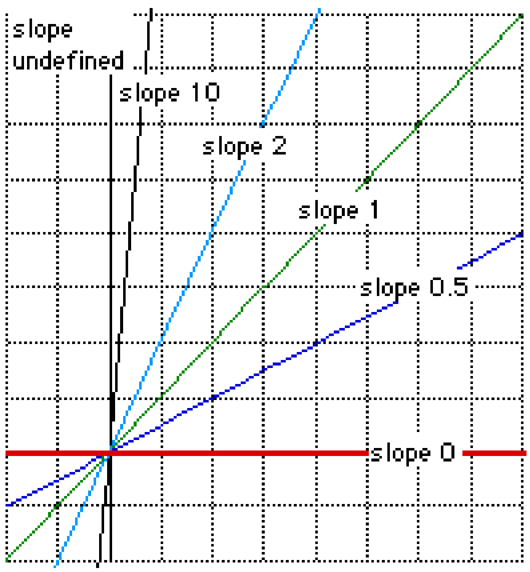
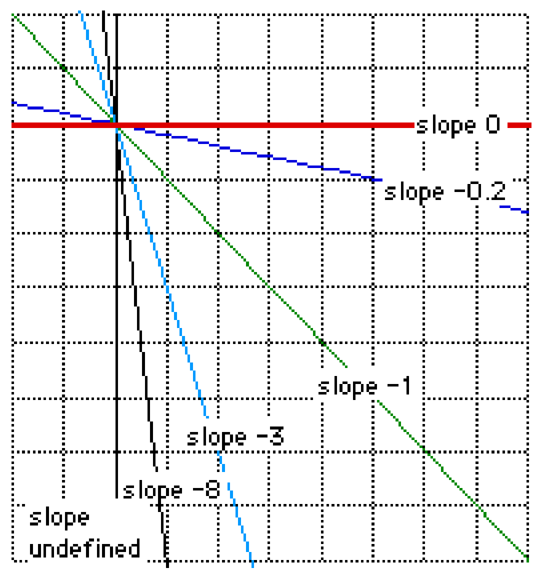
Lines and Linear Equations

Slopes





Consider walking on a line from left to right. The slope of a line is a measure of its steepness. A positive slope rises and a negative slope falls. A slope of zero means the line is horizontal. The slope of a line is undefined when the line is vertical.

The slope m of the line passes through the points (x_1,y_1) and (x_2,y_2) is given by $m = \dfrac{y_2 - y_1}{x_2 - x_1}$.

Example: Find the slope of the line passing though the given points:

a) (–1, 2) and (3, 5)

$$m = \frac{5 - 2}{3 - (-1)} = \frac{3}{4} \blacktriangleleft$$

b) (0, 1) and (2, 6)

$$m = \frac{6 - 1}{2 - 0} = \frac{5}{2} \blacktriangleleft$$

c) (5, 1) and (1, 3)

$$m = \frac{3 - 1}{1 - 5} = \frac{2}{-4} = -\frac{1}{2} \blacktriangleleft$$

d) (2, –3) and (–1, –9)

$$m = \frac{-9 - (-3)}{-1 - 2} = \frac{-6}{-3} = 2 \blacktriangleleft$$

e) (1, 3) and (4, 6)

$$m = \frac{6 - 3}{4 - 1} = \frac{3}{3} = 1 \blacktriangleleft$$

f) (3, 6) and (1, 6)

$$m = \frac{6 - 6}{1 - 3} = \frac{0}{-2} = 0 \blacktriangleleft$$

g) (–3, 2) and (–3, 5)

$$m = \frac{5 - 2}{-3 - (-3)} = \frac{3}{0}$$

slope undefined ◀

Various Linear Equations

Standard Form

An equation that can be written in the form

$$Ax + By = C, \quad (A, B \text{ not both zero})$$

is a linear equation in the variables x and y. This form is called the standard form.

Intercepts: The x–intercept is the point (if any) where the line crosses the x–axis; and the y–intercept is the point (if any) where the line crosses the y–axis.

To find the x–intercept, let $y = 0$ and solve the equation for x.
To find the y–intercept, let $x = 0$ and solve the equation for y.

Point–Slope Form

The equation of the line passing through the point (x_1, y_1) with slope m is given by

$$y - y_1 = m(x - x_1),$$

and we call this the point–slope form of the equation of the line. Since we need point information and slope information, this is the form used most often to find the equation.

Slope–Intercept Form

The equation of the line with slope m and y–intercept b is given by

$$y = mx + b,$$

and we call this the slope–intercept form of the equation of the line. Given the equation of a line, if we can write the equation of the line into this form (by solving for y), we can determine the slope and the y–intercept of the line.

