BOSNIA PROJECT

ELEMENTARY MATHEMATICS MODULE - PATTERNS, RELATIONSHIPS, & NUMBER SENSE

by

Nancy K. Mack
University of Pittsburgh

Running head: Elementary Math Module

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Introduction to the Module

The Elementary Mathematics Module - Patterns, Relationships, and Number Sense is a two-day workshop designed for prospective and inservice elementary school teachers and elementary school teacher educators. (Please note: These three groups of teachers will be referred to as "teacher participants" throughout the module.) The module has three main objectives. One objective is to help the teacher participants become familiar with current trends and recommendations related to the question "What mathematics should we teach in the elementary school?" A second objective is to help the teacher participants become familiar with current trends and recommendations related to the question "How can we teach mathematics for understanding in the elementary school?" The third objective of the module is to actively engage the teacher participants in learning experiences that reflect current trends and recommendations related to the curriculum, learning, and the teaching of mathematics in the elementary school.

The Elementary Mathematics Module is divided into two major sections. The first section focuses on current trends and recommendations for the curriculum and the teaching of mathematics in the elementary school. The second section focuses on an in-depth exploration of patterns, relationships, and number sense and their roles in the learning and teaching of mathematics in the elementary school. "Day One" of the workshop focuses on both the first section of the module and portions of the second section. More specifically, "Day One" focuses on the current trends and recommendations for elementary school mathematics, an introduction to patterns, relationships, and number sense, and activities related to patterns. "Day Two" of the workshop focuses on portions of the second section of the module related to activities

involving relationships, number sense, and connections between patterns, relationships, and number sense, as well as a review of current trends and recommendations for the curriculum, learning, and teaching of elementary school mathematics.

During the workshop, the teacher participants will engage in the activities presented in the module in small groups (3-4 people). The teacher participants will also discuss the activities in both small group and large group settings. The activities and related discussion questions are designed to actively involve the teacher participants in learning to help them gain insights into ways to involve elementary students in their own learning of mathematics. All activities and related small group discussion questions appear on reproducible pages in the module.

The Elementary Mathematics Module focuses on expanding views of the mathematics curriculum for the elementary school and ways to teach mathematics to actively involve students in their own learning. The ideas that are presented in this module may differ greatly from the teacher participants' views about the curriculum and the teaching of mathematics and their own past experiences with elementary school mathematics. Therefore, the workshop leader should consider the issues discussed below.

The Elementary Mathematics Module suggests that the mathematics curriculum for the elementary school should include more than computational procedures. It should also focus on areas that help students develop the ability to reason mathematically and solve a variety of problems, which includes real-world situation problems. Additionally, the module suggests that when teaching mathematics in the elementary school, the teacher should guide students in solving problems in their own meaningful ways rather than telling students specific steps to follow to solve problems. Some of the teacher participants may be uncomfortable with these ideas about the curriculum and the teaching of mathematics. The teacher participants should be allowed to express their disagreement and discomfort with these ideas during the workshop. However, the

workshop leader should encourage the teacher participants to try to view the mathematics curriculum, the learning, and the teaching of mathematics from a different perspective, at least during the time of the workshop.

The Elementary Mathematics Module also requires the teacher participants to actively explore ideas related to patterns, relationships, and number sense and to engage in discussions of their explorations and ways to involve elementary students in similar experiences. To encourage the teacher participants to freely explore and discuss their ideas, the workshop leader should attempt to create an environment where everyone is encouraged to talk and where everyone's views are accepted. Additionally, the workshop leader should attempt to create an environment where it is acceptable for the teacher participants to make mistakes and change, or revise, their thinking if they wish. In short, openness to ideas and non-judgmental statements on the part of **all** involved (teacher participants and workshop leader) help create an environment where individuals feel free to learn.

Lastly, the Elementary Mathematics Module focuses many activities on the use of manipulative materials. The teacher participants may not have previous experiences working with manipulative materials. Therefore, the workshop leader should provide some time for the teacher participants to "explore" with the materials before they are used in the activities. The explorations can take the form of providing the teachers with a specific amount of time to freely work with the materials and reflect on what they notice about the materials from a mathematical perspective. Additionally, patterns for the manipulative materials are provided in the appendix. The teacher participants can prepare the manipulatives during the workshop.

Many sources contributed to the ideas and activities presented in the Elementary Mathematics Module. A selected bibliography is presented at the end of the module.

Elementary Mathematics Module - Patterns, Relationships, & Number Sense

Day One

Focus:

- Introduction to current trends and recommendations for elementary school mathematics.
- Introduction to patterns, relationships, and number sense.
- Activities related to patterns.

Materials Needed:

- calculators (four functions: +, -, x, & ÷; one/2 people)
- colored markers (at least 5 different colors; one set/3-4 people)
- crayons (one box/3-4 people)
- glue (one bottle/3-4 people)
- pencils (one/person)
- scissors (one/person)
- construction paper (at least 5 different colors; 8 sheets/person)
- white paper (standard size; 25 sheets/person)
- white paper squares (12 cm square; one/person)
- white paper (large sheet - approximately 1m x 1.5 m, or larger; 50 sheets)
- ***** Pattern Blocks (one set/person)
- ***** Colored Squares (one set/person)
- ***** Addition Basic Facts Table (one/2 people)
- ***** Multiplication Basic Facts Table (one/2 people)
- ***** Hundreds Chart (one/2 people)
- ***** Fraction Circles (one set/person)
- ***** Fraction Strips (one set/person)
- various pieces of children's literature that involve patterns and numbers

***** Please note: Patterns for materials noted with a ***** can be found in the appendix.

I. Introduction to Current Trends and Recommendations
(approximate time: 1 1/2-2 hours)

A. Introduction to the Workshop (approximate time: 5-10 minutes)

The workshop leader begins the workshop by communicating the objectives of the workshop and the origin of the ideas that will be presented, which are described below. The leader should communicate the introductory information in whatever way s/he is most comfortable.

- The purpose of the workshop is for all those involved (teacher participants and workshop leader) to explore together two primary questions related to the learning and teaching of mathematics in the elementary school, "What mathematics should we teach in the elementary school?" and "How can we teach mathematics for understanding in the elementary school?".
- The ideas that will be shared about the curriculum, learning, and teaching of mathematics are from the United States. The United States does not have all the answers for how to teach mathematics. In fact, teachers in the United States are currently changing the ways they teach mathematics in the elementary school. Previous ways of teaching mathematics have not been particularly effective in helping students understand mathematics and in preparing students to function in society. In the past 5-10 years, there have been dramatic changes in the way mathematics has been taught in the United States. These changes have provided insights into more effective ways to teach mathematics in the elementary school. The ideas presented in this workshop focus on insights gained from changes in the way mathematics is taught in the elementary school in the United States.

B. Current Trends and Recommendations (approximate time: 1 hour)

1. Reflecting on Personal Experiences

To begin thinking about current trends and recommendations for mathematics in the elementary school, the workshop leader should ask the teacher participants to reflect on their own experiences with mathematics in the elementary school, whether it be as a teacher or student. The workshop leader should guide the teacher participants in reflecting on their experiences by engaging them in a discussion of the following questions.

- *What was (is) the mathematics curriculum like in the elementary school? (What was (is) the mathematical content covered? What was (is) the focus of the content on learning procedures and solving problems in meaningful ways?)*
- *What was (is) the students' role in learning mathematics?*
- *What was (is) the teachers' role in teaching mathematics?*

The questions listed above should be discussed in a large group setting. The teacher participants may have similar experiences related to the questions. If this is the case, the workshop leader should ask the teacher participants to also discuss the following questions.

- *How do you think mathematics could have been taught differently when you were in elementary school so that you and your classmates would have enjoyed learning mathematics more and you would have developed a stronger understanding of mathematics?*
- *Why do you think these different ways of teaching mathematics would have enhanced your and your classmates' learning?*

2. Introduction to Current Trends and Recommendations for Elementary School Mathematics

To help focus the teachers on ways to teach mathematics for understanding in the elementary school, the workshop leader should guide the teacher participants in a discussion of the following questions as a large group. If the

teacher participants do not offer some of the desired responses noted for each question, the leader should offer these ideas.

- *What is mathematics?*
  (desired responses include - a study of patterns and relationships, a way of thinking, an art form characterized by order and internal consistency, a language, a tool)

- *Why should we teach mathematics in the elementary school?*
  (desired responses include - a common human activity that is a part of everyday life, so students learn to value mathematics and become confident in their ability to do mathematics, to help students learn to reason logically and think abstractly)

- *What mathematics should we teach in the elementary school?*
  (desired responses include - more than computational procedures) During the discussion of this question, the workshop leader should present the National Council of Teachers of Mathematics (NCTM) Curriculum Standards for grades K-4 and 5-8 as the United States' current response to this question. The NCTM Curriculum Standards are listed below, and a reproducible master can be found in the appendix at the end of the module.

