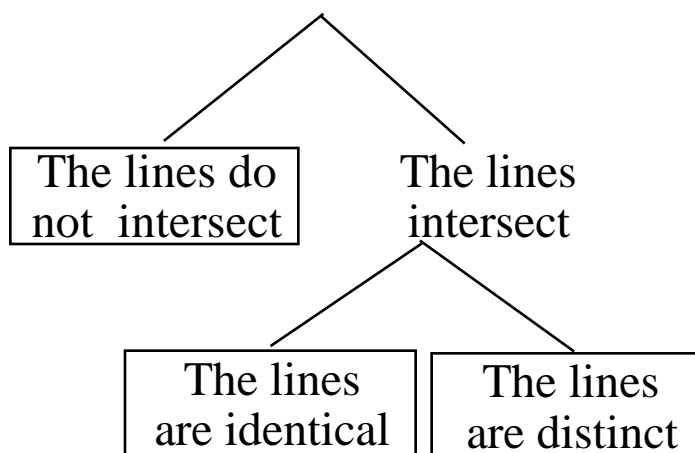


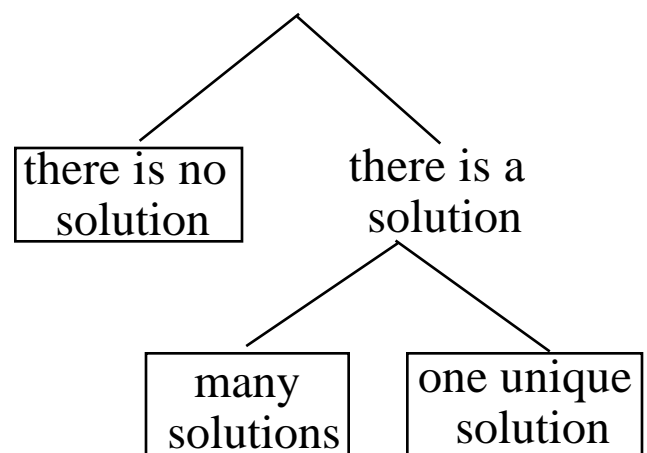
Systems of Linear Equations and Inequalities

Recall that every linear equation in two variables can be identified with a line. When we group two such equations together, we know from geometry what can happen with two lines:

graph of two lines in plane



solving systems of two equations



When solving a system of two equations of two unknowns, if you get an equation like $0 = 1$, then there can be no solution. If, on the other hand, you get an equation like $0 = 0$, then the system is (probably) dependent.

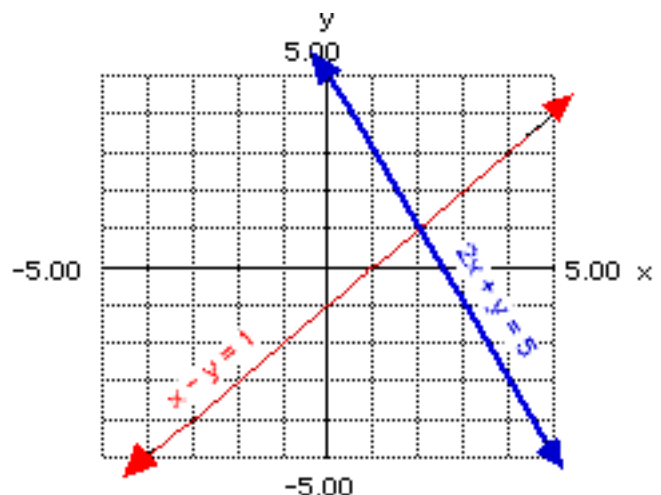
Example 1: Consider the system

$$\begin{aligned} 2x + y &= 5 \\ x - y &= 1 \end{aligned}$$

The solution is $x = 2, y = 1$.

The lines intersect at the point $(2,1)$.

We call the system **consistent**.