Horizontal Line Form

The equation of the horizontal line that passes through the point (a, b) is given by

$$y = b.$$

Vertical Line Form

The equation of the vertical line that passes through the point (a, b) is given by

$$x = a.$$

Finding Equations Of Lines

To find the equation of any line, you always need two types of information: slope information and point information.

Point information includes:
- any point the line passes through
- y–intercept of the line
- x–intercept of the line

Slope information includes:
- slope of line
- two points the line passes through
- knowing line is horizontal or vertical
- any parallel or perpendicular line

The basic equations of the line we use are listed:

1. **Point–Slope** $y - y_1 = m(x - x_1)$
2. **Slope–Intercept** $y = mx + b$
3. **Horizontal line** $y = b$
4. **Vertical line** $x = a$

Helpful information to remember:

a. The slope of the line passing through two points is $m = \dfrac{y_2 - y_1}{x_2 - x_1}$.

b. Slopes of parallel lines are equal.

c. Slopes of perpendicular lines are negative reciprocals.

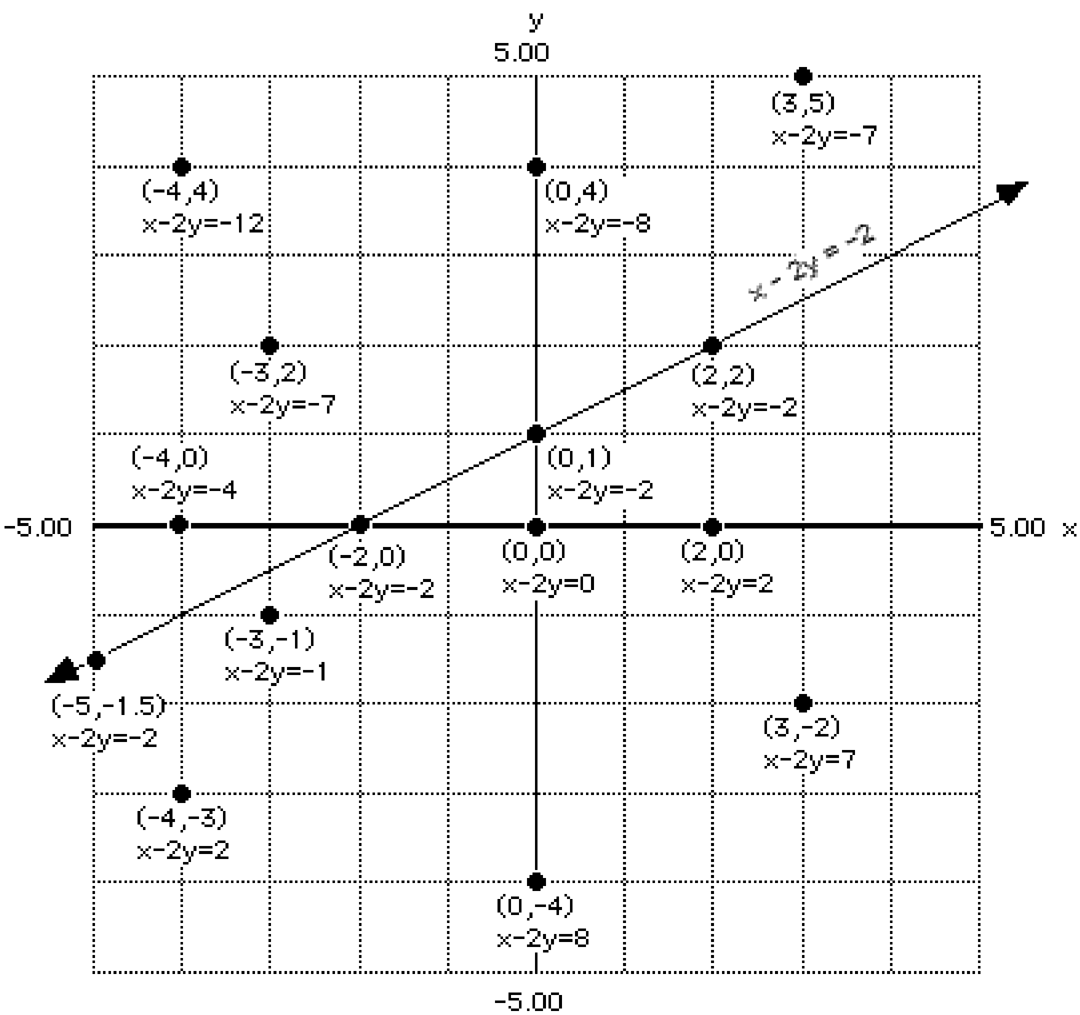
Linear Inequalities In Two Variables

Consider the linear equation x – 2y = –2 and the graph below. Careful observation reveals:

a) If the point (x,y) is on the line then x – 2y = –2, and if x – 2y = –2, then the point (x,y) is on the line.