  NCTM Curriculum Standards for Grades K-4
  Mathematics as Problem Solving
  Mathematics as Communication
  Mathematics as Reasoning
  Mathematical Connections
  Estimation
  Number Sense and Numeration
  Concepts of Whole-Number Operations
  Whole Number Computation
  Geometry and Spatial Sense
  Measurement
  Statistics and Probability
  Fractions and Decimals
  Patterns and Relationships

  NCTM Curriculum Standards for Grades 5-8
  Mathematics as Problem Solving
  Mathematics as Communication
  Mathematics as Reasoning
  Mathematical Connections
  Number and Number Relationships
  Number Systems and Number Theory
  Computation and Estimation
  Patterns and Functions
  Algebra
  Statistics
  Probability
  Geometry
  Measurement

The workshop leader should ask the teacher participants to discuss the first four Standards: mathematics as problem solving, mathematics as communication, mathematics as reasoning, and mathematical connections with respect to the following two questions.

- *What do you think each of these four Standards means?*
- *What role might each of these four Standards play in developing students' understanding of mathematics in the elementary school?*

- *How should we teach mathematics in the elementary school?*
  (desired responses include - in a way that helps students make sense of mathematics for themselves, in a way that focuses on concepts, processes, and connections rather than on rote learning, in a way that considers thought processes as important as the final answer)
  During the discussion of this question, the workshop leader should present the NCTM Professional Standards for Teaching Mathematics as the United States' current response to this question. The workshop leader should also present three guidelines for the learning and teaching of mathematics in the elementary school. The NCTM Professional Standards and the guidelines for learning and teaching appear below. Reproducible masters for both are found in the appendix at the end of the module.

NCTM Professional Standards for Teaching Mathematics

Worthwhile Mathematical Tasks
The Teacher's Role in Discourse
Students' Role in Discourse
Tools for Enhancing Discourse
Learning Environment
Analysis of Teaching and Learning

Guidelines for the Learning and Teaching of Mathematics

There is no one right way to solve a problem.
These is no one right way to teach mathematics; some ways are more effective than others in specific situations, but there is no one right way.
Children think about mathematics in ways that differ from adults. If we can gain insights into how children might think about and learn mathematics, we may be better able to help them learn mathematics with understanding.

3. Reflecting on Current Trends and Recommendations

The workshop leader should guide the teacher participants in a discussion of ways that their past experiences with elementary school mathematics

reflect current trends and recommendations related to the curriculum, learning, and teaching of mathematics in the elementary school.

C. Expanding Views of the Learning and Teaching of Elementary School Mathematics (approximate time: 30 minutes)

The workshop leader should communicate to the teacher participants that in order to change the way they teach mathematics in the elementary school, they may need to change, or broaden their perspectives on the curriculum, the learning, and the teaching of mathematics in the elementary school.

To help the teacher participants begin to expand their views of elementary school mathematics, the workshop leader should ask the teachers to work in small groups of 3-4 people and solve the following problem in **two** different ways. Each group should write their solutions on a large sheet of paper so they can share their solutions with the other small groups.

> ***Problem:***
> *Eight friends meet at a party. If every one of the eight friends shakes hands with each of the others, how many handshakes will occur?*

As the small groups of teachers are working on the problem, the workshop leader should assist those groups who are having difficulty solving the problem or difficulty solving the problem in more than one way by asking questions such as the following.

- *Could you draw a picture?*
- *Could you act out the problem?*
- *Could you make a table of some sort?*
- *Could you solve a simpler problem?*
- *Could you look for a pattern?*

After all of the small groups finish solving the problem, the teacher participants should gather back into the large group. Each small group should present its two solution strategies to the others in the large group by discussing the following two questions.

- *How did you solve the problem?*

- *Why do you think it was appropriate to solve the problem in each of these ways?*

Some of the small groups may solve the problem in the same way as other groups. When this happens, the workshop leader should ask the group to discuss the following question.

- *Why do you think this was a good way to solve the problem?*

After all small groups have presented their solution strategies, the workshop leader should guide the large group of teacher participants in discussing questions such as the following.

- *In what ways were the different small groups' solution strategies related to one another?*
  (possible responses include - different ways of representing the same situation, some solutions look at the situation one step at a time while others look for a pattern that ties the action together)

- *Why might it be helpful to have elementary students solve problems in different ways, share their solution processes, and accept and understand one anothers' solution processes?*

- *In what ways does this activity reflect current trends and recommendations for elementary school mathematics?*

- *What did you learn from this activity that you think might be helpful to you in teaching elementary school mathematics?*

II. Patterns, Relationships, and Number Sense (approximate time: 13 1/2 - 14 hours)

A. Introduction to Patterns, Relationships, and Number Sense (approximate time: 30 minutes)

The workshop leader should suggest that one important aspect of teaching mathematics for understanding in the elementary school is to focus on connections between mathematics and the real-world, between different forms of representations, and between different mathematical ideas. Three areas that can help elementary students focus on these connections are patterns, relationships, and number sense.

The workshop leader should guide the teacher participants in a discussion of the following questions. The leader should also suggest the noted desired responses if not suggested by the teachers.

- *What are patterns?*
  (desired responses include - regularity, can be repeating or growing)

- *What are relationships?*
  (desired responses include - study of similarities and differences)

- *What is number sense?*
  (desired responses include - characteristics include: thoroughly understand number meanings, recognize the relative magnitude of numbers, know the relative effect(s) of operating on numbers, develop multiple relationships among numbers, develop referents for measures of common objects and situations)

- *Why are patterns, relationships, and number sense important to elementary school mathematics?*
  (desired responses include - help students make connections between mathematics and the real-world, help students solve problems in ways that are meaningful to them, help students assess the reasonableness of answers)

- *What do students need to know to understand patterns, relationships, and number sense?*
  (desired responses include -
  patterns - core of pattern (e.g., ABABAB...; AB is the core)
  repeating pattern (e.g., ABABAB...; AB repeats)
  growing pattern (e.g., ABABBABBB...; one more B is added to each AB)
  relationships - concepts of same, different, greater than, and less than
  number sense - a sense of size of quantities and effects of operating on quantities)

- *How might we help students develop an understanding of patterns, relationships, and number sense in a way that actively involves them in their own learning?*

B. Activities to Help Develop Students' Understanding of Patterns, Relationships, and Number Sense

1. Patterns

The workshop leader should communicate to the teacher participants that patterns are encountered in daily life in many ways, such as in the spoken and written word, in musical forms, in visual images, and in daily routines. The workshop leader should also suggest that during the workshop, the teacher participants will explore three specific types of patterns - movement patterns, visual patterns, and number patterns.

Before leading the teacher participants through an exploration of patterns, the workshop leader should make sure that the teachers are familiar with the following ideas.

- *core of the pattern* (that part which stays the same and gives rise to the pattern - e.g., in the pattern ABABAB...; AB is the core)

- *repeating pattern* (a sequence of terms or images that repeats in the same manner - e.g., ABABAB... the core AB repeats continuously without alteration)

- *growing pattern* (a sequence of terms or images where each time the pattern is repeated, new terms or images are included - e.g., ABABBABBBABBBB..., each time AB repeats, another B is included in the pattern)

The workshop leader should also suggest that in all types of pattern activities, we want students to be able to do the following.

- *copy the pattern*
- *identify the core of the pattern*
- *describe the pattern in words*
- *identify what comes next*
- *extend the pattern*
- *create their own pattern*

a. Movement Patterns (for all elementary grade levels; approximate time: 30 minutes)

The workshop leader should introduce the idea of movement patterns to the large group of teachers by modeling a movement pattern, such as "clap hands, clap hands, stomp feet, clap hands, clap hands, stomp feet, ... The workshop leader should ask the teacher participants to copy the pattern, extend the pattern through movement, and describe the pattern in words.

The workshop leader should also guide the teacher participants in discussing how they might record the pattern symbolically. For example, the teachers might suggest that the pattern should be recorded as XX|XX|XX|..., where "X" represents "clap hands" and "|" represents "stomp feet".

After the large group introduction to patterns, the teacher participants should get into small groups and take turns creating their own movement patterns (both a repeating pattern and a growing pattern). As one person creates a pattern, the other group members should describe the pattern in words, extend the pattern through physical movement, and record the pattern with whatever symbols they wish.

After the small groups finish their movement patterns, the workshop leader should guide the teacher participants in a discussion of the following questions.

- *In what ways might these movement activities help elementary students understand the concept of patterns and help them identify the core of a pattern?*

- *What types of connections do students need to make to be able to relate movement patterns to symbolic representations of the patterns?*

- *In what ways might these connections help students learn mathematics with understanding in the elementary school?*

- *What other ways do you think you could use movement to help elementary students understand the concept of patterns?*

b. Visual Patterns (for all elementary grade levels)

The workshop leader should introduce the idea of visual patterns to the large group of teachers by using the **pattern blocks** or **colored squares** to create a repeating or growing pattern. The teacher participants should describe the leader's pattern in words and extend the pattern with the concrete materials. The workshop leader should also guide the teacher participants in a discussion of the following question.

- *Why might it be helpful to explore the idea of visual patterns when considering ways to help elementary school students understand mathematics?*

1. Concrete Materials and Visual Patterns (approximate time: 30 minutes)

After the suggested discussion above, the teacher participants should work in their small groups on the following activity.

Using Concrete Materials to Create Visual Patterns

This activity appears on the next page. The teacher participants should also discuss the "Small Group Discussion Questions" that are noted for the activity.

After the small groups finish creating visual patterns, the workshop leader should guide the teacher participants in a discussion of the following questions.