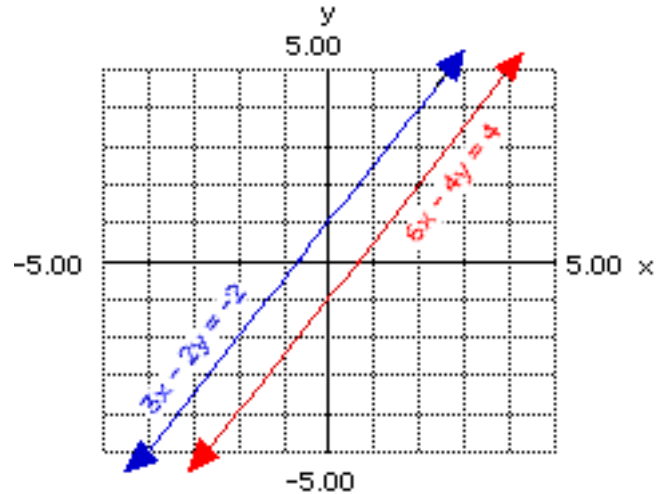
Example 2: Consider the system

$$\begin{aligned} 3x - 2y &= -2 \\ 6x - 4y &= 4 \end{aligned}$$

There is no solution.

The lines are parallel and distinct.

We call the system **inconsistent**.



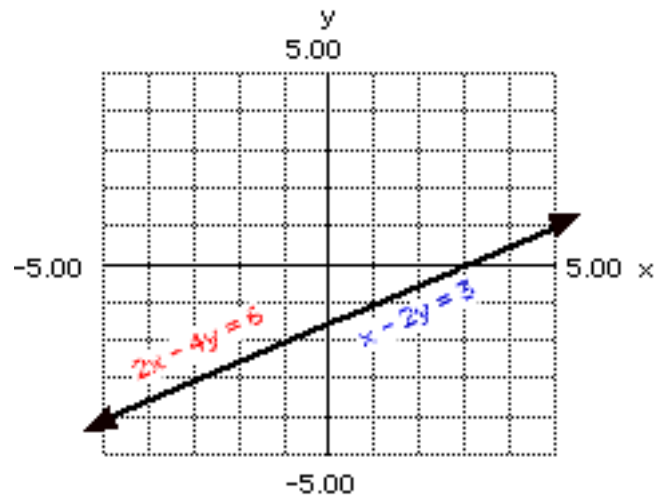
Example 3: Consider the system

$$\begin{aligned} 2x - 4y &= 6 \\ x - 2y &= 3 \end{aligned}$$

There are infinitely many solutions.
(Any values of x and y that satisfy $x - 2y = 3$)

The lines are identical.

We call the system **dependent**.



Solving Systems by Substitution

Step 1: Choose any equation and solve for one variable in terms of the other variable.

Step 2: Substitute that result into the other equation.

Step 3: Solve for the second variable.

Step 4: Choose the equation obtained in Step 1 to solve for the first variable.

Example: Solve
$$\begin{aligned} 2x + y &= 5 \\ x - y &= 1 \end{aligned}$$

Step 1: Choose any equation and solve for one variable in terms of the other variable.

We'll take the first equation
$$2x + y = 5$$

and we'll solve for y:
$$y = -2x + 5$$

Step 2: Substitute that result into the other equation.

$$\begin{aligned} x - y &= 1 \\ x - (-2x + 5) &= 1 \end{aligned}$$

Step 3: Solve for the second variable.

$$\begin{aligned} x + 2x - 5 &= 1 \\ 3x &= 6 \\ x &= 2 \end{aligned}$$

Step 4: Choose the equation obtained in Step 1 to solve for the first variable.

$$\begin{aligned} y &= -2x + 5 \\ &= -2(2) + 5 \\ &= 1 \end{aligned}$$

$\begin{aligned} x &= 2 \\ y &= 1 \end{aligned}$
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Solving Systems by Addition

Step 0: Write the system in standard form.

Step 1: Choose one variable and find the least common multiple of the coefficients of that variable.

Step 2: Multiply each equation by an appropriate factor to get the LCM as one coefficient and $-$ LCM as the other coefficient.

Step 3: Add the two equations. Solve for the remaining variable.

