Print your first and last name legibly above the line:

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Calculus III

Professor Piotr Hajłasz

Second Exam

November 8, 2013.

Problem	Possible points	Score
1	20	
2	20+10	
3	20	
4	20	
5	20	
Total	110	

Part (b) of Problem 2 is an extra bonus for 10 points. If you get 100 points it is 100%, so with the extra bonus you can get up to 110% from the exam.

Problem 1.

(a) Write $\iint_D f(x,y)\,dA$ as an iterated integral in **polar coordinates**, where D is the part of the unit disc in the first quadrant

$$D = \{(x,y) : x^2 + y^2 \le 1,\ x \ge 0,\ y \ge 0\}.$$

(b) Evaluate the integral

$$\int_0^2 \int_0^{\sqrt{2x-x^2}} y\sqrt{x^2+y^2}\, dydx$$

by converting it to polar coordinates. (In order to compute the integral it might be helpful to remember that $((\cos\theta)^5)' = -5(\cos\theta)^4 \sin\theta$.)

Problem 2. (20 for (a) + 10 for (b))

Rewrite the integral $\int_{-1}^{1}\int_{x^2}^{1}\int_{0}^{1-y} f(x,y,z)\,dz\,dy\,dx$ as

(a)

$$\int_{?}^{?}\int_{?}^{?}\int_{?}^{?} f(x,y,z)\,dx\,dy\,dz$$

(b)

$$\int_{?}^{?}\int_{?}^{?}\int_{?}^{?} f(x,y,z)\,dy\,dz\,dx$$

Problem 3. Evaluate the integral

$$\int_{-1}^{1} \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \int_{-\sqrt{1-x^2-y^2}}^{\sqrt{1-x^2-y^2}} e^{(x^2+y^2+z^2)^{3/2}} \, dz \, dy \, dx$$

by converting it to spherical coordinates.

Problem 4.

(a) Let $\mathbf{F} = \langle y^2e^z, 2xye^z + 1, 2z + xy^2e^z \rangle$. Find a function $f(x, y, z)$ such that $\nabla f = \mathbf{F}$.

(b) Use part (a) to evaluate the integral $\int_C \mathbf{F} \cdot d\mathbf{r}$ where $\mathbf{F}$ is as in part (a) and C has parametrization $\mathbf{r}(t) = \langle te^{\sin t}, te^{1+\cos t}, t(t-\pi) \rangle$, $0 \leq t \leq \pi$.

Problem 5.

(a) State the Green theorem.

(b) Evaluate the integral $\int_C (-y + \sin x)\, dx + (x + e^y)\, dy$ where the closed curve C is the triangle with vertices $(0, 0)$, $(1, 0)$ and $(1, 1)$ oriented **clockwise**.