TECHNICAL APPENDIX

ANALYTICAL FOUNDATIONS OF FORECAST HORIZON PROPOSITION
& STATIC AND DYNAMIC ORGANIZATIONAL STABILITY COROLLARIES

Explaining Bureaucratic Optimism: Theory and Evidence
from U.S. Executive Agency Macroeconomic Forecasts

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Our substantive problem involves analyzing the theoretical relationship between an agency’s reputation costs associated with bureaucratic competence and its level of forecast optimism.¹ This problem can be viewed as a modified application of a general autonomous terminal control problem originally advanced by Halkin (1974) (see also, Chiang 2000: 244–245).² In our setup of this infinite horizon dynamic control problem, we posit that a public

¹ We operationally define the concept of forecast optimism as the forecast error. That is, positive (negative) forecast errors represent greater forecast optimism (pessimism) than warranted by objective conditions (see Note 1: Manuscript). In the case of the organizational stability corollaries (and corresponding hypotheses H2 & H3), this is numerically equivalent to analyzing the forecasts by definition since the observed values for a given period cancel out across agencies (see also, Note 8: Manuscript, Note 9: Supplemental Technical Appendix).

² We adopt a decision–theoretic analytical modeling perspective rather than a game–theoretic approach for both substantive and practical reasons. The primary focus and contribution of this study is on intertemporal discounting in a dynamic setting over several periods. Incorporating these features into a game–theoretic model makes arriving at an analytically tractable model extremely difficult, if not infeasible (e.g., see Carpenter 2004: 625 who makes this very same point with respect to his theoretical analysis of regulatory review). This particular difficulty also manifests itself in sophisticated game–theoretic economic models of forecasting decisions which are, by definition, inherently static in nature, and thereby possess neither intertemporal discounting nor decision making by agents (Ehrbeck and Waldmann 1996; Graham 1999; Laster, Bennett, and Geoum 1999), as well as theoretical models of intertemporal choice which are not premised on strategic behavior involving two or more distinct players.
agency produces a level of forecast optimism that is determined by the rate at which bureau reputation costs incurred from these forecasts, in relation to (future) policy outcomes, are exponentially discounted in present value terms.\textsuperscript{3} The purpose of this modeling exercise is to merely demonstrate the analytical relationship between forecast horizon length, intertemporal discounting of a bureau’s reputation costs, and its implications for the level of agency forecast optimism.

In our analytical model, we posit that a public agency has an incentive to commit the most (relatively) optimistic forecasts possible if they do not experience any reputation costs associated with these decisions – i.e., they will not be operating under a reputational constraint. If we allow for the possibility that a public agency incurs nonzero reputation costs resulting from its forecasting decisions, then its decision problem can be characterized as the following constrained dynamic optimization problem\textsuperscript{4}:

\begin{itemize}
  \item \textsuperscript{3} We utilize exponential discounting since it is standard in theoretical models of intertemporal choice because of its intuitiveness and analytical ease. In addition, this method of discounting reputation costs is logically compatible with time-consistent rational behavior (e.g., see Laibson 1997; Loewenstein and Prelec 1992).
  \item \textsuperscript{4} The technical exposition of our model closely follows from Chiang (2000: 244–245).
\end{itemize}
Max \[ \int_0^\infty (1 - \overline{y}) \, c \, dt \]

Subject to \[ \frac{d\overline{y}}{dt} = (1 - \overline{y}) \, c; \quad \overline{y}(0) = 0; \quad \text{and} \quad c(t) \in [0, 1]^* \]  

where \( \overline{y}(t) \) is the agency’s choice variable concerning the level of forecast optimism for a given forecast horizon (t) and its value is assumed to lie in the interval [0, 1] with higher values representing greater forecast optimism reflected by its forecast errors (i.e., expectations superceding realizations), \( c(t) \) is the intertemporal discount factor corresponding to bureau reputation costs confronting the public agency when making a forecast concerning a future state or condition. For our purposes, higher (lower) values of \( \overline{y} \) imply more optimistic (pessimistic) forecast errors; whereas, higher (lower) values of \( c \) are indicative of a higher (lower) discount rate associated with lowering (raising) bureau reputation costs corresponding to a higher degree of forecast optimism. In other words, the \( c(t) \) term can be intuitively thought of as a bureaucratic reputation risk weight that an agency will assign to making optimistic forecasts. Higher values of \( c \) imply a greater willingness by agencies to respond to politicians demands for generating optimistic policy information, while lower values of \( c \) imply that public agencies are more averse about incurring a reputational loss pertaining to policy expertise and competence. Hence, this risk weight will determine the agency’s level of forecast optimism, \( \overline{y} \), for a particular forecast horizon, \( \overline{y}(t) \), ceteris paribus. The agency’s objective function expressed in

\footnote{Without loss of generality, the [0, 1] level of forecast optimism interval is an arbitrary interval that can allow for a scalar based transformation of empirically observed (data) values in order to account for any cases which might actually lie outside this range.}
(SA–1) can be solved as follows:

\[ \int_0^\infty \frac{d\bar{y}}{dt} \, dt = \left[ \bar{y}(t) \right]_0^\infty = \lim_{t \to \infty} \bar{y}(t) - \bar{y}(0) = \lim_{t \to \infty} \bar{y}(t) \quad . \]  