Step 4: Choose any original equation to solve for the other variable.

Example: Solve
$$\begin{aligned} 4x + 2y &= 10 \\ 6x - 6y &= 6 \end{aligned}$$

Step 1: Choose one variable and find the least common multiple of the coefficients of that variable.

To eliminate the x 's, the LCD is 12.

Step 2: Multiply each equation by an appropriate factor to get the LCM as one coefficient and $-$ LCM as the other coefficient.

We can multiply the first by 3 and the second by -2

$$\begin{aligned} 12x + 6y &= 30 \\ -12x + 12y &= -12 \end{aligned}$$

Step 3: Add the two equations. Solve for the remaining variable.

$$\begin{aligned} 18y &= 18 \\ y &= 1 \end{aligned}$$

Step 4: Choose any original equation to solve for the other variable.

$$\begin{aligned} 4x + 2y &= 10 \\ 4x + 2(1) &= 10 \\ 4x &= 8 \\ x &= 2 \end{aligned}$$

$\begin{aligned} x &= 2 \\ y &= 1 \end{aligned}$
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Hints for Solving Mixture-type Applications

When we solve mixture-type problems, we add mixture A to mixture B to get mixture C. A, B, and C are identified by being a percentage of some quantity. We usually get a chart as follows:

	amount	=	rate (%)	•	quantity
mixture A			a%		x
mixture B			b%		y
total			c%		x+y

which produces the system of equations

$$\begin{array}{rclclcl} x & + & y & = & (x + y) \\ (a\%)x & + & (b\%)y & = & (c\%)(x + y) . \end{array}$$

Example 1: Lynn Pike, a pharmacist, needs 70 liters of 50% alcohol solution. She has available a 30% solution and an 80% solution. How many of each solution should she mix to obtain 70 liters of a 50% solution?

$$\text{amount} = \text{rate (\%)} \cdot \text{quantity}$$

30% solution	.30	x
80% solution	.80	y
50% solution	.50	70

$$\text{amount} = \text{rate (\%)} \cdot \text{quantity}$$

30% solution	.30x	.30	x
80% solution	.80y	.80	y
50% solution	(.50)(70)	.50	70

$$\begin{array}{rclcl} x & + & y & = & 70 \\ .3x & + & .8y & = & 35 \end{array}$$

Example 2: A Candy Barrel shop manager mixes M&M's worth \$2.00 per pound with trail mix worth \$1.50 per pound. Find how many pounds of each she should use to get 50 pounds of a party mix worth \$1.80 per pound.

	amount	=	rate	•	quantity
M&M's			2.00		x
trail mix			1.50		y
party mix			1.80		50

	amount	=	rate	•	quantity
M&M's	2.00x		2.00		x
trail mix	1.50y		1.50		y
party mix	(1.80)(50)		1.80		50

$$\begin{array}{rclclcl}
 x & + & y & = & 50 \\
 2x & + & 1.5y & = & 90
 \end{array}$$

Example 3: Working his way through school, Richard works two part-time jobs for a total of 23 hours a week. Job A pays \$4.50 per hour and job B pay \$6.30 per hour. How many hours did he work on each job the week he made \$137.70?

	amount	=	rate	•	hours
Job A			4.5		A
Job B			6.3		B
total	137.70				23

	amount	=	rate	•	hours
Job A	4.5A		4.5		A
Job B	6.3B		6.3		B
total	137.7				23

$$\begin{array}{rclclcl}
 & A & + & B & = & 23 \\
 4.5A & & + & 6.3B & = & 137.7
 \end{array}$$

Example 4: An airplane takes 3 hours to travel a distance of 2160 miles with the wind. The return trip takes 4 hours against the wind. Find the speed of the plane in still air and the speed of the wind.

Speed of plane: p
 Speed of wind: w

	distance	=	rate	•	time
going	2160				3
returning	2160				4

	distance	=	rate	•	time
going	2160		720		3
returning	2160		540		4

$$\begin{array}{rclclcl}
 p & + & w & = & 720 \\
 p & - & w & = & 540
 \end{array}$$