- *In what ways were your experiences creating the visual patterns similar to and different from your experiences with the movement patterns?*

- *In what ways might activities with visual patterns help elementary students understand the concept of patterns?*

Elementary Mathematics Module - Patterns, Relationships, and Number Sense
Visual Patterns Activity

Using Concrete Materials to Create Visual Patterns

Materials Needed:
Pattern Blocks (at least 4 of each different shape)
Colored Squares (at least 5 of each color)

Use the pattern blocks or colored squares to create a visual repeating or growing pattern. Do not tell your small group members what pattern you have created. Other group members should try to describe your pattern in words, extend your pattern by four terms with the concrete materials, and explain why they think their extension is appropriate. Each group member should take a turn creating a visual pattern.

Create another visual pattern (repeating or growing) with the pattern blocks or colored squares. Do not tell your small group members what pattern you have created. Have your group members look in another direction while you take some elements of your pattern away. For example, your pattern may be red square, green square, red square, green square. You could take away a red square. After you secretly remove a part of your pattern, other small group members should try to determine the missing part of the pattern and explain why they think their responses are appropriate. Each group member should take a turn creating and removing a part of a visual pattern.

Small Group Discussion Questions:
In what ways were your experiences creating the visual patterns similar to and different from your experiences with the movement patterns?
In what ways might activities with visual patterns help elementary students understand the concept of patterns?

2. Patterns and Quilts (approximate time: 1 hour)

After discussing the activity involving using concrete materials to create visual patterns, the workshop leader should introduce the idea that patterns are a part of everyday life and that children's literature can be used to help elementary students see patterns in their world. One specific way in which patterns appear in the real-world is through quilts.

The workshop leader should introduce the idea of patterns and quilts by reading a children's book that focuses on quilts. Suggested children's books that are available in the United States include the following.

Flournoy, Valerie. *The Patchwork Quilt*. New York: Dial Books for Young Readers, 1985.
Johnston, Tony, *The Quilt Story*. New York: Putnam Publishing Group, 1985.
Jonas, Ann. *The Quilt*. New York: Greenwillow Books, 1984.

After reading the story, the teacher participants should work in their small groups on the following activity. This activity appears on the next page.

Creating a Quilt Square

After the small groups finish creating the "workshop quilt", the workshop leader should display the quilt and guide the teacher participants in a discussion of the following questions.

- *What patterns do you see in our "workshop quilt"?* (have the teacher participants identify at least 10 different patterns)

- *In what ways are the patterns in the individual quilt squares similar and different?*

- *What name can we give to our workshop quilt that would reflect the concept of patterns?*

Elementary Mathematics Module - Patterns, Relationships, and Number Sense
Visual Patterns Activity

Creating a Quilt Square

Materials Needed:

12 cm white paper squares (two squares for each small group)
colored construction paper (5 different colors)
white paper (standard size)
pattern blocks
colored squares
crayons
scissors
glue
1m x 1.5 m sheet of white paper (two pieces needed for the large group)

Work with others in your small group to create two quilt squares for the "workshop quilt." Your quilt square should contain an identifiable pattern.

Create each quilt square by using one 12 cm square of white paper. Use the pattern blocks and/or colored squares to create your quilt square. Create pieces for your quilt square pattern in one of the following ways.

Trace around your pattern blocks/colored squares onto colored construction paper to create the shapes to be included in your pattern. Cut out the shapes. Glue the shapes onto the 12 cm square into your desired pattern.

Trace around your pattern blocks/colored squares onto white paper to create the shapes to be included in your pattern. Color the shapes. Cut out the shapes. Glue the shapes onto the 12 cm square into your desired pattern.

After your group members create two quilt squares, work with the other small groups to create a "workshop quilt". Arrange all of the small group quilt squares onto a 1m x 1.5 m (or larger) sheet of white paper. Glue the squares onto the large sheet of white paper.

Determine a name for the quilt that will reflect the concept of patterns.

Small Group Discussion Questions:

In what ways do you think creating quilt squares and a class quilt could help elementary students understand the concept of patterns?
What other types of real-world activities might help elementary students understand the concept of patterns?

c. Extending Patterns to Everyday Life (all elementary grade levels)

1. Patterns in the Environment (approximate time: 15 minutes)

The workshop leader should communicate to the teacher participants that patterns are all around us and a part of our and our students' daily life experiences. The leader should solicit ideas about how patterns are involved in our daily lives from the teacher participants.

The workshop leader should ask the teacher participants to explore their environment for 5 minutes and identify at least **eight** different patterns in their environment. After the teachers find patterns in their environment, the workshop leader should guide the teacher participants in a discussion of the following questions.

- *What was your environment and what patterns did you find in your environment?*
- *What types of patterns do you think elementary students would notice in their environment? Why do you think they would notice these patterns?*
- *In what ways does this activity extend ideas related to patterns that were developed in earlier activities?*

2. Patterns in Children's Literature (approximate time: 1 1/2 hours)

The workshop leader should introduce the idea that one place patterns are found in daily life experiences is through children's literature. The workshop leader should read or tell a story that involves a pattern of events. For example, the story *Goldilocks and the Three Bears* involves a little girl finding a house in the woods and sitting in chairs that are too big, too small, and just right; eating porridge that is too hot, too cold, and just right; and sleeping in beds

that are too hard, too soft, and just right. After telling the story, the workshop leader should guide the teacher participants in a discussion of the following questions.

- *What was the pattern of events in the story?*
- *When did the pattern change or end?*
- *How might the story have been different if the pattern had changed in a different way or had not ended?*

The teacher participants should work in their small groups on the following activity. The activity appears on the next page.

Patterns in Children's Literature

After the small groups finish this activity, the small groups should share their stories and activities with the large group. Additionally, the workshop leader should guide the teacher participants in a discussion of the following questions.

- *What other types of activities could you use to help students "see" patterns in their environment and real-life experiences?*
- *In what ways might these activities related to patterns in children's literature be extended to help elementary students understand patterns in situations involving numbers?*

Elementary Mathematics Module - Patterns, Relationships, and Number Sense
Extending Patterns to Everyday Life Activity

Patterns in Children's Literature*

Materials Needed:
- pencils
- white paper (standard size)
- crayons
- markers
- pattern blocks
- colored squares
- children's books (2-3 books for each small group)

Work in pairs in your small group. As a pair, create a visual representation to describe the pattern of events in the story told or read by the workshop leader. Retell the story in your own words to the others in your small group by using the visual representation you created.

As a small group, choose one piece of children's literature that involves a pattern of events or write a story of your own that contains an identifiable pattern. Design an activity that would help elementary students focus on the pattern(s) in the story in a variety of ways.

Small Group Discussion Question:

Why do you think it might be helpful to use children's literature to encourage elementary students to become actively involved in their learning of patterns?

***** This activity was adapted from an idea generated by Ms. Melissa Lege, a graduate student at The University of Pittsburgh, Pittsburgh, PA, USA.

d. Extending Patterns to Number Situations

The workshop leader should guide the teacher participants in a discussion of the following question.

- *In what ways might elementary students be able to draw on the concept of patterns to solve problems involving numbers in meaningful ways?*

1. Patterns and Number Situations in the Primary Grades
(US grades K-3; approximate time: 1 hour)

The teacher participants should work on the following three activities in their small groups. These activities appear on the next page.

Patterns & Basic Facts

Patterns in the Hundreds Chart

What's My Rule

After the small groups finish these three activities, the workshop leader should guide the teacher participants in a discussion of the following questions.

- *What patterns did you identify for the addition and multiplication tables that you think might help elementary students learn their basic facts and see relationships between various facts?*

- *In what ways might you be able to build on the patterns you identified for addition and multiplication to help students learn their basic facts of subtraction and division?*

- *In what ways might finding patterns in the Hundreds Chart and in number sequences help primary grade level students learn mathematics with understanding?*

- *In what ways did these primary grade level pattern activities draw on ideas developed in earlier pattern activities?*

- *What other types of pattern activities with numbers might help primary grade level students solve mathematical problems in meaningful ways?*

Elementary Mathematics Module - Patterns, Relationships, and Number Sense

Patterns & Number Situations in the Primary Grades Activities

Patterns & Basic Facts

Materials Needed:

Addition Basic Facts Table
Multiplication Basic Facts Table

Work together as a small group. Examine the addition basic facts table. Identify 5-8 different patterns in the table that you think might help elementary students learn basic facts of addition. Explain why you think that recognizing these patterns would be helpful to elementary students. Repeat the activity with the multiplication basic facts table.

Patterns in the Hundreds Chart

Materials Needed:

Hundreds Chart

Work together as a small group. Examine the Hundreds Chart. Identify 5-8 different patterns in the chart that you think would be helpful to elementary students when learning ideas of place value and adding and subtracting two-digit numbers (e.g., 35+21). Explain why you think these patterns would help elementary students add and subtract two-digit numbers in meaningful ways.

What's My Rule

Work together as a small group. Examine the following number sequences. Determine a mathematical rule that describes the pattern in the sequence. Determine the next four terms in the sequence by applying your rule.

1, 3, 5, 7, 9, 11, _, _, _, _
2, 4, 8, 16, 32, _, _, _, _
1, 4, 13, 40, 121, _, _, _, _

Work in pairs. Create your own number sequence based on a mathematical rule. Exchange your sequence with others in your small group. Determine the rule for the new number sequence and extend the sequence by four terms.

