Analyzing Dimension-Adaptive Sparse Grid Stochastic Collocation Techniques for Partial Differential Equations with High-Dimensional Random Input Data

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ABSTRACT

This talk will propose and analyze a dimension-adaptive (anisotropic) sparse grid stochastic collocation method for solving partial differential equations with random coefficients and forcing terms (input data of the model). These methods have proven to have dramatic impact on several application areas, including statistical mechanics, financial mathematics, bioinformatics, and other fields that must properly predict certain model behaviors. The method consists of a Galerkin approximation in the space variables and a collocation, in probability space, on anisotropic sparse tensor product grids utilizing either Clenshaw-Curtis or Gaussian knots. Even in the presence of nonlinearities, the collocation approach leads to the solution of uncoupled deterministic problems, just as in sampling-based methods, such as Monte Carlo. This talk includes both *a priori* and *a posteriori* approaches to adapt the anisotropy of the sparse grids to each given problem.

This talk will also provide a rigorous convergence analysis of the fully discrete problem and demonstrate strong error estimates for the solution using L^q norms. In particular, our analysis reveals at least an algebraic convergence with respect to the total number of collocation points. The derived estimates are then used to compare the efficiency of the method with other ensemble-based methods. Numerical examples illustrate the theoretical results and are used to compare this approach with several others, including the standard Monte Carlo. In particular, for moderately large dimensional problems, the sparse grid approach with a properly chosen anisotropy is very efficient and superior to all examined methods.