

How to measure volume with a thread, erratum

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Our note *How to measure volume with a thread* (American Mathematical Monthly, February 2005) has appeared with an error. Namely, in the long process of corrections that had been suggested by the editors in order to make the text as smooth and as English as possible, the statement of the Homeomorphic Measures Theorem was changed from a correct one to an incorrect one. The authors have overlooked this while reading the proofs of the final version.

Here is the correct statement:

Homeomorphic Measures Theorem (von Neumann, Oxtoby, Ulam). *Let μ be a Borel measure on Q^n such that $\mu(Q^n) = 1$, $\mu(\{x\}) = 0$ for all points $x \in Q^n$, $\mu(\partial Q^n) = 0$, and $\mu(U) > 0$ for all nonempty open $U \subset Q^n$. Then, there exists a homeomorphism $h : Q^n \rightarrow Q^n$ such that $\mathcal{L}^n(A) = \mu(h(A))$ for every Borel set $A \subset Q^n$. Moreover, $h|_{\partial Q^n} = \text{id}$.*

Obviously, there are other homeomorphisms \tilde{h} of the unit cube Q^n which satisfy the condition $\mathcal{L}^n(A) = \mu(\tilde{h}(A))$ but are not equal to the identity on ∂Q^n . For example, one can compose the h from HMT with a reflection w.r.t. to the hyperplane $\{x_i = 1/2\}$.

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