

A Mixed-Bundling Pricing Strategy for the TV Advertising Market

Mihai Banciu

School of Management, Bucknell University, Lewisburg, Pennsylvania 17837, mihai.banciu@bucknell.edu

Esther Gal-Or, Prakash Mirchandani

The Joseph M. Katz Graduate School of Business, University of Pittsburgh, Pittsburgh, Pennsylvania 15260,
esther@katz.pitt.edu, pmirchan@katz.pitt.edu

Television networks rely on advertising sales for generating a substantial proportion of their annual revenues, but are facing increasing competition from both traditional and non-traditional media outlets for the annual \$150 billion US advertising market. Consequently, they are attempting to design better strategies for more effectively utilizing the limited advertising time that they have available. This paper develops an analytical, nonlinear programming model from the network's perspective and investigates the use of mixed bundling for revenue maximization. Two distinguishing characteristics of our research, limited availability of the advertising time, and a structural property of the television advertising market—a universally consistent preference order of the advertising products—lead to insightful results that help identify situations where different bundling strategies are optimal. The optimal pricing and bundling strategies depend on the relative availabilities of the advertising time resources. The analysis of the optimal solution also helps in assessing the network's incentives to improve ratings of advertising time, and in prioritizing its programming quality improvement initiatives. Our numerical analysis both extends our analytical results and demonstrates that our results are quite robust to the relaxation of our modeling assumptions.

Key words: TV advertising, bundle pricing, mixed bundling, value of bundling, revenue management.

1. Introduction

Advertising accounts for about two thirds of the total revenue¹ for a typical television broadcast network. While the quality of the programming affects the ratings and thus the demand for television advertising, effective strategies for selling the advertising time are an important determinant of the broadcaster's revenue. Determining such strategies is particularly important because the broadcaster's available advertising time is limited either by competitive reasons (as in the US, where commercials account for roughly eight minutes for every 30 minute block of time) or by government regulations (as in the European Union,² where commercials are limited to at most 20 % of the total broadcast time). Moreover, the advertising time is a perishable resource; if it is not used for showing a revenue-generating commercial, the time and the corresponding potential revenue is lost forever.

Broadcasters therefore use a multi-pronged strategy to capture revenues from the roughly \$150 billion dollar advertising market in the US.³ The market for selling television advertising time is split into two different parts: the *upfront* market, which accounts for about 60%-80% of airtime sold and takes place in May every year, and the *scatter* market which takes place during the remainder of the year. In the first stage of their strategy, broadcast networks make decisions about how much advertising time to sell in the upfront market and how much to keep for the scatter market. On their part, clients purchase advertising time in bulk, guided by their medium-term advertising strategy, during the upfront market (at prices that may eventually turn out to be higher or lower than the scatter market prices). The scatter market, on the other hand, allows advertisers to adopt a "wait-and-see" approach to verify the popularity of various network shows, and tailoring their decisions to match their short term advertising strategy.

Our work develops revenue maximizing strategies as they apply to broadcast networks making decisions during the scatter market period. The broadcaster makes available for sale limited amounts of advertising time during different categories of daily viewing times. Advertisers value these categories differently because television audience size varies by the time of the day. In particular, evening time, called *prime time*, traditionally attracts the most viewers, and as such is deemed more valuable by the advertisers, while the rest of the viewing time is referred to as *non-prime time*. A critical decision for the

¹ Ad Revenue Down, CBS Posts Profit Drop of 52%. *The New York Times*, February 18, 2009.

<http://www.nytimes.com/2009/02/19/business/media/19cbs.html>.

² <http://www.europarl.europa.eu/sides/getDoc.do?language=NL&type=IM-PRESS&reference=20071112IPR12883>

³ 2007 TNS media intelligence report (<http://www.tns-mi.com/news/03252008.htm>). Of this amount, television advertising accounts for roughly \$64 billion annually.

broadcaster is how to price these products (that is, the advertising time sold in the different categories) at levels that maximize revenue. Optimally aligning the prices with the advertiser's willingness to pay ensures that the network neither leaves "money on the table," nor uses the advertising resource inefficiently. Moreover, ad hoc pricing can lead to improper market segmentation: advertisers with a higher propensity to pay may end up buying a less expensive product. Likewise, some potential advertisers may be priced out of the market due to improper pricing, even though doing so may be unprofitable for the network. The broadcast network faces yet another decision which is based on an evaluation of the benefits of enhancing the programming quality. Improving quality requires effort (time and money), but can lead to higher ratings. However, the impact of better quality on the network's profitability may be different depending on whether it relates to prime or to non prime time programming. The question that broadcasters need to answer is the amount of effort they should apply to improve programming quality.