Small Group Discussion Questions:

In what ways might these three activities help elementary students become actively involved in their own learning and help them develop a deeper understanding of patterns and numbers?

2. Patterns and Number Situations in the Intermediate Grades (US grades 4-8; approximate time: 1 hour)

The workshop leader should guide the teacher participants in a discussion of the following questions.

- *What are some patterns in number situations that you think might help intermediate grade students learn mathematics with understanding? Why?*

- *What types of activities do you think might help intermediate grade students focus on these patterns in number situations?*

After this discussion, the teacher participants should work on the following three activities in their small groups. These activities appear on the next two pages.

Patterns in Multiplication

Why are Some Numbers Called "Squares"?

Patterns in Fractions

After the small groups finish these three activities, the workshop leader should guide the teacher participants in a discussion of the following questions.

- *In what ways might focusing on patterns help intermediate grade students solve mathematical problems in meaningful ways?*

- *What types of questions might we ask intermediate grade students to help them focus on patterns when solving mathematical problems?*

- *In what ways might we revise the pattern activities for the primary grade students to be appropriate for the intermediate grade students?*

- *What other types of pattern activities might be helpful to intermediate grade students? Why might these activities be helpful?*

Elementary Mathematics Module - Patterns, Relationships, and Number Sense
Patterns & Number Situations in the Intermediate Grade Activities

Patterns in Multiplication

Materials:
calculators

Use calculators for this activity, if available. Work together as a small group. Choose 5 different two-digit numbers (e.g., 37, 94, etc.) and 5 different three-digit numbers (e.g., 158, 736, etc.). Multiply each of the numbers by 11. What patterns do you notice with respect to your product (answer) and the number you multiplied by 11? Make a hypothesis. Use your hypothesis to predict the answer to three other problems involving multiplying by 11. Check your prediction with your calculator.

Repeat this activity by multiplying numbers by 99. Repeat this activity by multiplying numbers by 999.

Small Group Discussion Questions:
In what ways might these patterns help intermediate grade students solve multiplication problems in meaningful ways?
How might these patterns be extended to help students multiply decimal numbers in meaningful ways? (Try a few problems involving multiplying by 1.1 and multiplying by 0.99.)
What other ways might you encourage intermediate grade students to look for patterns when multiplying whole numbers and decimals?

Why Are Some Numbers Called "Squares"?

Materials Needed:
Colored Squares

Work together as a small group. List the first five square numbers (i.e., 1, 4, 9, 16, 25). Use the colored squares to construct a visual representation of each number. Each side of the visual representation should have the same length.

What do you notice about the shape of your representation? (If you did not make a square, can you do so?)

What pattern describes how many colored squares would be needed to make a representation of the next square number?

Create two patterns of your own that focus on patterns and square numbers.

Small Group Discussion Questions:
In what ways might this activity be helpful to intermediate grade students when learning mathematics?
What other types of pattern activities might help students focus on the meaning of different types of numbers, such as triangular numbers, even and odd numbers, prime and composite numbers, etc.?

Patterns in Fractions

Materials Needed:

Fraction Circles
Fraction Strips

Work together as a small group. Use either the fraction circles or fraction strips. Construct one whole for each division of the fraction materials (e.g., one whole made from two halves, one whole made from three thirds, & etc.). Examine all of the wholes. Identify 3-5 patterns that you see in the wholes made from the fraction materials. Be sure to focus on the size of the pieces in each whole and the number of pieces in each whole.

Small Group Discussion Questions:

In what ways might these patterns help intermediate grade students develop a sense of size of fractions?

In what ways might these patterns help students when working with symbolic representations (e.g., 1/2, 3/4, & etc.) to compare, add, and subtract fractions?

What other types of activities might help students recognize patterns and develop meaning for fractions?

e. Reflecting on Patterns in the Elementary School

The workshop leader should close Day One by guiding the teacher participants in a discussion of the following questions.

- *What did you learn about patterns today that you think may be useful to you when teaching mathematics in the elementary school?*

- *What did you learn today about teaching mathematics in ways that actively involves students in their own learning that you think may be useful to you?*

- *In what ways does what you learned today reflect your own experiences with learning and teaching mathematics in the elementary school?*

End of Day One

Day Two

Focus:

- Activities related to relationships and number sense.
- Activities connecting patterns, relationships, and number sense.
- Reviewing current trends and recommendations for elementary school mathematics.
- Reviewing ways to actively engage elementary students in their own learning of mathematics.

Materials Needed:

- calculators (four function: +,-, x, & ÷)
- dice (regular six sided dice; one/3-4 people)
- tape or paper clips
- index cards (regular size; 30/person)
- white paper (standard size; 30 sheets/person)
- 2 glass jars (one filled with pieces of candy or some other item where each individual piece is the same size, one jar filled approximately 1/4 full with the same item as the first jar)
- ***** Fraction Circles
- ***** Fraction Strips
- ***** Pattern Blocks
- ***** Colored Squares
- ***** Grid Paper (one sheet/person)
- ***** Base 10 Materials (one set/person)
- various pieces of children's literature that involve patterns and situations that could be described with numbers

***** Please note: Patterns for materials noted with a * can be found in the appendix.

II. Patterns, Relationships, and Number Sense (continued)

The workshop leader should begin Day Two by asking if the teacher participants have any questions related to the activities from Day One. The group as a whole should discuss any questions raised. The workshop leader should also guide the teacher participants in a discussion of the following question.

- *In what ways might ideas related to patterns be built upon to help elementary students understand ideas related to relationships and number sense?*
  (desired responses include - determining relationships and building number sense are applications of patterns)

2. Relationships

The workshop leader should communicate to the teacher participants that for elementary students to be able to solve mathematical problems in meaningful ways, the students need to be able to focus on various types of relationships between numbers and quantities. The leader should suggest that during the workshop, the teacher participants will focus on four specific types of relationships - the same as, different from, greater than, and less than.

a. Relationship Situations Without Numbers (all elementary grade levels; approximate time: 45 minutes)

The workshop leader should guide the large group of teacher participants in the following activity, called "Tibby".

"Tibby"

> Focus on one common characteristic of some of the people in the large group (e.g., has on glasses, wearing blue shirt, has on stripes, does not have on red, etc.). Do not tell the people in the group the characteristic you chose. People having your chosen characteristic will be called a “Tibby”.
>
> One-by one, ask some of the people in the group with your chosen characteristic to stand to one side of the room and tell them "You are a Tibby". Ask some of the others in the group without your chosen characteristic to stand on the other side of the room and tell them "You are not a Tibby". After 4-6 "Tibbies" and "Not a Tibby" have been designated, ask the large group to determine what makes a person a "Tibby" or "Not a Tibby" (e.g., all Tibbies are wearing glasses and no "Not a Tibby" has on glasses).

Repeat "Tibby" with 2-3 of the teacher participants taking turns designating who is and is not a Tibby. The workshop leader should stress that it is important to vary the "Tibby" and "Not a Tibby" each time so that one is not perceived as being more desirable than the other.

After "Tibby" has been performed 2-3 times, the workshop leader should guide the teacher participants in a discussion of the following questions.

- *In what ways might this activity, “Tibby”, help elementary students understand the concept of relationships?*
- *In what ways can this activity involve elementary students in their own learning of mathematics?*

After this discussion, the teacher participants should work in their small groups on the following activity. This activity appears on the next page.

Relationships and Visual Representations

After the small groups finish this activity, the workshop leader should guide the teacher participants in a discussion of the following questions.

- *Why is it important for elementary students to focus on the ideas of same and different when learning mathematics?*

- *In what ways might this activity help students focus on the concepts of same and different?*

- *What other types of activities might you use to help elementary students focus on the concepts of same and different when learning mathematics?*

Elementary Mathematics Module - Patterns, Relationships, and Number Sense
Relationship Situations Without Numbers Activity

Relationships and Visual Representations

Materials Needed:
Pattern Blocks

Work together in your small group. Use the pattern blocks. One member of your group should sort the pattern blocks into smaller groups in whatever way s/he wishes without telling the other small group members. After the blocks are sorted, the other group members should discuss the ways in which the smaller groups of pattern blocks are similar and different. They should also try to determine a rule that describes how the blocks are sorted.

Switch roles among small group members and repeat the activity.

As a group, design one activity that you could use with primary grade students to help them focus on the ideas of same and different. Design another activity that would be appropriate for intermediate grade students.

Small Group Discussion Question:
Why do you think the activities you designed would be effective in helping primary and intermediate grade students focus on the ideas of same and different?

b. Relationships and Number Situations in the Primary Grades (U.S. grade K-3; approximate times: 45 minutes)

The workshop leader should introduce the idea of relationships and number situations in the primary grades by guiding the teacher participants in a discussion of the following two questions.

- *What do primary grade students need to know in order to understand the concepts of more (greater than), less (less than), and the same as (equal to) when working with number situations?*

- *What difficulties might primary grade students have with the concepts of greater than and less than when working with number situations?*

After this discussion, the teacher participants should work in their small groups on the following three activities. These activities appear on the next page.

Creating Similarities & Differences

Comparing Names

Problems in Children's Literature

After the small groups finish these activities, the workshop leader should guide the teacher participants in a discussion of the following questions.

