Position vector $\mathbf{r}$

- Has Magnitude and Direction
-Locates a Point Relative to Another Point

\[
\mathbf{r}_{AB} = x\mathbf{\hat{x}} + y\mathbf{\hat{y}} + z\mathbf{\hat{z}}
\]

\[
\mathbf{r}_{BA} = -x\mathbf{\hat{x}} - y\mathbf{\hat{y}} - z\mathbf{\hat{z}}
\]
Write the vector $r_{AB}$ in Cartesian vector notation:

$$\vec{r}_{AB} = -4\hat{i} + 5\hat{j} + 3\hat{k}$$

Answer:

$$r_{AB} = 7.071$$

A(4, 0, 3)

B(0, 5, 6)

Write the force vector $F$ in Cartesian vector notation:

$$\vec{F} = 10\text{lb}$$

$$\vec{F} = 10\hat{i}\text{lb}$$

$$\vec{u}_{AB} = \frac{r_{AB,x}}{r_{AB}}\hat{i} + \frac{r_{AB,y}}{r_{AB}}\hat{j} + \frac{r_{AB,z}}{r_{AB}}\hat{k}$$

$$\vec{u}_{AB} = -\frac{4}{7.071}\hat{i} + \frac{5}{7.071}\hat{j} + \frac{3}{7.071}\hat{k}$$

$$\vec{u}_{AB} = -0.566\hat{i} + 0.707\hat{j} + 0.424\hat{k}$$

$$\vec{F} = (10)(-0.566\hat{i} + 0.707\hat{j} + 0.424\hat{k})$$

$$\vec{F} = -5.66\hat{i} + 7.07\hat{j} + 4.24\hat{k}$$

[16]
Section 2.8 Force vector directed along a line

Problem 1 (Reference 3D Model)
Given: A person is positioned as shown below. He pulls the cable with a force of 70 lb.
Required: Write this force in terms of a vector.

Solution:

Approach:
Step 1 Determine the position vector \( \mathbf{r}_{AB} \)

\[
\mathbf{r}_{AB} = 12\hat{i} - 8\hat{j} - 24\hat{k} \text{ [ft]}
\]

Also, magnitude of \( \mathbf{r}_{AB} \) is

\[ r_{AB} = 28 \text{ ft.} \]

Step 2: Determine the unit vector of this position vector

\[
\mathbf{u}_{AB} = \frac{\mathbf{r}_{AB}}{r} = \frac{12}{28} \hat{i} + \frac{-8}{28} \hat{j} + \frac{24}{28} \hat{k} \text{ [ft]}
\]

\[= 0.429\hat{i} - 0.286\hat{j} - 0.857\hat{k} \text{ [ft]} \]

Step 3: Multiply the magnitude of this force to the vector \( \mathbf{u}_{AB} \)

\[
\mathbf{F}_{AB} = F \mathbf{u}_{AB} = (70)(0.429\hat{i} - 0.286\hat{j} - 0.857\hat{k}) \text{ [lb]}
\]

\[= 30.03\hat{i} - 20.02\hat{j} - 59.99\hat{k} \text{ [lb]} \]
Problem 2.86. Determine the position vector $\mathbf{r}$ directed from point $A$ to point $B$ and the length of cord $AB$. Take $z=4$ m.

Given: Above

Required: $\mathbf{r}_{AB}$

Solution:

$$\mathbf{r}_{AB} = -3\hat{x} + 6\hat{y} + 2\hat{z}\text{ m}$$

Answer
Problem 2-101. The cable \( AO \) exerts a force on the top of the pole of \( F = \{-120\mathbf{i} - 90\mathbf{j} - 80\mathbf{k}\} \text{ lb} \). If the cable has a length of 34 ft, determine the height \( z \) of the pole and the location \((x, y)\) of its base.

**Given:**
\[
F = \frac{-120\mathbf{i} - 90\mathbf{j} - 80\mathbf{k}}{\sqrt{(-120)^2 + (-90)^2 + (-80)^2}} \\
F = 170 \text{ lb} \\
\mathbf{r}_{AO} = 34 \text{ ft}
\]

**Required:**
\[
(x, y) = ? \\
z = ?
\]

**Solution:**

The coordinates \((x, y)\) and the height \(z\) can be found using the Cartesian vector notation of vector \( \mathbf{r}_{AO} \):

\[
\mathbf{r}_{AO} = r_{AO,x}\mathbf{i} + r_{AO,y}\mathbf{j} + r_{AO,z}\mathbf{k}
\]

Write each term in terms of \( r_{AO} \) and the components of the unit vector \( \mathbf{u}_{AO} \):

\[
r_{AO,x} = r_{AO} \mathbf{u}_{x} = \frac{r_{AO, \mathbf{F}_x}}{F} = \frac{-120}{170} = \frac{-12}{17} \text{ ft}
\]

\[
r_{AO,y} = r_{AO} \mathbf{u}_{y} = \frac{r_{AO, \mathbf{F}_y}}{F} = \frac{-90}{170} = \frac{-9}{17} \text{ ft}
\]

\[
r_{AO,z} = r_{AO} \mathbf{u}_{z} = \frac{r_{AO, \mathbf{F}_z}}{F} = \frac{-80}{170} = \frac{-8}{17} \text{ ft}
\]

\[
\mathbf{r}_{AO} = (-24\mathbf{k}) + (-18\mathbf{j}) + (-16\mathbf{k})
\]

\[
x = | -24 | = 24 \text{ ft} \\
y = | -18 | = 18 \text{ ft} \\
z = | -16 | = 16 \text{ ft}
\]

**Answer:**

\[
(x, y) = (24, 18) \text{ ft} \\
z = 16 \text{ ft}
\]
TEAM PROBLEM

F2–22. Express the force as a Cartesian vector.

Step 1: Determine the position vector \( \mathbf{r}_{AB} \).

Step 2: Determine the unit vector, \( \mathbf{u}_{AB} \), of the position vector \( \mathbf{r}_{AB} \).

Step 3: Multiply the magnitude of this force to the vector \( \mathbf{u}_{AB} \): \( F_{AB} = F \mathbf{u}_{AB} \)