The complexity in the analysis for the situations described above gets amplified significantly if the network decides to use bundling—the strategy of combining several individual products for sale as a package (Stigler, 1963). In this regard, the broadcaster has several options available (Adams and Yellen, 1976): (i) *pure components* strategy, that is, offer for sale the different categories of advertising time as separate items only; (ii) *pure bundling* strategy, that is, offer for sale advertising time from the different categories only as a package; and (iii) *mixed bundling* strategy, that is, offer for sale both the bundle and the pure components. Mixed bundling offers an opportunity to the broadcaster to more precisely segment the market. However, as the number of constituent components increases, the number of bundles that we can offer in a mixed-bundling strategy increases exponentially. As a consequence, the number of pricing relationships that need to hold also increases exponentially. Specifically, the broadcaster needs to ensure that the price of each bundle should be no more than the price of its constituent parts. Otherwise, the advertiser can simply buy the constituent parts instead of the bundle (Schmalensee, 1984). If the number of bundles is exponential, so is the number of such pricing constraints. To keep the problem tractable, and since our intent is to draw out qualitative managerial insights to help the broadcaster make decisions regarding the available advertising time resources during the scatter market, we begin by assuming that the components each consist of one unit of prime and non-prime time respectively, and the bundle consists of one unit each of the two components. We later show that under some situations these earlier results apply with a simple recalibration of the units of measurement of the components. When the bundle composition can be chosen by the advertiser, one might consider potentially using an elegant approach proposed by Hitt and Chen (2005). This approach, *customized* bundling, allows buyers to themselves create for a fixed price idiosyncratic bundles of a specified cardinality from a larger set of available items. Wu et al. (2008) use nonlinear programming to further explore the properties of

customized bundling. The customized bundling approach is not needed for the equal proportions television advertising case that we are considering; moreover, as we discuss later, we assume that the available resources are limited, and so the customized bundling model does not directly apply. Therefore, we focus on the seller (that is, the network broadcaster) creating and offering the bundle for sale.

In the television advertising case (as opposed to other bundling situations), the two components have a fundamental structural relationship. Since viewership during prime time hours exceeds the viewership during non-prime time hours, all advertisers prefer to advertise during prime time as compared to advertising during non-prime time hours. Therefore, the prime time product offered is more attractive than the non-prime time product. This natural ordering of the advertising products offered by the broadcaster implies that, given suitably low prices for the three products, all advertisers prefer the non-prime time product to no advertising, the prime time product to the non-prime time product, and the bundle to the prime time product. In the bundling context, this type of preference ordering between the components does not always exist. Indeed, the traditional bundling literature has focused on independently valued products (e.g., Adams and Yellen, 1976, Schmalensee, 1984, McAfee, McMillan, and Whinston, 1989, Bakos and Brynjolfsson, 1999) or assumed that the bundle consists of substitutable or complementary components (Venkatesh and Kamakura, 2003). Products are independently valued if the reservation price of the bundle is the sum of the reservation prices of the components. When the relationship is complementary, the reservation price of the bundle may exceed the sum of the reservation prices of its constituents (Guiltinan, 1987), and when the constituents are substitutable, the bundle's reservation price may (though not necessarily) be lower than the sum of the reservation prices of the two constituents. (Marketers may still offer the bundle to exploit market segmentation benefits, and because the variable cost of the bundle may be a subadditive function of the component variable costs.) Substitutable products may (as in the case of a slower versus a faster computer system) or may not (as in the case of Coke versus Pepsi, or a slower versus a faster automobile) be amenable to a universally consistent ordering. Regardless, independent and complementary products clearly lack the natural ordering that we see for television advertising, where all advertisers prefer prime time advertising to non-prime time advertising.

This type of ordering in the advertisers' preferences also exists in some other commercially important practical situations. Radio or news magazine advertising are obvious examples. Additionally, in internet advertising, advertisers prefer placing an advertisement on the front page of a website to placing it on a lower ranking page. Billboard advertising also exhibits this relationship. Here, placing a billboard advertisement featured along an interstate highway is preferred to placing the same advertisement on a secondary road, where the exposure to the advertisement may be more limited. While

in this paper we use television advertising as a prototypical example, our model and results apply to other situations that exhibit the preference ordering. As we will see, this preference ordering in the products leads to some counter-intuitive and insightful results.

Another distinctive feature of our research concerns the total amounts of each type of advertising time available for sale. As is the case in practice, we assume that these amounts are limited, and investigate how the broadcast network's decisions change as the availabilities change. In contrast, previous bundling literature has not modeled resource availabilities.

This paper is organized as follows. Section 2 discusses our modeling assumptions and develops a nonlinear pricing model for a bundling situation when the resources have limited availability. The output from this model is a set of optimal product prices that automatically segments the market, and correspondingly sets the fraction of the market that is covered by each product. Advertisers decide on the product they wish to purchase based on the prices they are offered and their willingness to pay—which in turn depends on the “efficiency” with which they can generate revenues from viewers of their advertisements. In Section 3, assuming that the distribution of the advertiser's efficiency parameter (which measures the effectiveness with which the advertiser translates viewers into revenue) is uniform, we analyze the properties of the optimal prices, and shadow prices. Interestingly, the tightness *and* the relative tightness of the advertising resources plays a pivotal role in not only affecting the product prices but also influencing whether or not to offer the bundle, and if the bundle is offered, the type of bundling strategy to adopt. When prime and non-prime time resource availability is unconstrained, the broadcaster offers only the bundle. On the other hand, the broadcaster offers the bundle in conjunction with some components only when there is “enough” prime and non-prime time advertising resource. We also analyze the shadow prices of advertising resources, and evaluate how the broadcast network should focus its quality improvement efforts to improve total revenue. Due to bundling, the shadow price of the prime time resource (non-prime time resource) can decrease or remain the same even when its availability is kept unchanged but the availability of only the non-prime time resource (prime time resource) is increased. Our analysis shows that when the relative availability of the two resources is comparable, it always makes more sense for the network to improve the ratings of the prime time product. This section also explores the value of bundling. Section 4 relaxes two of the assumptions in our original model. Using specific instances from the Beta family of distributions to model the density function of advertiser efficiencies, we show numerically that the general nature of our conclusions is quite robust. We also investigate how to implement, and the impact of, a generalization of the definition of the bundle to allow for an unequal mix its constituent components. Section 5 concludes the paper by identifying some future research directions.