- *What types of activities or problems did you develop from pieces of children's literature to help primary grade students understand concepts of relationships?*

- *What other types of activities might you use to help primary grade students understand concepts of relationships?*

- *What types of activities do you think might help intermediate grade students understand concepts of relationships?*

Elementary Mathematics Module - Patterns, Relationships, and Number Sense
Relationships and Number Situations in the Primary Grades Activities

Creating Similarities & Differences

Materials Needed:
Pattern Blocks
Colored Squares

Work together as a small group. Use the pattern blocks or colored squares. Create 5 different problem situations that would help primary grade students focus on who has more, who has less, how many more or less, and who has the same as.

Small Group Discussion Question:
In what ways might your problem situations help primary grade students understand the concepts of greater than, less than, and equal to?

Comparing Names

Materials Needed:
Grid paper

Work together as a small group. Each group member prints his/her name on the grid paper, one letter per box (e.g., H E I D I). Each group member also prints the names of 2-3 friends on grid paper (a total of 10-12 names are needed for the small group). Refer to the names on the grid paper and answer the following questions.

- Who has the longest name?
- Who has the shortest name?
- Can you find someone with a name the same length as yours?
- Can you find someone whose name has one more (less) letter than your name?
- One question that you create.

Organize the names into some type of graph. Refer to the graph and answer the following questions.

- Which name length is the most popular?
- (Fill in name) was not here today. Where should her/his name go on our graph?
- Can you think of anyone you know who has a shorter name than (fill in name)?
- Two questions that you create.

Small Group Discussion Question:
In what ways might this activity help primary grade students understand the concepts of greater than, less than and equal to?

Problems in Children's Literature

Materials Needed:

various pieces of children's literature that involve focusing on specific characteristics

suggested books available in the US are:

How Many Snails? by Paul Giganti (1987)
Caps for Sale by Esphyr Slobodkina (1984)
So Many Cats by Beatrice Schenk de Regniers (1985)

Work together as a small group. Choose one children's book. Read the book. Create 5-8 word problems related to the story that you could ask primary grade students. The problems should focus on the concepts of more, less, and the same as.

Small Group Discussion Questions:

Why do you think your problems would help primary grade students focus on the concepts of more, less, and the same as?

Why might it be beneficial to use children's literature to help primary grade students understand concepts of relationships?

How might you encourage primary grade students to develop their own mathematical problems from pieces of children's literature to further their own understanding of mathematical relationships?

c. Relationships and Number Situations in the Intermediate Grades (US grades 4-8; approximate time: 45 minutes)

The workshop leader should introduce the idea of relationships and number situations in the intermediate grades by guiding the teacher participants in a discussion of the following questions.

- *How might intermediate grade students' understanding of concepts of relationships be similar to and different from primary grade students' understanding?*
- *What specific mathematical ideas could, or should, be addressed through activities to help intermediate grade students understand concepts of relationships?*

After this discussion, the teacher participants should work in their small groups on the following three activities. These activities appear on the following three pages.

Whole Numbers, Fractions, & Decimals - Same or Different?

Fractions Near 0, 1/2, & 1

Comparing Fractions Through Reasoning

After the small groups finish these activities, the workshop leader should guide the teacher participants in a discussion of the following questions.

- *What are some relationships between whole numbers, fractions, and decimals that are helpful for intermediate grade students to know?*
- *What difficulties might intermediate grade students have with the concepts of greater than, less than, and equal to when working with whole numbers, fractions, and decimals?*
- *What other types of activities might you use to help intermediate grade students understand concepts of relationships?*

Elementary Mathematics Module - Patterns, Relationships, and Number Sense
Relationships and Number Situations in the Intermediate Grade Activities

Whole Numbers, Fractions, & Decimals - Same or Different?

Work together as a small group. Think about whole numbers, fractions, and decimals. Describe 5 ways in which whole numbers, fractions, decimals, and operations on these three types of numbers numbers are similar and five ways in which they are different. It may be helpful to consider the following questions when describing the similarities and differences.

- Do the numbers answer the question "How many?", "How much", or both? When do they answer each question?

- How are the numbers represented with concrete materials? With number symbols?

- What operations can be performed on each type of number?

- What are the procedures for operating on each type of number?

Small Group Discussion Questions:

Why might it be helpful for intermediate grade students to explore similarities and differences between whole numbers, fractions, and decimals?

What other types of activities might help students explore these similarities and differences?

Fractions Near 0, 1/2, & 1

Materials Needed:
Fraction Circles
Fraction Strips

Work together as a small group. Use the fraction circles or fraction strips. Form a whole with each of the different divisions (e.g., one whole made from two halves, one whole made from three thirds, and etc.). Refer to your fraction materials and answer the following questions.

- Which division required the most pieces to form a whole? Which required the least?

- What can you say about the relationship between the number of pieces in a whole and the size of the pieces?

Work with one division of your fraction materials at a time. Represent fractions near 0, 1/2, and 1. Record your answers in a chart.

Example:	Fractions Near 0	Fractions Near 1/2	Fractions Near 1
	1/8	3/8, 4/8, 5/8	7/8, 8/8

Continue with other divisions of your fraction materials.

Look at your chart, what patterns do you notice within each column that describe the group? (What is similar about all the fractions that you listed as being close to 1?)

Look at the fractions in the column "Fractions Near 1/2". Sort these into three groups (use your fraction materials if needed): those less than 1/2, those equal to 1/2, and those greater than 1/2. Explain how you know that your sorting is accurate.

Small Group Discussion Questions:
What mathematical relationships might this activity help intermediate grade students understand? Why?
How might this activity be revised to help students understand relationships between decimal numbers?

Comparing Fractions Through Reasoning

Materials Needed:

Fraction Circles
Fraction Strips

Work together as a small group. Draw on what you learned from the activity "Fractions Near 0, 1/2, and 1 to determine which fraction in each pair listed below is of greater or lesser value. Use mathematical reasoning rather than an algorithm or step-by-step procedures. Justify your reasoning with the fraction materials.

1/4 & 1/5	2/3 & 3/4	1/2 & 3/5	two pairs of fractions you create
5/11 & 6/10	6/8 & 4/5	3/8 & 3/6	

Small Group Discussion Questions:

In what ways might this activity help strengthen intermediate grade students' understanding of the concepts of greater than, less than, and equal to?

How might this activity be revised to strengthen students' understanding of decimal numbers?

d. Equivalent Expressions (for all elementary grade levels, approximate time: 45 minutes)

The workshop leader should communicate to the teacher participants that there is a special type of relationship that all students need to understand, ***equivalent expressions***. The leader should guide the teacher participants in a discussion of the following questions.

- *What are equivalent expressions?*
  (desired responses include - different expressions with the same value, e.g., 4 + 2 = 54 ÷ 9)

- *Why might it be helpful for elementary students to understand equivalent expressions?*
  (desired responses include - helpful when renaming numbers, e.g., 42 = 4 tens 2 ones, 3 tens 12 ones, 42 ones, etc.)

Before the teacher participants break into their small groups for activities related to equivalent expressions, the workshop leader should make sure that the teachers are familiar with Base 10 materials and how to use them. If the teachers are unfamiliar with the materials, the workshop leader should introduce the materials by communicating that the small square (unit) represents a "one", the stick-like object called a "long" (1 unit by 10 units) represents a "ten", and the large square called a "flat" (10 units by 10 units) represents a "hundred". The workshop leader should stress that the materials are based on powers or ten - 10 ones equal one long, 10 longs equal one flat, 100 ones equal one flat. The materials can be used to represent whole numbers in the following way - 34 can be represented by 3 longs and 4 ones, 2 longs and 14 ones, 34 ones, etc. The leader should also communicate that the Base 10 materials can be used to represent decimals numbers. However, when the materials

represent decimals, a flat must represent "one", a long must represent "one tenth", and a unit must represent "one hundredth".

The teacher participants should work in their small groups on the following two activities. These activities appear on the next two pages.

Fish For Equivalent Expressions

Can You Make This?

After the small groups finish these activities, the workshop leader should guide the teacher participants in a discussion of the following questions.

- *When do elementary students need to draw on the concept of equivalent expressions in elementary school mathematics?* (desired responses include - when regrouping to add and subtract numbers and when renaming numbers to operate on fractions)

- *How can understanding equivalent expressions deepen elementary school students' understanding of mathematics?* (desired responses include - helps students see connections between various types of numbers and gives meaning to the equal sign)

- *What other types of activities might help elementary students understand the concept of equivalent expressions?*

Elementary Mathematics Module - Patterns, Relationships, and Number Sense
Equivalent Expressions Activities

Fish for Equivalent Expressions

Materials Needed:
20 index cards (or 1/4 pieces of paper) for each small group member

Each group member should have 20 index cards or 5 sheets of paper that have been cut into quarters. Sixty to 80 cards are needed for the game.

Write one mathematical expression of the types listed below on each card. Try to make sure that some of the different facts have the same value.

5 basic facts of addition (e.g., 3 + 4; 5 + 7, 8 + 2, etc.)
5 basic facts of subtraction (e.g., 12 - 3, 8 - 1, 15 - 9, etc.)
5 basic facts of multiplication (e.g., 3 x 4, 5 x 8, 7 x 3, etc.)
5 basic facts of division (e.g., 24 ÷ 4, 8÷2, 42 ÷ 7, etc.)

This activity is similar to the card game "Go Fish". After each group member prepares his/her cards, all group members combine their cards and shuffle them. One group member deals everyone seven cards and places the rest of the cards face down on the table.