(SA–2)

Thus, (SA–2) simply integrates the agency’s objective function in (SA–1) over the permissible range at various forecast horizons consistent with the path, \( \bar{y}(t) \). Therefore, the objective function as constituted above depends solely on the terminal state at the infinite forecast horizon – i.e., upper bound of \( \bar{y} \) as \( t \to \infty \). Solving for the upper bound of the agency’s level of forecast optimism, however, requires us to find this variable’s time path, \( \bar{y}(t) \), in terms of the following first–order differential equation containing both a variable coefficient and a variable term:

\[ \frac{d\bar{y}}{dt} + c(t)\bar{y} = c(t) \quad . \]  

(SA–3)

Integrating (SA–3) yields its corresponding general solution:

\[ \bar{y}(t) = \delta e^{\int_0^t c \, dt} + 1 \quad (SA–4) \]

where \( \delta \) is an arbitrary constant arising from integration of (SA–3). Using the initial condition of the agency’s level of forecast optimism at time zero, \( \bar{y}(0) \), as well as setting \( t = 0 \) in the general solution yields:

\[ 0 = \bar{y}(0) = \delta e^0 + 1 = \delta + 1 \quad , \]

\[ \therefore \delta = -1 \quad (SA–5) \]

and thus the definite solution is given by:

\[ \bar{y}(t) = 1 - e^{-\int_0^t c \, dt} \equiv 1 - e^{-Z(t)} \quad . \]  

(SA–6)

the definite integral of \( c, Z(t) \), represents the intertemporal reputation cost discount factor.
For purposes of both notational convenience and mathematical simplification, we treat $t + h$ and $t$ found in our manuscript document as $t$ and $0$ respectively in equations (SA–1) – (SA–6). Our theoretical results and subsequent predictions are robust to these pair of scalar–based notational transformations involving the inclusion of the $h$ term (i.e., forecast horizon) that is adopted in our subsequent discussion of the empirical implications of the theoretical model. Consider that these respective terms are equivalent because if $h = 0$ ($t = 0$ in our manuscript document’s notation), then $t + h = t$. If $h > 0$ ($t > 0$ in our manuscript document’s notation), then $t + h > t$ by definition. Furthermore, if $h \to \infty$ (i.e., $t \to \infty$ in our manuscript document’s notation), likewise $t + h \to \infty$ since $t$ is nonnegative by definition.

Further, because $e^{-Z(t)} \in [0,1]$ and $\overline{y}(t) \in [0,1]$ maximizes the definite solution of the agency’s objective function whenever any $c(t)$ path is consistent with $Z(t) \to \infty$ as $t \to \infty$, these values of $c(t)$ and $Z(t)$ will, in turn, yield $e^{-Z(t)} \to 0$ and $\overline{y}(t) \to 1$, respectively. This particular limiting case reveals that an agency has an incentive to commit the most optimistic forecast errors possible when its reputation costs are discounted at the highest possible rate (i.e., the agency’s present value discounted reputation costs are close to zero), ceteris paribus. The relationship between forecast horizon (denoted by $h$) corresponding to the present value of exponentially discounted future reputation costs (denoted by $e^{-Z(t)}$), and an agency’s level of forecast optimism (denoted by $\overline{y}_{t,h}$) that is made at time $t$ with respect to period $t + h$ leads to the following theoretical proposition:

\[ \overline{y}_{t,h} \]

\[ e^{Z(t)} \]

\[ \overline{y}(t) \]

\[ t + h \]

\[ t \]

\[ h \]

\[ 0 \]

\[ 1 \]

\[ c(t) \]

\[ Z(t) \]

\[ e^{-Z(t)} \]

\[ \overline{y}(t) \]

\[ \overline{y}_{t,h} \]

\[ c(t) \]

\[ Z(t) \]

\[ h \]

\[ 0 \]

\[ 1 \]

\[ t + h \]

\[ t \]

\[ h \]

\[ 0 \]

\[ 1 \]

\[ \overline{y}_{t,h} \]

\[ e^{Z(t)} \]

\[ \overline{y}(t) \]

\[ t + h \]

\[ t \]

\[ h \]

\[ 0 \]

\[ 1 \]

\[ c(t) \]

\[ Z(t) \]

\[ h \]

\[ 0 \]

\[ 1 \]

\[ \overline{y}_{t,h} \]

\[ e^{Z(t)} \]

\[ \overline{y}(t) \]

\[ t + h \]

\[ t \]

\[ h \]

\[ 0 \]

\[ 1 \]

\[ c(t) \]

\[ Z(t) \]

\[ h \]

\[ 0 \]

\[ 1 \]

\[ \overline{y}_{t,h} \]

\[ e^{Z(t)} \]

\[ \overline{y}(t) \]

\[ t + h \]

\[ t \]

\[ h \]

\[ 0 \]

\[ 1 \]

\[ c(t) \]

\[ Z(t) \]

\[ h \]

\[ 0 \]

\[ 1 \]

\[ \overline{y}_{t,h} \]

\[ e^{Z(t)} \]

\[ \overline{y}(t) \]

\[ t + h \]

\[ t \]

\[ h \]

\[ 0 \]

\[ 1 \]

\[ c(t) \]

\[ Z(t) \]

\[ h \]

\[ 0 \]

\[ 1 \]

\[ \overline{y}_{t,h} \]

\[ e^{Z(t)} \]

\[ \overline{y}(t) \]