2. The Model

A monopolist television broadcasting network, which we refer to as the *broadcaster*, considers offering for sale on the scatter market its available advertising time, that is, its advertising *inventory*. This inventory is of two types: prime time and non-prime time. The availability of both of these inventories, which we interchangeably refer to also as *resources*, is fixed, with q_P denoting the amount of advertising time available during prime time hours, and q_N denoting the amount of advertising time available during non-prime time hours. The broadcaster's objective is to maximize the total revenue it generates from selling its inventory. As in the information goods situation in Bakos and Brynjolfsson (1999), we can assume that the variable costs of both resources is zero for our situation, and so maximizing the revenue is equivalent to maximizing the contribution. In order to do so, the broadcaster sells three products corresponding to selling one unit of each of the two resources separately, and selling a bundle which consists of one unit of each resource.

The market consists of *advertisers* interested in purchasing advertising time from the broadcaster. In line with the bundling literature (e.g., Adams and Yellen, 1976; Schmalensee, 1984), we assume that the marginal utility of a second unit of a product is zero for all advertisers. Advertisers have a strict ordering of their preferences: They consider advertising during non-prime time to be more desirable than not advertising, prime time advertising to be more desirable than non-prime time advertising, and the bundle that combines both prime and non-prime time advertising to be the most desirable. This preference is a consequence of prime time ratings being higher than non-prime time ratings. We designate the ratings of the non-prime time, prime time, and the bundle options by α , β , and γ , respectively, where, $0 < \alpha < \beta < \gamma$. We also assume that the relationship between the ratings is “concave” in nature, that is, $\alpha + \beta \geq \gamma$. This assumption is reasonable because of diminishing returns seen in advertising settings: in this case, the same individual might see an advertisement shown during both prime and non-prime time periods, and so the rating of the bundle is less than the sum of the ratings of the prime and non-prime advertisements.

Advertisers differ in their willingness to pay for the three advertising products due to their varied ability to translate eyeballs into purchase decisions of viewers and the consequent profits. Advertisers who are more successful in generating higher profits have a greater willingness to pay for the more desirable products—which are also more expensive. We designate by the parameter t the intrinsic efficiency of an advertiser to generate profits out of advertisements, and assume that this efficiency is distributed on the unit interval according to some probability density function $f(t)$ and cumulative distribution function $F(t)$. The willingness to pay of an advertiser with efficiency t for an advertisement

placed in time period i is thus equal to $t r_i$, where r_i is the rating of the i^{th} product, i equal to prime, non-prime or the bundle.

Given the above distribution of the efficiency parameter of advertisers and their willingness to pay function, an optimal strategy for the broadcaster segments the population of advertisers into at most four groups as described in Figure 1, with the thresholds T^* , T^{**} , and T^{***} demarcating the different market segments.⁴ With this strategy, advertisers in the highest range of efficiency parameters (interval $[T^*, 1]$) choose to purchase the bundle. Those in the second highest range of efficiency parameters (interval $[T^{**}, T^*)$) choose to advertise during prime-time. Those in the third highest range (interval $[T^{***}, T^{**})$) choose the non-prime product, and those in the lowest range refrain from advertising altogether. An interval of zero length implies that it is not optimal for the broadcaster to offer the corresponding product. The values of the threshold parameters T^* , T^{**} , and T^{***} are determined to guarantee that the advertiser located at a given threshold level is indifferent between the two choices made by the advertisers in the two adjacent intervals separated by this threshold parameter.

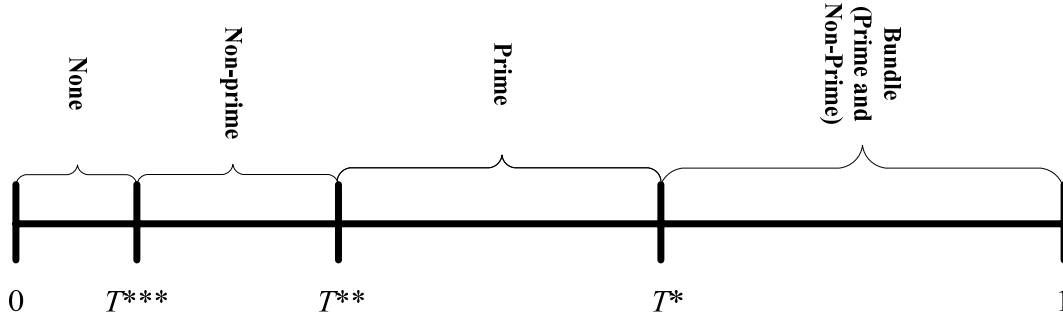


Figure 1. Market segmentation

To set up the model we define the selling prices for the bundle, prime, and non-prime products by p_B , p_P and p_N , respectively. The revenue optimization with mixed bundling model (*ROMB*), from the broadcaster's perspective, is:

