

Bifurcation of periodic solutions in nonlinear evolution problems with periodic forcing

This lecture will address the bifurcation of 2π -periodic solutions in nonautonomous semilinear PDEs of evolution type with 2π -periodic forcing. For expository purposes, we shall center the discussion on the “model” problem

$$\begin{cases} u_t = \Delta_r u + f(\lambda, t, u), \\ u = 0 \text{ on } \mathbb{R} \times \partial\Omega, \end{cases}$$

where Ω is a bounded open subset of \mathbb{R}^N , λ is a real parameter, Δ_r is the r -dimensional Laplace operator and $f : \mathbb{R}^{r+2} \rightarrow \mathbb{R}^r$ is a given function such that $f(\lambda, t, \xi)$ is 2π -periodic in t for all $\lambda, \xi \in \mathbb{R}$ and $f(\lambda, t, 0) = 0$. Thus, $u = 0$ is a 2π -periodic solution for all λ and bifurcation will be investigated from this trivial branch. The lecture will identify the key steps and important concepts without going into minute technical arguments.