When it is a group members' turn, s/he asks any other member of the group "Do you have an expression that is equivalent to (supply the number)?". If the member that was asked has one or more cards with the requested value, s/he must give the person asking for the card only *one* of his/her cards. If the member who was asked does not have a card with the requested value, s/he says "Go fish for an equivalent expression" and the asking member draws a card from the pile of face down cards.

When a group member has a pair of cards with the same value, s/he may lie them face up on the table at any time during his/her turn.

The game ends when no more equivalent expressions can be found. The player with the most pairs of equivalent expressions on the table wins.

Small Group Discussion Questions:
In what ways might this activity help elementary students understand the concept of equivalent expressions?
How might this activity be revised to focus on fractions or decimals?

Can You Make This?

Materials Needed:
Base 10 materials

Work together as a group. Use the Base 10 materials. Represent each of the numbers listed below in 3 different ways. Example: 124 can be represented as 12 tens 4 ones, 1 hundred 2 tens 4 ones, and 1 hundred 1 ten 14 ones.

145 97 286

Represent each of the numbers listed in the chart below by using the total number of Base 10 blocks given. Record your answers in the chart.

	Number	Hundreds	Tens	Ones	Total Number of Blocks
Ex.	148	1	4	8	13
Ex.	148	1	3	18	22
	237				12
	237				30
	237				66
	361				10
	361				37
	361				91
	418				49
	418				22
	418				85

Small Group Discussion Questions:

How might this activity help strengthen elementary students' understanding of equivalent expressions?

How might this activity be revised to help students understand equivalent expressions with decimals?

3. Number Sense

The workshop leader should guide the teacher participants in a discussion of the following questions.

- *What is number sense?*
  (desired responses include - characteristics include: thoroughly understand number meanings, recognize the relative magnitude of numbers, know the relative effect(s) of operating on numbers, develop multiple relationships among numbers, develop referents for measures of common objects and situations)

- *Why is number sense important to students' understanding of elementary school mathematics?*
  (desired responses include - helps students solve problems in ways that are meaningful to them, helps students assess the reasonableness of answers)

- *How is number sense connected to mathematical patterns and relationships?*
  (desired response include - recognizing mathematical patterns and relationships helps build number sense, patterns and relationships involve applying number sense)

- *How might we help elementary students develop number sense in a way that actively involves them in their own learning?*

a. Developing a Sense of Size of Numbers (all elementary grade levels; approximate time: 1 hour)

The workshop leader should introduce the idea of developing a sense of size of numbers by guiding the teacher participants in a discussion of the following questions.

- *What are some numbers you have heard elementary students use that they may not truly understand with respect to how big or how small the number is?*

- *How might you help students understand the largeness or smallness of these numbers?*

After discussing the above questions, the workshop leader should guide the teacher participants in the following activity, called "Estimation and Number Sense". This activity illustrates one way to help elementary students understand how big or small numbers are.

"Estimation and Number Sense"

Two clear glass jars are needed for this activity. One jar (the first jar) should be completely filled with items of the same size, such as candy or popcorn kernels. The other jar (the second jar) should be approximately one quarter full with the same items that are in the first jar.

Show the first jar to the teacher participants and ask them to estimate the number of items that are in the jar. Have several of the teachers explain their estimates.

After the teachers have estimated the number of items in the first jar and explained their estimates, show the teachers the second jar. Ask the teachers to estimate the number of items in the second jar and explain their estimates.

Have 2-3 teachers count the number of items in the second jar. Inform the other teacher participants how many items were contained in the second jar. Ask the teacher participants to use this information to revise their estimates of the number of items in the first jar if they wish. Ask several teachers to explain why they revised or did not revise their estimates after gaining information about the second jar.

After the large group finishes the activity "Estimation and Number Sense", the workshop leader should guide the teacher participants in a discussion of the following questions.

- *How might this activity help elementary students develop a sense of size of numbers?*

- *What other types of activities involving visual representations might help elementary students develop a sense of size of numbers?*

The workshop leader should communicate that children's literature, as well as visual representations, can be used to help elementary students develop number sense. The workshop leader should introduce this idea through a children's book such as *How Much is a Million?* by David M. Schwartz (1985). The workshop leader could read the book and ask the teacher participants to make predictions about various number situations in the story. The teachers could also explain the reasoning behind their predictions.

After being introduced to the idea of using children's literature to develop number sense, the teacher participants should work in their small groups on the following activity. This activity appears on the next page.

How Big Is __?

After the small groups finish this activity, each group should share their activity with the large group. Additionally, the workshop leader should guide the teacher participants in a discussion of the following questions.

- *Why might these activities help elementary students develop a sense of size of specific numbers?*
- *What is the teacher's role and the student's role in these activities?*
- *How might these activities help students draw on number sense to solve a variety of mathematical problems in meaningful ways?*

Elementary Mathematics Module - Patterns, Relationships, and Number Sense
Developing a Sense of Size of Numbers Activity

How Big Is __?

Work together as a small group. Choose a specific number that you think elementary students use but have little understanding of its size (e.g., 100, 500, 1000, et.). Develop a mathematical classroom celebration around this number. Your celebration should include making predictions and collecting or representing the number you chose.

For example, you might choose the number 100. You could ask students to predict when the hundredth day of school will occur. You could also ask students to collect 100 of different items and bring them to school for the "100 Celebration". You could ask students to predict such things as how large a container would need to be to hold 100 math books, 100 coins, 100 erasers, and etc. You could ask students to predict how far 100 m is from their desk, and then have the students measure 100 m. You could ask students to write a story about 100. You could ask students to predict if 100 people could fit inside the classroom and then determine how to check their prediction. All of the daily activities should focus around 100.

Small Group Discussion Questions:

Why do you think your activity would help elementary students develop a sense of size of the number you chose?

What other types of activities could you use to help students develop a sense of size of numbers?

b. Extending Ideas of Number Sense

The workshop leader should suggest that number sense can help elementary students solve mathematical problems in meaningful ways by giving meaning to numbers represented symbolically (e.g., 27, 386, 1/4, 2.57, etc.). Number sense functions in the same way for both primary and intermediate grade students; however, the types of numbers that primary and intermediate grade students encounter in the elementary school often differ. Therefore, the teacher participants will first engage in activities focused specifically on number sense in the primary grades and then activities focused on number sense in the intermediate grades.

1. Primary Grade Level Activities (US grades K-4; approximate time: 1 hour)

The workshop leader should introduce the idea of number sense in the primary grades by guiding the teacher participants in a discussion of the following questions.

- *What types of numbers do primary grade level students work with most often?*
  (In the US, students work with whole numbers.)

- *What meaning do students associate with these numbers and operations on these numbres when they are represented symbolically (e.g., 23, 587, etc.)?*

- *What types of activities might help primary grade students develop meaning for numbers represented symbolically?*

After this discussion, the teacher participants should work in their small groups on the following four activities. These activities appear on the next two pages.

Creating Whole Numbers

Using Number Sense to Describe Solutions

Using Number Sense to Compute Answers

Problem Solving and Number Sense

After the small groups finish these activities, the workshop leader should guide the teacher participants in a discussion of the following questions.

- *What other types of activities might be effective in helping primary students extend and apply ideas related to number sense?*

- *How might the activities above be revised to help intermediate grade students extend and apply ideas related to number sense?*

Elementary Mathematics Module - Patterns, Relationships, and Number Sense
Extending Ideas of Number Sense - Primary Grade Level Activities

Creating Whole Numbers

Materials Needed:
dice (3)

Work in pairs within your small group. Roll **two (2)** dice. Take the numbers that appear on the dice and make the largest two-digit number possible. For example, if a "3" appears on one die and a "5" appears on the other die, the largest number would be "53". After each pair makes its number, compare the two numbers and determine how much larger and smaller the numbers are in relation to one another. Repeat three times.

Work in pairs within your small group. Choose a three-digit target number (e.g., 356, 982, 154, etc.). Roll **three (3)** dice. Take the numbers that appear on the dice and make a three-digit number that is as close to the target number as possible. After each pair has made a three-digit number, compare the numbers to determine which one is closest to the target number and how much closer it is. Repeat three times.

As a small group, design one activity involving creating whole numbers that focuses on extending ideas of number sense for primary grade level students.

Small Group Discussion Questions:
What difficulties do primary grade students have with ideas related to number sense and multi-digit numbers?
What mathematical ideas would these activities help primary grade students understand as they develop ideas of number sense?

Using Number Sense to Describe Solutions

Work together as a small group. Without solving the following problems, tell 3-5 things that you know about the answer to each problem.

27 + 95 = 1284 ÷ 16 = 25 x 72 =

2784 - 1956 = two problems that you create

Small Group Discussion Questions:
Why might it be effective to use an activity such as this with primary grade level students?
What other types of activities might help primary grade level students draw on their number sentence to describe solutions to mathematical problems?

Using Number Sense to Compute Answers

Work together as a small group. Draw on number sense to solve the following problems rather than solving the problems by applying a step-by-step procedure. For example, 99 x 27 = (100 x 27) - (1 x 27).

97 + 105 =	158 - 49 =	25 x 17 =
50 x 32 =	95 x 41 =	367 + 285 =

Create three problems of your own that you could solve by drawing on number sense. Exchange your problems with another group. Solve the new problems by drawing on ideas of number sense.