$$[\mathbf{ROMB}] \quad \max_{p_B, p_P, p_N \geq 0} \pi = p_B \int_{T^*}^1 f(t) dt + p_P \int_{T^{**}}^{T^*} f(t) dt + p_N \int_{T^{***}}^{T^{**}} f(t) dt \quad (1)$$

subject to:

$$p_B \leq p_P + p_N, \quad (2)$$

⁴ The willingness to pay function satisfies the ‘‘single crossing property’’ and therefore facilitates segmentation and guarantees the uniqueness, as well as the monotonicity ($0 \leq T^{***} \leq T^{**} \leq T^* \leq 1$) of the thresholds.

$$\int_{T^*}^1 f(t)dt + \int_{T^{**}}^{T^*} f(t)dt \leq q_P, \text{ and} \quad (3)$$

$$\int_{T^*}^1 f(t)dt + \int_{T^{***}}^{T^{**}} f(t)dt \leq q_N. \quad (4)$$

The broadcaster's revenue from a market segment equals its size multiplied by the price of the product it corresponds to; the total revenue, π , in the objective function (1) is the sum of the revenues from each of the three segments that the broadcaster serves. Constraint (2), the "price-arbitrage" constraint, prevents arbitrage opportunities for an advertiser to compose a bundle by separately buying a prime and a non-prime time products separately.⁵ Constraints (3) and (4) model the limited prime and non-prime time available.

Advertisers self-select their purchases (or they may decide to not purchase any of the offered products) based on their willingness to pay and the product prices. (See Moorthy, 1984, for an analysis of self-selection based market segmentation.) Consider the difference between an advertiser's willingness to pay and the price of the product he⁶ purchases. This difference equals the *premium* the advertiser derives from the purchase. An advertiser will purchase a product only if his premium is nonnegative. Moreover, an advertiser will be indifferent, say, between buying only prime time and buying a bundle consisting of prime and non-prime time, if he extracts the same premium from either purchase. The following relationships between the purchasing premiums are invariant boundary conditions, regardless of the efficiency distribution $f(t)$.

$$\gamma T^* - p_B = \beta T^* - p_P \Leftrightarrow T^* = \frac{p_B - p_P}{\gamma - \beta}, \quad (5)$$

$$\beta T^{**} - p_P = \alpha T^{**} - p_N \Leftrightarrow T^{**} = \frac{p_P - p_N}{\beta - \alpha}, \text{ and} \quad (6)$$

$$\alpha T^{***} - p_N = 0 \Leftrightarrow T^{***} = \frac{p_N}{\alpha}. \quad (7)$$

Notice that the non-negativity of the thresholds implies

$$p_P \leq p_B, \text{ and} \quad (8)$$

$$p_N \leq p_P. \quad (9)$$

⁵ Unless a systematic secondary market exists, an intermediary cannot purchase a bundle and then sell its components individually at a profit.

⁶ Where necessary, we use masculine gender for the advertiser and feminine gender for the broadcaster.

Moreover, it is easy to see that p_N , as well as the premium for customers in each of the three categories, is nonnegative.

Before we analyze the situations that arise when at least one of the capacity constraints is binding, Proposition 1 considers the case when neither capacity constraint is binding. Appendix I gives the proof of this and all subsequent results.

Proposition 1. *If the prime and non-prime resource availability is sufficiently high, the optimal strategy for the broadcaster is pure bundling. The corresponding optimal threshold is the fixed point of the reciprocal of the hazard rate function of the distribution of advertisers, that is, $T^* = (1 - F(T^*)) / f(T^*)$.*

The following corollary uses Markov's inequality to establish an upper bound on the optimal revenue when the problem is not constrained by the inventory availability.

Corollary 2. *An upper bound on the broadcaster's total revenue π is $\gamma \mathbf{E}[T]$, where $\mathbf{E}[T]$ is the expected value of the efficiency, t . The actual revenue collected under the pure bundling strategy is $\gamma(1 - F(T^*))^2 / f(T^*)$.*

The result in Proposition 1 seems to contradict previous bundling literature (for example, Schmalensee, 1984, McAfee, McMillan, Whinston, 1989) which demonstrates that the mixed bundling strategy weakly dominates both the pure bundling and pure components strategies. However, a critical difference between our model and previous work is that the advertisers have a common preferred ordering of the three products. In contrast, the previous research stream does not assume any such ordering of the products. Since the bundle is the most desirable option for every advertiser, the broadcaster offers only the bundle when the available prime and non-prime advertising time is unconstrained. This unconstrained case is unlikely to arise in reality, since all broadcasters are usually heavily constrained by the prime time resource availability.

