Print your first and last name legibly above the line:

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Calculus III

Professor Piotr Hajłasz

First Exam

October 7, 2013.

Problem	Possible points	Score
1	20	
2	20	
3	20	
4	20	
5	20+10	
Total	110	

The total score is 100. The problem 5 is the most difficult one, so for a complete solution without any mistakes you will get an extra bonus of 10 points. However, do not get stuck on that problem. Solve the problems 1-4 first.

Problem 1. (20p=4×5p)

(a) Find an equation of the plane passing through the points $A(1, 2, 3)$, $B(0, 1, 3)$ and $C(1, 1, 1)$.

(b) What is the area of the triangle ΔABC with the vertices A, B, C as in (a)?

(c) For what value of the parameter a are the planes $3x + 6y + 2z = 7$ and $x + 2y + az = 24$ perpendicular?

(d) Find parametric equations of the line through $(1, 0, 6)$ that is perpendicular to the plane $x + 3y + z = 5$.

Problem 2. (20p=2×10p)

(a) Find the tangent plane to the surface $x^3y^7 + \sin(\pi x)y - xyz = 0$ at $(1, 1, 1)$.

(b) Let $\psi(x, y, z) = \sin(x) + \cos(y) + 2z$. Find the direction in which the function ψ **decreases** most rapidly at the point $(0, \pi, 1)$.

Problem 3. (20p) Show that the function $u(x, y, z) = (x^2 + y^2 + z^2)^{-1/2}$ is harmonic, i.e. $u_{xx} + u_{yy} + u_{zz} = 0$.

Problem 4. (20p) Find the absolute maximum and minimum values of $f(x, y) = xy - x - 2y + 1$ on the closed triangular region with vertices $(0, 0)$, $(1, 0)$ and $(0, 1)$.

Continuation of the solution of Problem 4.

Problem 5. (20p+10p) Use the Lagrange multipliers to find the point on the sphere $x^2+y^2+z^2 = 4$ that is closest to the point $(3, 1, -1)$. (You **must** use the Lagrange multipliers; other solutions will not be accepted.)

Continuation of the solution of Problem 5.