Print your first and last name legibly above the line:

Calculus III

Professor Piotr Hajłasz

First Exam

October 12, 2015.

Problem	Possible points	Score
1	20	
2	20	
3	20	
4	20	
5	20	
Total	100	

Problem 1. (20p=4×5p)
(a) For what values of a are the vectors $\langle a-1, 2\rangle$ and $\langle a-4, 1\rangle$ orthogonal?

(b) Find the angle between the planes $x+2=y-z$ and $2x-y=z$

(c) Find the equation of a plane passing through the points $A(1, 1, 1)$, $B(2, 2, 2)$, $C(1, 2, 3)$

(d) Find the area of the triangle with vertices $A(1, 1, 1)$, $B(2, 2, 2)$, $C(1, 2, 3)$.

Problem 2.

(a) Find the length of the curve $\mathbf{r}(x) = \langle x, f(x) \rangle$, $a \leq x \leq b$, where f is a given function.

(b) Show that the limit $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2 - y^2}{x^2 + y^2}$ does not exist.

Problem 3. (20p=2×10p)
(a) Find the equation of the tangent plane to the surface $x^2 + y^2 + z^2 - 8x - 6y - 8z + 24 = 0$ at the point $(1, 1, 2)$.

(b) Classify the surface $x^2 + y^2 + z^2 - 8x - 6y - 8z + 24 = 0$ (i.e. is it ellipsoid, paraboloid, cylinder,...?)

Problem 4. (20p) Find the maximum and minimum values of $f(x, y) = (x - 1)^2 + (y - 2)^2$ on the disc $x^2 + y^2 \leq 45$.

Problem 5. (20p=2×10p) Using the method of Lagrange multipliers find the distance of the point $(17, -4, -3)$ to the plane $6x - 3y + 2z = 10$.