In the next section we derive the analytical solution of the constrained optimization problem under the simplifying assumption that the efficiency parameter of advertisers is uniformly distributed. In Section 4, we extend the results numerically using a Beta Distribution.

3. Revenue Maximizing Strategies when Capacity is Binding

Clearly, the capacity constraints in the *ROMB* model play a significant role in determining the broadcaster's optimal strategy. Particularly, the relative scarcity of the two resources, prime time and non-prime time, is the main driver of the analysis. In the television advertising market, the prime time

resource availability constraint (3) is far more likely to be binding than the non-prime time resource availability constraint (4). Prime time on television is usually the slot from 8:00 pm until 11:00 pm Monday to Saturday, and 7:00 pm to 11:00 pm on Sunday. Hence, the ratio of prime to non-prime time availability is about 1:8 (or 1:6 on Sunday). In other media markets, the relative scarcity of the non-prime time constraint may also become an issue. For instance, in the billboard advertising market, the “non-prime time” resource is the limited availability of billboards on secondary roads (which are less traveled), whereas the “prime time” is the extensive availability of billboard advertisement space on major roads (which have more travelers). In such a market, it is the “non-prime time” capacity that is more likely to be binding. Finally, in internet advertising both types of capacity constraints may be binding. Each website has limited space for banner advertisements irrespective of whether it is the front page (“prime time”) or a secondary page (“non-prime time”). In this section, we specify the distribution of the efficiency parameter of advertisers to be uniform and identify the impact of the two capacity constraints on the type of strategy followed by the broadcaster. We find that the following strategies can arise as the optimal solution of *ROMB*: no bundle is offered, that is, the pure components strategy, *PC*; only the bundle is offered, that is, the pure bundling strategy, *PB*; the bundle, as well as each separate time product is offered, that is, the full spectrum mixed bundling strategy, *MBPN*; the bundle and the prime time product are offered, that is, the partial spectrum mixed bundling strategy, *MBP*; the bundle and the non-prime time product are offered, that is, the partial spectrum mixed bundling strategy, *MBN*. Throughout the remainder of the paper we will refer to these abbreviations.

In our derivations, we will demonstrate that the optimal strategy critically depends upon the relative availability of q_P and q_N . We will show that, for instance, that the *MBN* strategy is optimal when q_P is scarce relative to q_N , and the *MBP* strategy arises in the opposite case. The *MBPN* strategy is the optimal strategy when the ratio of q_P to q_N is close to one, but they are both sufficiently large.

We will also show that the characterization of the solution when the partial spectrum mixed bundling strategies (*MBP* or *MBN*) are optimal is further contingent upon the overall availability of the more abundant resource. Specifically, even though the strategy itself, say *MBP*, remains the same, the solution characteristics (product prices and the shadow prices of the resources) depend on whether q_P is less than or greater than a half. Similarly, the characteristics of the solution corresponding to *MBN* depend on whether q_N is less than or greater than a half. To distinguish between these two cases, we designate by MBP^+ and MBP^- the partial spectrum mixed bundling strategies when q_P is greater than and when q_P is less than a half, respectively. We define the subcategories MBN^+ and MBN^- of *MBN* in a similar manner depending on the availability of q_N .

Figure 2 depicts the regions corresponding to the various strategies that we discussed above. For the uniform distribution, the unconstrained solution that we described in Section 2 arises when q_P and q_N are both at least a half. In this case, at the optimal solution, the broadcaster never sells more than an aggregate quantity of one, split equally between the prime and non-prime advertising times. To depict the constrained solution, therefore, in Figure 2, we restrict attention only to the case when $q_P + q_N \leq 1$. The unconstrained solution in the figure is designated by the point, PB , where $q_P = q_N = \frac{1}{2}$. The boundaries for the regions in Figure 2 will be explained in detail once the solution to model *ROMB* is derived.

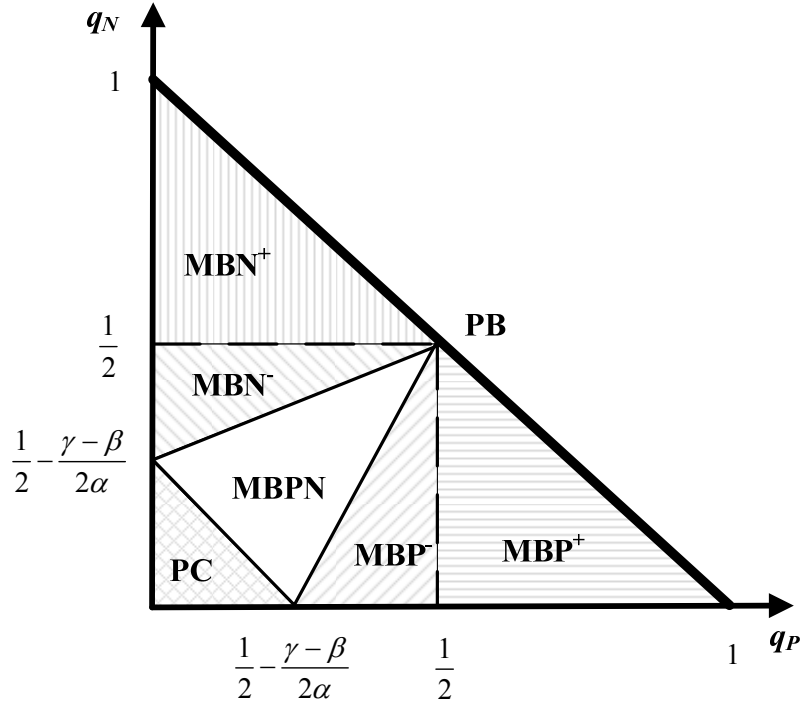


Figure 2. Representation of the optimal strategies

Replacing the general distribution by a uniform distribution in the model *ROMB* yields the following model, which we refer to as *ROMB_U*.