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\[ c(t) \]

\[ Z(t) \]

\[ h \]

\[ 0 \]

\[ 1 \]

\[ \overline{y}_{t,h} \]

\[ e^{Z(t)} \]

\[ \overline{y}(t) \]

\[ t + h \]

\[ t \]

\[ h \]

\[ 0 \]

\[ 1 \]

\[ c(t) \]

\[ Z(t) \]

\[ h \]

\[ 0 \]

\[ 1 \]

\[ \overline{y}_{t,h} \]

\[ e^{Z(t)} \]

\[ \overline{y}(t) \]

\[ t + h \]

\[ t \]

\[ h \]

\[ 0 \]

\[ 1 \]

\[ c(t) \]

\[ Z(t) \]

\[ h \]

\[ 0 \]

\[ 1 \]

\[ \overline{y}_{t,h} \]

\[ e^{Z(t)} \]

\[ \overline{y}(t) \]

\[ t + h \]

\[ t \]

\[ h \]

\[ 0 \]

\[ 1 \]

\[ c(t) \]

\[ Z(t) \]

\[ h \]

\[ 0 \]

\[ 1 \]

\[ \overline{y}_{t,h} \]

\[ e^{Z(t)} \]

\[ \overline{y}(t) \]

\[ t + h \]

\[ t \]

\[ h \]

\[ 0 \]
Proposition 1 (Policy Forecast Horizon Proposition): As $h \to \infty$, 
\[ e^{-Z(t)} \to 0, \text{ and hence, } \frac{\bar{y}}{1 + h} \to 1. \]

Proposition 1, which serves as the logical basis for H1, states that as the forecast horizon increases, the present value of exponentially discounted future reputation costs will drop, and hence, an agency’s level of forecast optimism will rise, ceteris paribus.\(^7\)

Further, the relationship described above also serves as the theoretical basis for both Corollaries A & B that highlight the varying intertemporal discounting of bureau reputation costs arising from organizational stability differences discussed in the manuscript. Suppose that the present value of exponentially discounted future reputation costs associated with a public agency’s forecasts are positively related to their level of organizational stability such that the discount factors for a pair of bureaus can be represented by $Z(t)_L$ and $Z(t)_H$, where the $L$ and $H$ notation denotes agencies with low and high levels of organizational stability, respectively. This line of reasoning produces Corollary A to Proposition 1:

\[ \text{Corollary A} \]

\(^7\) For purposes of brevity, we do not present the general proofs of the maximum principle for this type of infinite horizon problem since they have already been established elsewhere (see Halkin 1974: 270–271). To summarize, these proofs consist of demonstrating that (1) optimality of infinite horizon problem (IHP) implies finite horizon clamped end-point (FHCEP) optimality but not vice-versa (Halkin 1974: Theorem 4.1, 270), and also that (2) the maximum principle holds for the IHP when FHCEP is optimal for IHP (Halkin 1974: Theorem 4.2, 270–271). The motivation from these class of infinite horizon problems is merely an extension of those solutions developed in the mathematics of optimal control under finite time horizons (Halkin 1964; Pontriagin et al; 1962).
Corollary A (Static Organizational Stability Corollary): For a given $h$, if
\[ e^{-Z(t)_L} < e^{-Z(t)_H}, \text{ then } \overline{Y}_{t+h}^L > \overline{Y}_{t+h}^H. \]

Corollary A, which serves as the logical basis for H2, states that for a given forecast horizon, $h$, if a less stable agency incurs reputation costs which are intertemporally discounted at a higher rate for a given forecast horizon compared to a more stable agency, then the present value of exponentially discounted reputation costs should be lower for the less stable agency. As a result, we can infer that a less stable agency will possess a higher level of forecast optimism vis-à-vis a high stable agency. Corollary B, directly follows from both Proposition 1 and Corollary A.

Corollary B (Dynamic Organizational Stability Corollary): As $h \to \infty$, if
\[ e^{-Z(t)_H} - e^{-Z(t)_L} \to 1, \text{ then } \overline{Y}_{t+h}^L - \overline{Y}_{t+h}^H \to 1. \]

Corollary B, which serves as the logical basis for H3, states that if the difference in the present value of discounted future reputation costs between the more stable agency and the less stable agency grows monotonically with respect to forecast horizon, then we should likewise observe a growing divergence between the level of forecast optimism produced by a less stable agency and a more stable agency.

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8 In both the static and dynamic organizational stability hypotheses (H2 & H3), the observed outcome appears in the forecast error for both agencies, so the relative level of forecast optimism is simply the difference in the level of the forecasts across agencies (H2) and the change in the difference in the level of the forecasts across agencies as the forecast horizon extends (H3).
References