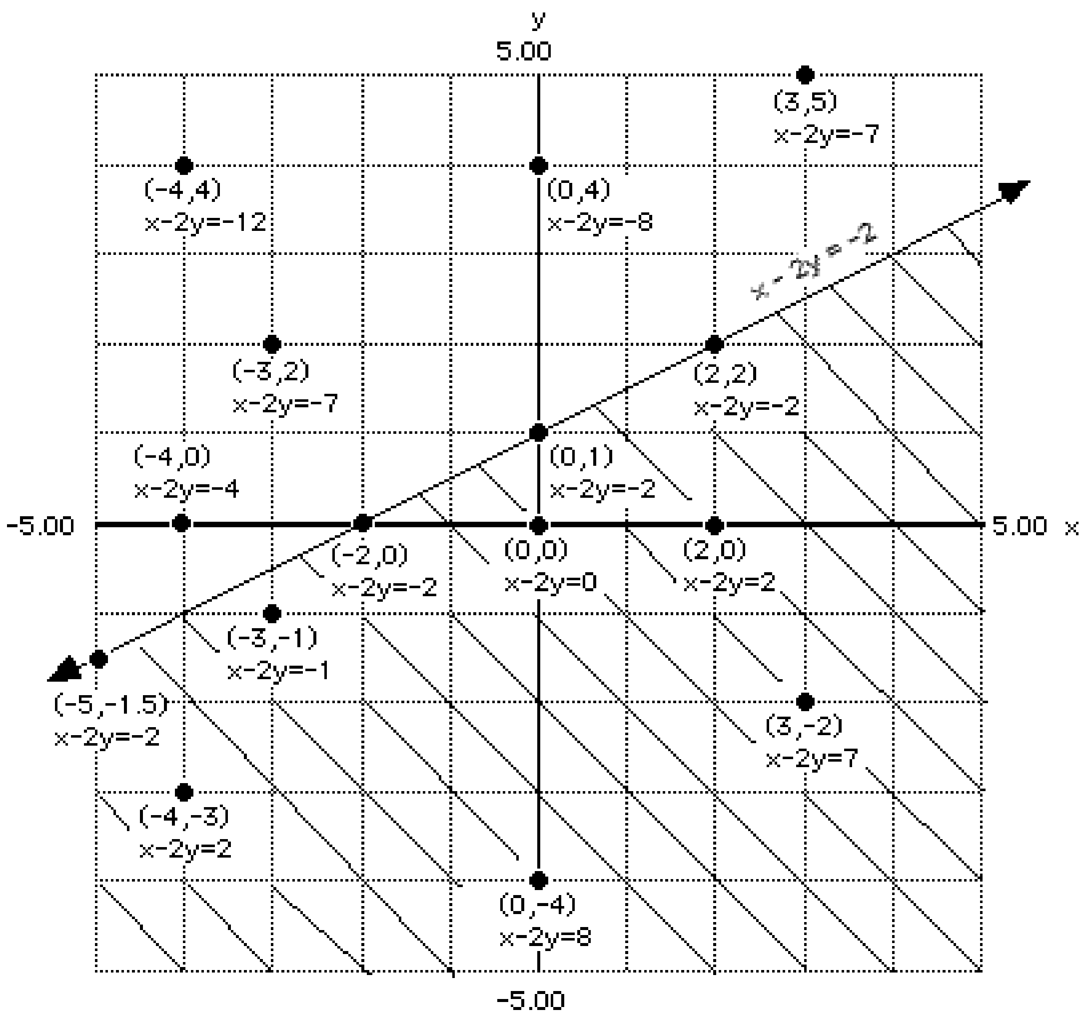
b) Every point (x,y) not on the line is in a region such that every point in that region satisfies either x – 2y < –2 or x – 2y > –2.

c) If x – 2y < –2 or x – 2y > –2, then the point (x,y) is is in a region such that every point in that region satisfies the same inequality.



Graphing Linear Inequalities In Two Variables (GTS)

1. Graph the boundary line (solid for ≤ and ≥, dotted for < and >).
2. Pick any test point not on the line. Does it satisfies the inequality?
3. If yes, shade that side of the line. If not, shade the other side.



The graph of $x - 2y \geq 0$ consists of the line and the shaded region.

