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Calculus III

Professor Piotr Hajłasz

First Exam

October 10, 2016.

Problem	Possible points	Score
1	20	
2	10	
3	10	
4	20	
5	10	
6	10	
7	20	
Total	100	

Problem 1. (20p)

(a) For what values of a are the planes $x + 2y = 3 - z$ and $x + ay - 2z = 5$ orthogonal?

(b) Find the equation of a plane passing through the point $(1, 2, 3)$ that is parallel to the plane $x = 2y + 3z$.

(c) Find the equation of a plane passing through the points $A(1, 1, 1)$, $B(0, 2, 5)$, $C(-1, 0, 1)$

(d) Find the angle between a diagonal of a cube and one of its edges.

Problem 2. (10p) Find the limit

$$\lim_{(x,y) \rightarrow (0,0)} \frac{e^{-x^2-y^2} - 1}{x^2 + y^2}.$$

Problem 3. (10p)

(a) Find a parametrization of the curve of intersection of the surfaces $x^2 + y^2 = 1$ and $z = 4x^2$

(b) Find parametric equations of the tangent line to the curve in (a) at the point $(\sqrt{2}/2, \sqrt{2}/2, 2)$.

Problem 4. (20p) (a) Calculate $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ at $(1, 1, 1)$, where $x^2 + y^2 - 2z^2 + 12x - 8z - 4 = 0$.

(b) Find the equation of the tangent plane to the surface $x^2 + y^2 - 2z^2 + 12x - 8z - 4 = 0$ at $(1, 1, 1)$.

Problem 5. (10p) Find the curvature of $\mathbf{r}(t) = \langle \sin t, 1 - \cos t, t^2 \rangle$ at $t = 2\pi$, i.e. find $\kappa(2\pi)$.

Problem 6. (10p) Find the critical points of $f(x, y) = (x^2 + y^2)e^{-x}$ and analyze them using the Second Derivative Test.

Problem 7. (20p) Find the absolute maximum and minimum of the function $f(x, y) = x^2 + y^2 - 8y + 3$ on the disc $x^2 + y^2 \leq 9$.