Adding and Subtracting Polynomials

Example: Perform the indicated operation and / or simplify.

$$(2x^2 + 4x - 10) - (6x^2 - x - 1) + (5x^2 - 6x + 3)$$

$$= \quad 2x^2 + 4x - 10 \quad - \quad 6x^2 + x + 1 \quad + \quad 5x^2 - 6x + 3$$

$$= \quad (2x^2 - 6x^2 + 5x^2) + (4x + x - 6x) + (-10 + 1 + 3)$$

$$\boxed{= \quad x^2 - x - 6}$$

Multiplying Polynomials

The product of two polynomials can be found by using some form of the distributive property $a(b + c) = ab + ac$.

Example: Multiply the given polynomials and simplify.

a) $(x + 4)(2x - 3)$

$= x(2x - 3) + 4(2x - 3)$

$= 2x^2 - 3x + 8x - 12$

$\boxed{= 2x^2 + 5x - 12}$

b) $(3x - 5)(x - 2)$

$= 3x(x - 2) - 5(x - 2)$

$= 3x^2 - 6x - 5x + 10$

$\boxed{= 3x^2 - 11x + 10}$

c) $(2x - 3y)(x + 2y)$

$= 2x(x + 2y) - 3y(x + 2y)$

$= 2x^2 + 4xy - 3xy - 6y^2$

$\boxed{= 2x^2 + xy - 6y^2}$

d) $(z - 1)(z - 2)$

$= z(z - 2) - 1(z - 2)$

$= z^2 - 2z - z + 2$

$\boxed{= z^2 - 3z + 2}$

e) $(x + 2y - 3)(2x - y - 2)$

$= x(2x - y - 2) + 2y(2x - y - 2) - 3(2x - y - 2)$

$= 2x^2 - xy - 2x + 4xy - 2y^2 - 4y - 6x + 3y + 6$

$\boxed{= 2x^2 - 2y^2 + 3xy - 8x - y + 6}$

f) $(x + 2)(x^2 - 2x + 4)$

$= x(x^2 - 2x + 4) + 2(x^2 - 2x + 4)$

$= x^3 - 2x^2 + 4x + 2x^2 - 4x + 8$

$\boxed{= x^3 + 8}$

We use the table below for reference to help with multiplication.

$(F + S)(F - S)$	$=$	$F^2 - S^2$	Difference of squares
$(F + S)^2$	$=$	$F^2 + 2FS + S^2$	Perfect square trinomial
$(F - S)^2$	$=$	$F^2 - 2FS + S^2$	Perfect square trinomial

Example: Perform the indicated products and simplify.

a) $(x + 3y)(x - 3y)$

$= (F + S)(F - S)$

$= F^2 - S^2$

$\boxed{= x^2 - 9y^2}$

b) $(2xy - 5z)(2xy + 5z)$

$= (F - S)(F + S)$

$= F^2 - S^2$

$\boxed{= 4x^2y^2 - 25z^2}$

c) $(6w^2 + 7v^5)(6w^2 - 7v^5)$

$= (F + S)(F - S)$

$= F^2 - S^2$

$\boxed{= 36w^4 - 49v^{10}}$

d) $(x - 4)^2$

$= (F - S)^2$

$= F^2 - 2FS + S^2$

$\boxed{= x^2 - 8x + 16}$

e) $(4x - 3y)^2$

$= (F - S)^2$

$= F^2 - 2FS + S^2$

$\boxed{= 16x^2 - 24xy + 9y^2}$

f) $(7w + 5)^2$

$= (F + S)^2$

$= F^2 + 2FS + S^2$

$\boxed{= 49w^2 + 70w + 25}$

g) $(x^3 + 8y)^2$

$= (F + S)^2$

$= F^2 + 2FS + S^2$

$\boxed{= x^6 + 16x^3y + 64y^2}$

h) $(2x^2z - 3y^4)^2$

$= (F - S)^2$

$= F^2 - 2FS + S^2$

$\boxed{= 4x^4z^2 - 12x^2y^4z + 9y^8}$

Special Products:
Difference of Squares

Complete the table:

F	**S**	**F^2**	**S^2**	**$(F + S)(F – S)$**	**=**	**$F^2 – S^2$**
x	3			$(x + 3)(x – 3)$	=	_____
z	4			$(z + 4)(z – 4)$	=	_____
w	5			$(w + 5)(w – 5)$	=	_____
x	2			$(x – 2)(x + 2)$	=	_____
y	1			$(y – 1)(y + 1)$	=	_____
z	6			$(z – 6)(z + 6)$	=	_____
$2x$	$3y$			$(2x + 3y)(2x – 3y)$	=	_____
$3x$	$5y$			$(3x – 5y)(3x + 5y)$	=	_____
$4x$	y^2			$(4x – y^2)(4x + y^2)$	=	_____
$2x^2$	$5yz^3$			$(2x^2+5yz^3)(2x^2–5yz^3)$	=	_____