Small Group Discussion Questions:

Why might it be helpful for primary grade level students to solve mathematical problems by drawing on ideas of number sense rather than by applying step-by-step procedures to solve problems?

What are some ideas of number sense that can help primary grade level students solve mathematical problems in meaningful ways?

Problem Solving and Number Sense

Materials Needed:

Calculators

Work together as a small group. Solve each of the following problems. Use a calculator to check your answers, if one is available.

Use the digits 1, 2, 3, 4, & 5 to create two- and three-digit numbers that will result in the given product when multiplied. Use each digit once, and only once, in each problem.

Example: _ _ _ x _ _ = 3672 153 x 24 = 3672

_ _ _ x _ _ = 9635 _ _ _ x _ _ = largest product possible

_ _ _ x _ _ = 16524 _ _ _ x _ _ = smallest product possible

_ _ _ x _ _ = 4640 two problems that you create

Use the digits 5, 6, 7, 8, & 9 to create two- and three-digit numbers that will result in the given product when multiplied. Use each digit once, and only once, in each problem.

_ _ _ x _ _ = 39695 _ _ _ x _ _ = largest product possible

_ _ _ x _ _ = 39032 _ _ _ x _ _ = smallest product possible

_ _ _ x _ _ = 50690 two problems that you create

Small Group Discussion Question:

In what ways might this activity help primary grade students develop and apply ideas of number sense?

2. Intermediate Grade Level Activities (US grades 5-8; approximate time: 1 hour)

The workshop leader should introduce the idea of number sense in the intermediate grades by guiding the teacher participants in a discussion of the following questions.

- *What types of numbers do intermediate grade students work with most often?*
  (In the US, students work with whole numbers, fractions, and decimals.)

- *What meaning do students associate with these numbers and operations on these numbers when they are represented symbolically (e.g., 2856, 5/8, 0.0378)?*

- *What types of activities might help intermediate grade level students develop meaning for numbers represented symbolically?*

The teacher participants should work in their small groups on the four of the five following activities. These activities appear on the next four pages.

Using Number Sense to Create Sums, Differences, Products, and Quotients

MMBDMS?

Creating Decimal Numbers

Multiplying by Decimals Near 0, 1/2, & 1

Using Number Sense to Describe Decimal Solutions

After the small groups finish these activities, the workshop leader should guide the teacher participants in a discussion of the following questions.

- *What types of difficulties do intermediate grade level students have related to number sense, fractions, and decimals? Why do you think they have these difficulties?*

- *How might drawing on number sense help intermediate grade students solve mathematical problems in meaningful ways?*

- *What other types of activities might be effective in helping intermediate grade level students extend and apply ideas related to number sense?*

Elementary Mathematics Module - Patterns, Relationships, and Number Sense
Extending Ideas of Number Sense - Intermediate Grade Level Activities

Using Number Sense to Create Sums, Differences, Products, and Quotients

Work together as a small group. Use the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, & 9 to construct the sums, differences, products, and quotients that satisfy the equations given below. All sums and quotients should be close to, but not equal to 1. All differences and products should be close to, but not equal to 1/2. You do not have to use all of the digits in each equation. You may only use a digit once in each equation. Construct at least three different sums, three different differences, three different products, and three different quotients. Be sure to explain how you created each sum, difference, product, and quotient.

Small Group Discussion Questions:

In what ways might this activity help intermediate grade students draw on number sense to understand addition, subtraction, multiplication, and division of fractions?

What other types of activities might help students draw on number sense to understand operations with fractions?

MMBDMS?
(Multiplication Makes Bigger Division Makes Smaller?)

Materials Needed:
Calculators

Work together as a small group. Create three different computation problems involving multiplication of whole numbers and three different computation problems involving division of whole numbers. Solve each problem; use a calculator if available. Examine the multiplication problems. What do you notice about the size of the factors compared to the size of the product? Examine the division problems. What do you notice about the size of the quotient compared to the size of the dividend? Make a hypothesis based on your examination.

Create three different computation problems involving multiplication of proper fractions (fractions less than one). Create three different computation problems involving division of proper fractions. One division problem must involve a divisor that is larger than the dividend, and one problem must involve a divisor that is smaller than the dividend. Solve each problem. Examine the multiplication problems. What do you notice about the size of the factors compared to the size of the product? Examine the division problems. What do you notice about the size of the quotient compared to the size of the dividend? When is the quotient greater than one? When is the quotient less than one? Make a hypothesis based on your examination.

Create three different computation problems involving multiplication of decimals. One multiplication problem must involve decimal numbers less than one, and one problem must involve decimal numbers greater than one. Create three different computation problems involving division of decimals. One division problem must involve decimal numbers less than one, and one problem must involve decimal numbers greater than one. Solve each problem; use a calculator if available. Examine the multiplication problems. What do you notice about the size of the factors compared to the size of the product? Examine the division problems. What do you notice about the size of the quotient compared to the size of the dividend? When is the quotient greater than one? When is the quotient less than one? Make a hypothesis based on your examination.

Reflect on your examinations of the different multiplication and division problems involving whole numbers, fractions, and decimals to answer the question below. If needed, create and solve additional problems.

When does multiplication make bigger and division make smaller?

Small Group Discussion Questions:

Why do intermediate grade level students often think that "multiplication always makes bigger and division always makes smaller"?

How might this activity help students understand that multiplication does not always make bigger and division does not always make smaller?

How does understanding when multiplication makes bigger and when division makes smaller extend intermediate grade level students' number sense?

Creating Decimal Numbers*

Materials Needed:

dice (3)
3 quarter sheets of paper or index cards
pencils
paper

Write "tenths" on one sheet of paper or index card. Write "hundredths" on a second card and "thousandths" on a third card. Place these cards face down in the center of the table.

Each person in the group is considered a "player". When it is a players turn, s/he rolls the dice and makes the largest three-digit number possible from the numbers that appear. For example, if "3", "2", and "7" appear on the dice, the player should make 732. After making the three-digit number, the player draws one card from the center of the table. If the player draws "tenths", s/he must change the number into a decimal number whose smallest digit is in the tenths place (e.g., 73.2). If the player draws "hundredths", s/he must change the number into a decimal number whose smallest digit is in the hundredths place (e.g., 7.32). Similarly for "thousandths" (e.g., 0.732).

Each time it is a players turn, s/he must add the new number to his/her previous number, or total of previous numbers (e.g., 7.32 + 56.1). The first player to reach 100 wins.

Repeat the game by starting at 100 and subtracting decimal numbers to reach 0.

Small Group Discussion Questions:

How might this activity help develop intermediate grade level students' number sense related to decimals?
What other types of activities might help students develop number sense for decimals?

***** This activity was shared by Ms. Susan Roehrig, a 5th grade teacher at Heritage Elementary School, Murrysville, PA, USA.

Multiplying by Decimals Near 0, 1/2, and 1

Materials Needed:
Calculators

Work together as a small group. What happens when you multiply a whole number by a number less than one? Explore this question by completing the table below. Use a calculator if one is available.

Choose a Whole Number	Multiply by 0.05	Multiply by 0.48	Multiply by 0.9

In general, what happens when you multiply a whole number by 0.05?
In general, what happens when you multiply a whole number by 0.48?
In general, what happens when you multiply a whole number by 0.9?

Revise this activity to help intermediate grade students explore what happens when they multiply a whole number by other common decimal numbers.

Small Group Discussion Questions:
What mathematical ideas do intermediate grade students need to know in order to understand multiplication of decimal numbers?
In what ways does this activity involve students in drawing on number sense to construct their own understanding of multiplication of decimal numbers?

Using Number Sense to Describe Decimal Solutions

Work together as a small group. Without calculating the answers, tell at least three (3) things that you know about the solution to each of the following problems.

146 x 0.76 = 7.8 x 0.978 = 45.1 x 1.5 =
16 ÷ 0.497 = 39.5 ÷ 0.95 = 436.2 ÷ 0.0302 =

Create two problems involving multiplication and division of decimals. Do not solve the problems, but tell three (3) things about the solution to each problem.

Small Group Discussion Questions:
In what ways might this activity help develop intermediate grade level students' number sense and meaning for multiplication and division of decimals?
What other types of activities might help develop students' number sense related to multiplication and division of decimals?

4. Connecting Patterns, Relationships, and Number Sense (all elementary grade levels; approximate time: 1 1/2 hours)

The workshop leader should guide the teacher participants in a discussion of the following questions.

- *Why might it be important for elementary students to connect ideas related to patterns, relationships, and number sense?*
- *In what ways have you explored connections between patterns, relationships, and number sense in earlier activities in this workshop?*
- *What other types of activities might we use to help elementary students form connections between ideas of patterns, relationships, and number sense?*

After this discussion, the teacher participants should work in their small groups on at least three of the following seven activities. These activities appear on the next five pages.

Guess My Number

The Number on My Back

What This Number Means To Me

If the Answer is ___, What Might the Question Be?

Do You Agree or Disagree?

Folktales, Fairytales, and Numbers

Number Adventures

After the small groups finish these activities, the workshop leader should guide the teacher participants in a discussion of the following questions.