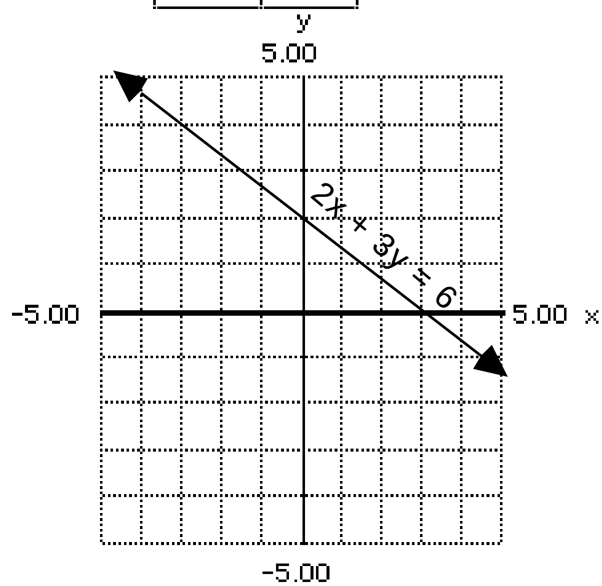
Example 1: Graph the following inequality: $2x + 3y \geq 6$.

Step 1. Graph the boundary line:
$2x + 3y = 6$

1a) Find (at least) three points on the line:

x	y
0	2
3	0
–3	4

1b) Graph the line (solid line since we have "≥"):



Step 2. Test any side

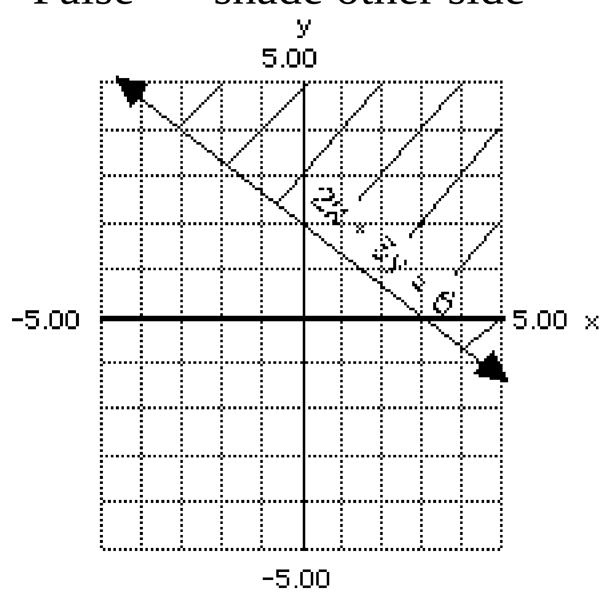
2a) Pick any test point NOT on the line.

Test point: (0,0)

2b) Does it satisfies the inequality?

T or F? $2(0)+3(0) \geq 6$
T or F? $0 \geq 6$ F!!
False => shade other side

Step 3. Shade the appropriate side:



Example 2: Graph the following inequality: $x - 2y < 4$.

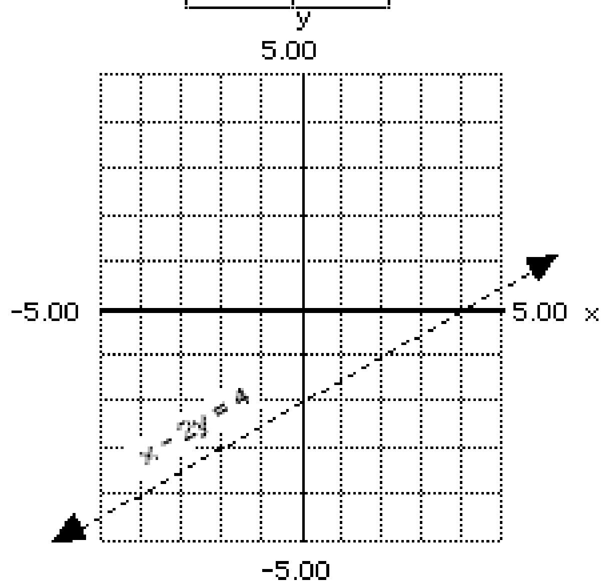
Step 1. Graph the boundary line:

$x - 2y = 4$

x	y
0	–2
4	0
–4	–4

1a) Find (at least) three points on the line:

1b) Graph the line (dotted or dashed line since we have “>”):



Step 2. Test any side

2a) Pick any test point NOT on the line.

Test point: (0,5)

2b) Does it satisfies the inequality?

T or F? $(0)-2(5) \le 4$

T or F? $-10 \le 4$ T!!

True => shade same side

Step 3. Shade the appropriate side: