Print your first and last name legibly above the line:

Calculus III

Professor Piotr Hajłasz

First Exam

February 18, 2015.

Problem	Possible points	Score
1	20	
2	20	
3	20	
4	20	
5	20	
Total	100	

Problem 1. (20p=4×5p)

(a) Find the equation of the plane that passes through the points $P(1,1,1)$, $Q(1,2,3)$, $R(3,2,1)$.

(b) Find the area of the triangle with the vertices $P(1,1,1)$, $Q(1,2,3)$, $R(3,2,1)$.

(c) For what values of the parameter a are the planes $x+ay+2z=2015$ and $3x+(a+2)y+6z=1410$ parallel?

(d) Find the equations of the line of intersection of the planes $x+y+z=1$ and $x-2y+3z=1$.

Problem 2. (20p=2×10p)

(a) Show that the limit $\lim_{(x,y)\to(0,0)} \dfrac{\sin(xy)+y^2}{x^2+y^2}$ does not exist.

(b) Show that the limit exists and find it. $\lim_{(x,y)\to(0,0)} \dfrac{e^{xy}(x^2+y^2)+y^2\sin(xy^2)}{2(x^2+y^2)}$.

Problem 3. (20p=2×10p)
(a) Find the curvature of the helix $\mathbf{r}(t) = \langle 3\cos t, 3\sin t, 4t \rangle$.

(b) Find the normal and the binormal vectors to the helix $\mathbf{r}(t) = \langle 3\cos t, 3\sin t, 4t \rangle$.

Problem 4. (20p=2×10p)

(a) Find ∇z where z is a function of variables x and y implicitly defined by the equation

$$xyz = 1 + \pi^3 + \cos(x + y + z).$$

(b) Find the equation of the tangent plane to the surface $xyz = 1 + \pi^3 + \cos(x + y + z)$ at the point (π, π, π). **Hint:** *You can use the results of the part (a), but you do not have to.*

Problem 5. (20p) Find the absolute maximum and minimum values of

$$x^3 - y^3 + 6xy, \quad 0 \leq x \leq 1, \quad 0 \leq y \leq x.$$