

COLLOQUIUM
UNIVERSITY OF PITTSBURGH
FRIDAY, SEPTEMBER 07, 2007
704 THACKERAY HALL
4:00 P.M.

SPEAKER: PROFESSOR TIMOTHY HEALEY

**DEPARTMENT OF THEORETICAL & APPLIED MECHANICS
FIELDS OF MATHEMATICS & APPLIED MATHEMATICS
CORNELL UNIVERSITY**

**TITLE: SOME TWO-PHASE PROBLEMS OF
NONLINEAR ELASTICITY VIA BIFURCATION METHODS**

ABSTRACT: Experiments on real materials often involve the careful observation of developing micro-structure or pattern formation on nominally homogeneous specimens subject to slowly changing loadings. Bifurcation analysis is an ideal tool for the study of such problems. We consider two such examples here from nonlinear elasticity, motivated by ample experiments: (1) shear-induced twinning in shape-memory solids; (2) pressure driven non-spherical deformations of closed fluid shells or lipid-bilayer vesicles.

In (1) we consider a 2-phase anti-plane shear model, which provides the simplest 2-dimensional setting for such problems. Here our main contribution is in our approach to finding stable equilibria corresponding to local energy minima. A popular method for such problems is the inference of micro-structure via minimizing sequences of the free energy in the absence of interfacial energy. In spite of its success, that approach allows for infinite refinement of phase mixtures, typically violates equilibrium conditions, ignores local energy minima and is not readily generalized to problems under external loading. In contrast we propose the use of global bifurcation methods, in the presence of small interfacial energy, to determine paths of stable equilibria in a rather unorthodox manner. The unorthodoxy here refers to the fact that we treat the capillarity coefficient of the interfacial energy, as well as the usual loading parameter, as a control variable. We obtain a-priori bounds and perform a global bifurcation analysis, deducing the existence of equilibria corresponding to arbitrarily small capillarity. We then compute such solutions to find branches of locally stable equilibria, exhibiting phase nucleation (and anti-nucleation) and fine layering of phases - with definitive scales - all in qualitative agreement with experiment.

In (2) we consider the equilibrium of 2-phase fluid spherical shells under the influence of osmotic pressure. Here our results are less complete than those of (1), and our main contribution is one of modeling. In particular, we present a van der Waals type model for the membrane, accounting also for an experimentally observed change in thickness, over the phase transition, in a novel way. We perform a rigorous nonlinear bifurcation analysis, using well-known group-theoretic strategies, showing the existence of bifurcated non-spherical equilibria. We find such states, provided that the change in bending stiffness over the spinodal regime is sufficiently steep. The bifurcating equilibria represent phase-nucleated states from the homogeneous membrane, and bear striking similarity to many of the non-spherical patterns observed in experiments.

**Refreshments served at 3:30 p.m.
in the Math Dept. COMMON ROOM, Thackeray 705**