowed position errors and momentum errors are unrelated. (e) The position error for A is either 4 times smaller or 4 times bigger than for B.

2. The Heisenberg uncertainty principle allows all of the following measurements EXCEPT: (a) Exact measurement of momentum pf a particle (b) Exact measurement of position of a particle (c) Exact, simultaneous measurements of the position and momentum of a particle. (d) Exact measurement of energy.

3. According to Heisenberg's uncertainty principle, the longer a particle's lifetime (a) the better defined is its mass (b) the larger a particle's mass (c) the smaller a particles mass (d) the lifetime and the certainty in the mass are unrelated.