$$[\mathbf{ROMB_U}] \quad \max \pi = p_B \left(1 - \frac{p_B - p_P}{\gamma - \beta} \right) + p_P \left(\frac{p_B - p_P}{\gamma - \beta} - \frac{p_P - p_N}{\beta - \alpha} \right) + p_N \left(\frac{p_P - p_N}{\beta - \alpha} - \frac{p_N}{\alpha} \right) \quad (10)$$

subject to:

$$p_B - (p_P + p_N) \leq 0, \quad (11)$$

$$1 - \frac{p_P - p_N}{\beta - \alpha} \leq q_P, \text{ and} \quad (12)$$

$$1 - \frac{p_B - p_P}{\gamma - \beta} + \frac{p_P - p_N}{\beta - \alpha} - \frac{p_N}{\alpha} \leq q_N. \quad (13)$$

It is easy to see that the solution to the unconstrained case (when $q_P \geq \frac{1}{2}$ and $q_N \geq \frac{1}{2}$) is $p_B = \gamma/2$, $p_P = \beta/2$, and $p_N = \alpha/2$. This solution guarantees that only pure bundling arises since $T^* = T^{**} = T^{***} = \frac{1}{2}$, and the broadcaster's revenues are $\gamma/4$. Note that this solution also guarantees that the arbitrage constraint $p_B \leq p_P + p_N$ is satisfied since $\gamma \leq \alpha + \beta$ by the concavity assumption.

3.1. Characterization of the Different Strategies

We now discuss the characterization of the constrained case. Proposition 3 describes the boundaries of the regions corresponding to the different strategies depicted in Figure 2, and Propositions 3 and 4 derive the optimal product and the shadow prices, respectively.

Proposition 3. *The optimal strategies as a function of the availability of q_P and q_N are as follows:*

(i) *The pure component strategy, PC, is optimal if*

$$0 < q_N + q_P < \frac{1}{2} - \frac{\gamma - \beta}{2\alpha}.$$

(ii) *The full spectrum mixed bundling strategy, MBPN, is optimal if*

$$\frac{\gamma - \beta}{\alpha} \leq \frac{1 - 2q_N}{1 - 2q_P} \leq \frac{\alpha}{\gamma - \beta}.$$

(iii) *The partial spectrum mixed bundling strategies, MBN^- and MBN^+ , are optimal if $0 < q_P < \frac{1}{2}$, and*

a. $\frac{1 - 2q_N}{1 - 2q_P} < \frac{\gamma - \beta}{\alpha}$ and $q_N < \frac{1}{2}$ for MBN^- ,

b. $q_N \geq \frac{1}{2}$ for MBN^+ .

(iv) *The partial spectrum mixed bundling strategies, MBP^- and MBP^+ , are optimal if $0 < q_N < \frac{1}{2}$ and*

a. $\frac{1 - 2q_N}{1 - 2q_P} > \frac{\alpha}{\gamma - \beta}$ and $q_P < \frac{1}{2}$ for MBP^- ,

b. $q_P \geq \frac{1}{2}$ for MBP^+ .

(v) *The pure bundling strategy, PB, is optimal at a single point $q_P = q_N = \frac{1}{2}$.*

According to Proposition 3 when the available aggregate capacity is small (lower than $\frac{1}{2} - \frac{\gamma - \beta}{2\alpha}$), the broadcaster follows a pure component strategy where each advertiser can choose between advertising on prime time or on non-prime time but not both. Offering the bundle is suboptimal in this case given the extreme scarcity of the advertising time availability. When the aggregate capacity is larger

than $\frac{1}{2} - \frac{\gamma - \beta}{2\alpha}$, and the discrepancy between the capacities available on prime time and non-prime time is relatively moderate (that is, $\frac{\gamma - \beta}{\alpha} \leq \frac{1 - 2q_N}{1 - 2q_P} \leq \frac{\alpha}{\gamma - \beta}$), it is optimal for the broadcaster to choose full segmentation of advertisers by offering all three different products. Notice that the size of the region expands as the relationship between the ratings parameters becomes more concave (that is, the fraction $(\gamma - \beta)/\alpha$ becomes smaller.) On the other hand, the *MBPN* region becomes smaller as $(\gamma - \beta)/\alpha$ becomes larger, that is, as $\gamma - \beta$ approaches α . In the extreme case, when $\gamma - \beta$ equals α (that is, the ratings are additive), the *MBPN* region becomes a line (and the *PC* region disappears). In this case, the *MBPN* strategy applies only when $q_P = q_N$. This is obvious since the ratings of the bundle exactly equal the sum of the prime and non-prime ratings.