Special Products:
Perfect Square Polynomials

Complete the table:

F	**S**	**F^2**	**2FS**	**S^2**	**$(F + S)^2$**	**=**	**$F^2 + 2FS + S^2$**
x	3				$(x + 3)^2$	=	_____
z	4				$(z + 4)^2$	=	_____
w	5				$(w + 5)^2$	=	_____
x	–2				$(x – 2)^2$	=	_____
y	–1				$(y – 1)^2$	=	_____
z	–6				$(z – 6)^2$	=	_____
2x	3y				$(2x + 3y)^2$	=	_____
3x	–5y				$(3x – 5y)^2$	=	_____
4x	–y				$(4x – y)^2$	=	_____
$2x^2$	$5yz^3$				$(2x^2 + 5yz^3)^2$	=	_____

Common Factors & Negative Exponents

Examples: Factor the following and use mathematics writing style:

a) $x^2 + 5x$

$= x(x + 5)$

b) $8z^7 + 12z^4$

$= 4z^4(2z^3 + 3)$

c) $12x^3y^2 - 32x^2y^3 + 20xy^4$

$= 4xy^2(3x^2 - 8xy + 5y^2)$

d) $9a^9b^4 - 12a^8b^6 - 6a^7b^8$

$= 3a^7b^4(3a^2 - 4ab^2 + 2b^4)$

e) $x^{-5} + 5x^{-7}$

$= x^{-7}(x^2 + 5)$

f) $8x^{-9} - 6x^{-8}$

$= 2x^{-9}(4 - 3x)$

g) $x^{-3}y^2 - x^{-4}y^6$

$= x^{-4}y^2(x - y^4)$

h) $12a^{-6}b^{-8} - 16a^{-8}b^{-6}$

$= 4a^{-8}b^{-8}(3a^2 - 4b^2)$

Factoring By Grouping

Problems: Factor the following and use mathematics writing style:

a) $ax + ay + bx + by$

$= a(x + y) + b(x + y)$

$\boxed{= (a + b)(x + y)}$

b) $ax + ay – x – y$

$= a(x + y) – 1(x + y)$

$\boxed{= (a – 1)(x + y)}$

c) $x^2 + 4xy – 2x – 8y$

$= x(x + 4y) – 2(x + 4y)$

$\boxed{= (x – 2)(x + 4y)}$

d) $xy – 3x – 4y + 12$

$= x(y – 3) – 4(y – 3)$

$\boxed{= (x – 4)(y – 3)}$

e) $x^3 – 4x^2 + 3x – 12$

$= x^2(x – 4) + 3(x – 4)$

$\boxed{= (x^2 + 3)(x – 4)}$

f) $2x^2 – 7xy + 6y^2$

$= 2x^2 – 4xy – 3xy + 6y^2$

$= 2x(x – 2y) – 3y(x – 2y)$

$\boxed{= (2x – 3y)(x – 2y)}$

Factoring Using The ac–Method

Problems: Factor the following using the ac–method:

a) $4x^2 – x – 18$

$= 4x^2 – 9x + 8x – 18$

$= x(4x – 9) + 2(4x – 9)$

$\boxed{= (x + 2)(4x – 9)}$

b) $12x^2 – 23x – 24$

$= 12x^2 – 32x + 9x – 24$

$= 4x(3x – 8) + 3(3x – 8)$

$\boxed{= (4x + 3)(3x – 8)}$

Factoring With Substitution

Problems: Factor the following and use mathematics writing style:

a) $x^2y^6 - 81z^{10}$

$= F^2 - S^2$

$= (F - S)(F + S)$

$\boxed{= (xy^3 - 9z^5)(xy^3 + 9z^5)}$

b) $x^3 + 8$

$= F^3 + S^3$

$= (F + S)(F^2 - FS + S^2)$

$\boxed{= (x + 2)(x^2 - 2x + 4)}$

c) $x^2 - (y + 2)^2$

$= F^2 - S^2$

$= (F - S)(F + S)$

$= [x - (y + 2)][x + (y + 2)]$

$\boxed{= (x - y - 2)(x + y + 2)}$

d) $(x + 2y)^2 - 25$

$= F^2 - 25$

$= (F - 5)(F + 5)$

$= [(x + 2y) - 5][(x + 2y) + 5]$

$\boxed{= (x + 2y - 5)(x + 2y + 5)}$

e) $(2x - 5)^2 - 2(2x - 5) - 8$

$= F^2 - 2F - 8$

$= (F - 4)(F + 2)$

$= [(2x - 5) - 4][(2x - 5) + 2]$

$= [2x - 5 - 4][2x - 5 + 2]$

$\boxed{= (2x - 9)(2x - 3)}$

f) $(x - 3)^2 - 9(y + 2)^2$

$= F^2 - 9S^2$

$= (F - 3S)(F + 3S)$

$= [(x-3) - 3(y+2)][(x-3) + 3(y+2)]$

$= [x - 3 - 3y - 6)][x - 3 + 3y + 6]$

$\boxed{= (x - 3y - 9)(x + 3y + 3)}$

Factoring Completely

Problems: Factor completely the following and use mathematics writing style:

a) $w^{-7} - 4w^{-9}$

$= w^{-9}(w^{22} - 4)$

$\boxed{= w^{-9}(w - 2)(w + 2)}$

b) $4 + 4z^{-1} + z^{-2}$

$= z^{-2}(4z^2 + 4z + 1)$

$\boxed{= z^{-2}(2z + 1)^2}$

c) $(x^2 - x)^2 - 18(x^2 - x) + 72$

$= F^2 - 18F + 72$

$= (F - 6)(F - 12)$

$= (x^2 - x - 6)(x^2 - x - 12)$

$\boxed{= (x - 3)(x + 2)(x - 4)(x + 3)}$

d) $x^8 - y^8$

$= (x^4 - y^4)(x^4 + y^4)$

$= (x^2 - y^2)(x^2 + y^2)(x^4 + y^4)$

$\boxed{= (x-y)(x+y)(x^2+y^2)(x^4+y^4)}$

e) $(x^2 + 3x - 10)^2 - (x - 2)^2$

$= F^2 - S^2$

$= (F + S)(F - S)$

$= [(x^2 + 3x - 10) + (x - 2)][(x^2 + 3x - 10) - (x - 2)]$

$= [x^2 + 3x - 10 + x - 2][x^2 + 3x - 10 - x + 2]$

$= (x^2 + 4x - 12)(x^2 + 2x - 8)$

$= (x + 6)(x - 2)(x + 4)(x - 2)$

$\boxed{= (x + 6)(x + 4)(x - 2)^2}$

Problems: Solve the following equations by factoring:

a) $y^2 - 6y = 27$

$y^2 - 6y - 27 = 0$

$(y - 9)(y + 3) = 0$

$y - 9 = 0 \qquad y + 3 = 0$

$y = 9 \qquad y = -3$

$\boxed{y = 9, -3}$

b) $(3z + 6)(4z + 12) = -3$

$12z^2 + 60z + 72 = -3$

$12z^2 + 60z + 75 = 0$

$3(4z^2 + 20z + 25) = 0$

$3(2z + 5)^2 = 0$

$2z + 5 = 0$

$\boxed{z = -5/2}$

c) $4x^3 = 100x$

$4x^3 - 100x = 0$

$4x(x^2 - 25) = 0$

$4x(x - 5)(x + 5) = 0$

$4x = 0 \qquad x - 5 = 0 \qquad x + 5 = 0$

$x = 0 \qquad x = 5 \qquad x = -5$

$\boxed{x = 0, 5, -5}$

d) $y^2 - 4y = 21$

$y^2 - 4y - 21 = 0$

$(y - 7)(y + 3) = 0$

$y - 7 = 0 \qquad y + 3 = 0$

$y = 7 \qquad y = -3$

$\boxed{y = 7, -3}$

e) $3x^2 - 16x + 5 = 0$

$(x - 5)(3x - 1) = 0$

$x - 5 = 0 \qquad 3x - 1 = 0$

$x = 5 \qquad x = 1/3$

$\boxed{x = 5, 1/3}$

f) $z^2 + 6z = -9$

$z^2 + 6z + 9 = 0$

$(z + 3)^2 = 0$

$z + 3 = 0$

$\boxed{z = -3}$

g) $x(3x + 5) = x(x + 2) + 14$

$3x^2 + 5x = x^2 + 2x + 14$

$2x^2 + 3x - 14 = 0$

$(x - 2)(2x + 7) = 0$

$x - 2 = 0 \qquad 2x + 7 = 0$

$x = 2 \qquad x = -7/2$

$\boxed{x = 2, -7/2}$

h) $144x^3 + 10x^2 = 50x$

$144x^3 + 10x^2 - 50x = 0$

$2x(72x^2 + 5x - 25) = 0$

$2x(8x + 5)(9x - 5) = 0$

$2x = 0 \quad 8x+5 = 0 \quad 9x-5=0$

$x = 0 \quad x = -5/8 \quad x = 5/9$

$\boxed{x = 0, -5/8, 5/9}$