- *In what ways do these activities tie together ideas related to patterns, relationships, and number sense?*
- *In what ways do these activities involve students' in their own learning of mathematics in the elementary school?*

- *What other types of activities might help elementary students form connections between ideas of patterns, relationships, and number sense?*

Elementary Mathematics Module - Patterns, Relationships, and Number Sense
Connecting Patterns, Relationships, and Number Sense Activities

Guess My Number

Materials Needed:
Pencil
Paper

Work together in pairs in your small group. This activity is similar to the game "Twenty Questions."

One person thinks of a number (whole number, fraction, or decimal) and writes the number on a piece of paper. The second person tries to determine his/her partner's number by asking questions that can be answered with only "yes" or "no" (e.g., Is your number a fraction?, Is your fraction greater than one half?, etc.).

Each time the second person asks a question, s/he records the question on a piece of paper. The question may be recorded symbolically (e.g., $x > 1/2$?) or in words (e.g., Is the fraction greater than one half?).

The second person tries to find his/her partner's number in 20 questions or less. The first partner keeps track of the number of questions asked.

After the number is found, the partners examine the list of questions asked and try to identify places where the second person could have asked a different question to get more information about the number. The partners should also suggest 1-2 questions that could have been asked to gain important information.

Change roles and perform the activity again.

Small Group Discussion Questions:
In what ways might this activity help elementary students form connections between patterns, relationships, and number sense?
To what extent does this activity involve students in constructing their own knowledge of mathematics?

Number On My Back

Materials Needed:

Markers
Index Cards or one quarter sheets of paper
Tape or Paper Clips

This activity should be performed in a large group. Each person writes a number (whole number, fraction, decimal, negative number, number raised to a power, or etc.) on an index card or quarter sheet of paper. The person writing the number should keep it a secret.

Everyone participating in the activity should stand in a circle, face counter-clockwise or clockwise (it does not matter), and attach his/her number to the back of the person in front of him/her. After everyone has a number on his/her back, the goal is for each person to determine what number is on his/her back in the exact form that it is written on the card or paper.

There are some rules to follow when trying to find your number. You may only ask questions that can be answered with "yes" or "no" (e.g., Is my number expressed as a decimal?), and you may ask only one question of a particular person at a time. After you have asked everyone in the group a question, you may begin asking questions of those people you have previously questioned.

When you determine your number, take your number off your back and put it on your front side. Continuing answering questions of other people until everyone in the group has determined his/her number.

Small Group Discussion Questions:

What strategies did you use to find your number?
What mathematical ideas did you need to know to find your number?
How might this activity help elementary students form connections between patterns, relationships, and number sense?

What This Number Means to Me

Materials Needed:

Pencil
Paper

Work together as a small group. Choose a number less than 10. What does this number mean to you? Think of at least 10 different meanings and record them on your paper. You do not need to limit your meanings to number sentences (e.g., 3 + 2 = 5). You can include diagrams, word descriptions, or anything else. Explain why you chose these 10 meanings. Choose a different number and repeat the activity.

Small Group Discussion Questions:

What meanings do you think elementary students would have for the numbers you chose? Why?
If you used an activity such as this with elementary students, what information might you gain about the students' understanding of patterns, relationships, and number sense?

If the Answer is ___, What Might the Question Be?

Materials Needed:

Pencil
Paper
Markers or Crayons

Work together in pairs in your small group. Choose an answer to a mathematical problem (e.g., 3/4, 2.75, 85, or etc.). Determine 10 questions that would have the answer you choose. Two of your questions must involve numbers expressed as fractions. Two of your questions must involve numbers expressed as decimals. Two of your questions must involve word problems, and two of your questions must involve diagrams.

After determining your answer and the 10 questions, explain to the others in your small group why you chose the answer that you did and why you wrote the specific questions that you did.

Challenge the others in your group by asking them to write 5 questions for a specific number of your choice. You may add any restrictions you wish on the types of questions they should generate. After the questions are written, discuss the answer and questions in the same manner as above.

Small Group Discussion Questions:

Why might it be helpful to use an activity such as this with elementary students?
In what ways might this activity help elementary students form connections between patterns, relationships, and number sense?
What other types of connections might this activity help elementary students form?

Do You Agree or Disagree?

Materials Needed:

Pencil
Paper

Work together in pairs in your small group. Generate a problem and an answer arrived at by an imaginary person. The problem may be a computation problem or an open-ended problems solving problem. Additionally, the answer provided by the imaginary person may be either correct or incorrect.

Sample problem: Which number does not belong?
Sarah thinks the answer is 6.

6	12
10	13

Switch problems and answers with the other pair in your small group. Write a letter to the person who solved this problem explaining if you agree or disagree with his/her answer and why.

As a small group, share your letters with the others. Discuss any questions or concerns that arise as you share your letters.

Small Group Discussion Question:

In what ways does this activity involve elementary students in constructing their own knowledge of mathematics?

Folktales, Fairytales, and Numbers

Materials Needed:

Pencil
Paper
various pieces of children's literature involving folktales and fairytales

Work together as a small group. Choose one folktale or fairytale that would be familiar to elementary students. Revise the folk or fairytale in a way that involves making numbers explicit. Write 5 word problems based on the story that would be appropriate for primary grade level students to solve. The word problems should focus on patterns, relationships, and number sense. Also, write 5 word problems that would be appropriate for intermediate grade level students.

Share your revised folktale or fairytale and word problems with another small group. Discuss ways that other pieces of children's literature could be revised to include numbers. Discuss what types of word problems could be generated from other stories that could help elementary students form connections between patterns, relationships, and number sense.

Small Group Discussion Questions:

How might this activity help elementary students form connections between patterns, relationships, and number sense?
How might you actively involve elementary students in revising folktales or fairytales and generating their own problems from the stories?

Number Adventures

Materials Needed:

Pencil
Paper

Work together in pairs in your small group. Choose one specific type of number (e.g., fractions, decimals, etc.) and three specific mathematical ideas (e.g., equivalent fractions, adding fractions, comparing fractions). Write an adventure story involving the type of number and mathematical ideas you chose. The numbers should possess human, or super human, characteristics and have adventures that involve the mathematical ideas you chose. For example, if you choose to write about fractions, you may have characters named “One Half”, “One Fourth”, and “One Eighth” who all live in “Wholeville”. Their adventures could involve such things as working together to form larger fractions to accomplish some task.

Your story should be one that you could use with elementary school students to help them understand the mathematical ideas you chose.

Share your story with others in your small group. Explain why you chose your particular type of number and the three mathematical ideas. Also, explain what you would want students to learn from your story.

Small Group Discussion Questions:

How might you actively involve elementary students in creating their own number adventure stories?

What information might you gain about elementary students’ understanding of patterns, relationships, and number sense from the number stories they create?

III. Closing (approximate time: 30 minutes)

The workshop leader should close the workshop by communicating to the teacher participants that the trends, recommendations, and activities they have experienced during the workshop are the current focus of mathematics education in the elementary school in the United States. The leader should suggest that although these activities and directions for elementary school mathematics have proven effective in the United States, only the teacher participants themselves can determine if the information presented and the activities they have engaged in will be helpful to them in their own teaching.

The workshop leader should suggest that one way to reflect on what aspects of the workshop might be helpful to each individual is by considering the questions listed below. The workshop leader should guide the teacher participants in a discussion of the following questions.

- *What have you learned from this workshop about current tends and recommendations for the curriculum, learning, and the teaching of mathematics in the elementary school?*
- *What have you learned from this workshop about ways to actively engage students in their own learning of mathematics in the elementary school?*
- *How might you extend the ideas presented in this workshop to other areas of mathematics in the elementary school?*
- *How might you extend the ideas presented in this workshop to other areas of the curriculum for the elementary school?*
- *What have you learned from this workshop that you think will be helpful to you as you teach elementary students? How might you incorporate this into your own classroom?*
- *What other things would you like to know about the curriculum, the learning, and the teaching of mathematics in the elementary school? How might you gain this information?*

The workshop leader should thank all of the teacher participants for their participation in the workshop. The workshop leader should also share the "Selected Bibliography" with the teacher participants if they desire more

information about ways to teach to actively involve elementary students in constructing their own knowledge of mathematics.

End of Workshop

Selected Bibliography

National Council of Teachers of Mathematics. *Addenda Series: Grade K.* Reston, VA: The Council, 1991.

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Payne, Joseph N. *Mathematics for the Young Child.* Reston, VA: National Council of Teachers of Mathematics, 1990.

Reys, Robert E., Marilyn N. Suydam, and Mary Montgomery Lindquist. *Helping Children Learn Mathematics (4th Edition).* Boston: Allyn and Bacon, 1995.

Thiessen, Diane and Margaret Matthias. *The Wonderful World of Mathematics.* Reston, VA: National Council of Teachers of Mathematics, 1992.

Welchman-Tischler, Rosamond. *How to Use Children's Literature to Teach Mathematics.* Reston, VA: National Council of Teachers of Mathematics, 1992.

Appendix

Masters

- NCTM Curriculum Standards for Grades K-4
- NCTM Curriculum Standards for Grades 5-8
- NCTM Professional Teaching Standards
- Guidelines for the Learning and Teaching of Mathematics
- Pattern Blocks
- Colored Squares
- Addition Basic Facts Table
- Multiplication Basic Facts Table
- Hundreds Chart
- Fraction Circles
- Fraction Strips
- Base 10 Materials
- Grid Paper