When the availability of the non-prime time resource is much greater than that of the prime time resource ($\frac{1 - 2q_N}{1 - 2q_P} < \frac{\gamma - \beta}{\alpha}$), the broadcaster offers both the non-prime product and the bundle.

Conversely, when the availability of the prime time resource is much bigger than that of the non-prime time ($\frac{\alpha}{\gamma - \beta} < \frac{1 - 2q_N}{1 - 2q_P}$), the broadcaster offers a choice between advertising just on prime time or buying a

bundle. With significant abundance of one resource relative to the other, it pays to utilize the entire capacity of the more scarce resource as part of the bundle. Since advertisers have a higher willingness to pay for the bundle than for each component sold separately, the broadcaster uses the entire capacity of the scarcer resource in the form of the product that can command a higher price. Any remaining quantity of the more abundant resource, not sold as part of the bundle, is offered separately to the customers.

According to part (v) of the Proposition, pure bundling arises only when the capacity of each category is large enough to obtain the solution of the unconstrained optimization (equal to $\frac{1}{2}$). Notice, in fact, that the scarcity of the two resources, which determines the boundaries of the regions in Figure 2, is expressed in terms of $(1 - 2q_N)$ and $(1 - 2q_P)$. These expressions measure the extent to which the individual capacities fall short of the unconstrained optimal value of $\frac{1}{2}$.

3.2. Optimal Product Prices and Shadow Prices

Having studied how and why the differing relative availabilities of the prime and non-prime time resources impact the regions where the different strategies apply, we now investigate the optimal pricing structure under the different strategies.

Proposition 4. *The optimal prices charged by the broadcaster in the different regions of Figure 2 are as follows:*

STRATEGY	OPTIMAL PRICES
PC	$p_B = \gamma - \beta q_P - \alpha q_N$ $p_P = \beta(1 - q_P) - \alpha q_N$ $p_N = \alpha(1 - q_P - q_N)$ (14)
MBPN	$p_B = \frac{\gamma}{2} + \frac{\beta(\gamma - \beta) + \alpha(\gamma - \alpha)}{\gamma - \beta + \alpha} \left(\frac{1}{2} - q_P \right) + \frac{2\alpha(\gamma - \beta)}{\gamma - \beta + \alpha} \left(\frac{1}{2} - q_N \right)$ $p_P = \frac{\beta}{2} + \frac{\beta(\gamma - \beta) + \alpha(\beta - \alpha)}{\gamma - \beta + \alpha} \left(\frac{1}{2} - q_P \right) + \frac{\alpha(\gamma - \beta)}{\gamma - \beta + \alpha} \left(\frac{1}{2} - q_N \right)$ $p_N = \frac{\alpha}{2} + \frac{\alpha(\gamma - \beta)}{\gamma - \beta + \alpha} \left[\left(\frac{1}{2} - q_P \right) + \left(\frac{1}{2} - q_N \right) \right]$ (15)
MBN⁻	$p_B = \gamma(1 - q_P) - \alpha(q_N - q_P)$ $p_P = \beta(1 - q_P) - \alpha(q_N - q_P)$ $p_N = \alpha(1 - q_N)$ (16)
MBN⁺	$p_B = \frac{\gamma}{2} + (\gamma - \alpha) \left(\frac{1}{2} - q_P \right)$ $p_P = \max \left(\frac{\beta}{2} + (\beta - \alpha) \left(\frac{1}{2} - q_P \right), \frac{\gamma - \alpha}{2} + (\gamma - \alpha) \left(\frac{1}{2} - q_P \right) \right)$ $p_N = \frac{\alpha}{2}$ (17)
MBP⁻	$p_B = \gamma(1 - q_N) - \beta(q_P - q_N)$ $p_P = \beta(1 - q_P)$ $p_N = \alpha(1 - q_P)$ (18)
MBP⁺	$p_B = \frac{\gamma}{2} + (\gamma - \beta) \left(\frac{1}{2} - q_N \right)$ $p_P = \frac{\beta}{2}$ $p_N = \max \left(\frac{\alpha}{2}, (\gamma - \beta)(1 - q_N) \right)$ (19)

It is noteworthy that ignoring the arbitrage constraint (11) (that is, $p_B \leq p_P + p_N$) and solving for the optimal prices yields a solution that automatically satisfies the constraint except possibly in the MBN^+ and the MBP^+ regimes. If $\alpha < 2(\gamma - \beta)$ (that is, when the concavity of the ratings parameters is moderate), the constraint might be violated when $q_P < \alpha / (2(\gamma - \beta))$ under MBN^+ or $q_N < \alpha / (2(\gamma - \beta))$

under MBP^+ . Since the arbitrage constraint is binding in this case, incorporating it (that is, setting $p_P = p_B - p_N$ under MBN^+ or $p_N = p_B - p_P$ under MBP^+) still results in the desired outcome for the broadcaster. Specifically, no advertiser chooses to buy the prime time product under MBN^+ or the non-prime time product under MBP^+ .

In Proposition 5, we solve for the Lagrange multipliers of the resource constraints. This analysis provides the foundation for our subsequent investigation into the relative marginal values of the two resources.

Proposition 5. *The shadow prices of the resources in the different regions are specified as follows.*

STRATEGY	OPTIMAL SHADOW PRICES
PC	$\lambda_P = \beta(1 - 2q_P) - 2\alpha q_N$ $\lambda_N = \alpha(1 - 2q_P - 2q_N)$ (20)
$MBPN$	$\lambda_P = \frac{\beta(\gamma - \beta) + \alpha(\beta - \alpha)}{\gamma - \beta + \alpha}(1 - 2q_P) + \frac{\alpha(\gamma - \beta)}{\gamma - \beta + \alpha}(1 - 2q_N)$ $\lambda_N = \frac{2\alpha(\gamma - \beta)}{\gamma - \beta + \alpha}(1 - q_P - q_N)$ (21)
MBN^-	$\lambda_P = (\gamma - \alpha)(1 - 2q_P)$ $\lambda_N = \alpha(1 - 2q_N)$ (22)
MBN^+	$\lambda_P = (\gamma - \alpha)(1 - 2q_P)$ $\lambda_N = 0$ (23)
MBP^-	$\lambda_P = \beta(1 - 2q_P)$ $\lambda_N = (\gamma - \beta)(1 - 2q_N)$ (24)
MBP^+	$\lambda_P = 0$ $\lambda_N = (\gamma - \beta)(1 - 2q_N)$ (25)

To understand the relationship between the shadow prices that we report in Proposition 5, and the product prices in Proposition 4, note that the availability of one additional unit of a scarce resource results in both a direct effect of generating additional revenues from the sale of this unit (perhaps, partly separately and partly in the bundle), and an indirect effect of depressing the prices that the broadcaster can charge for the products. For instance, the availability of an additional unit of the prime time resource, when the strategy PC is employed, has the direct effect of generating extra revenues equal to p_P and an indirect effect of reducing the price of the prime time product at the rate of β and the price of the non-prime time resource at the rate of α . Hence, $\lambda_P = p_P - \beta q_P - \alpha q_N$. Substituting for p_P from Proposition

4 yields the expression for λ_P reported in (20). The explanation for the shadow price of the non-prime resource, λ_N , is similar. For the *MBPN* strategy, establishing the relationship between shadow prices and product prices is a bit more complicated since an additional unit of the scarce resource is partially allocated to the bundle and partially sold separately. Specifically, an additional unit of the prime time resource is allocated to the bundle at the rate of $\alpha/(\gamma - \beta + \alpha)$ and is sold separately at the rate of $(\gamma - \beta)/(\gamma - \beta + \alpha)$. Hence an additional unit of prime time resource generates direct extra revenues equal to $\alpha/(\gamma - \beta + \alpha) p_B + (\gamma - \beta)/(\gamma - \beta + \alpha) p_P$. The extra unit depresses prices according to Proposition 4 as follows:

- the price p_B at the rate of $((\gamma - \beta)(\alpha + \beta) + \alpha(\beta - \alpha)) / (\gamma - \beta + \alpha)$,
- the price p_P at the rate of $(\beta(\gamma - \beta) + \alpha(\beta - \alpha)) / (\gamma - \beta + \alpha)$, and
- the price p_N at the rate of $\alpha(\gamma - \beta) / (\gamma - \beta + \alpha)$.

Combining the direct and indirect effects yields the desired expression for λ_P in (21), and similarly for λ_N . We observe that an increase in either the prime or the non-prime time resource availability lowers the optimal prices of all three products even though the broadcaster offers a lower amount of the prime time product when q_N increases and of the non-prime time product when q_P increases. An argument that combines the direct and indirect effect similarly applies for the expressions (22) – (25).

3.3. Relative Shadow Prices

The shadow prices in Proposition 5 measure the extra cost the broadcaster might be willing to incur in order to obtain one additional unit of the scarce resource. In the case of television advertising, an increase in the available advertising time comes at the expense of programming time, and thus can potentially decrease the ratings and hence the advertiser's profits. The shadow prices in Proposition 5 provide an upper bound on the reduction in ratings that the broadcaster might be willing to tolerate in order to increase advertising time by one unit.

A related question that arises is how much more, or less, valuable to the broadcaster an additional unit of prime time is vis-à-vis an additional unit of non-prime time. In addition, how does this comparison change as we move from one regime to another? We summarize this comparison in Corollary 6.

Corollary 6. *The shadow price of a unit of the prime time resource is greater than a unit of non-prime time resource if the strategies PC, MBPN, and MBN (both MBN^- and MBN^+) are optimal. If strategy*

MBP is optimal, an extra unit of the non-prime time resource may become more valuable than an extra unit of the prime time resource. Specifically, the difference $\lambda_P - \lambda_N$ for the different regions is as follows.

STRATEGY	DIFFERENCE IN THE OPTIMAL SHADOW PRICES
<i>PC</i>	$(\beta - \alpha)(1 - 2q_P)$ (26)
<i>MBPN</i>	$(\beta - \alpha)(1 - 2q_P)$ (27)
<i>MBN⁻</i>	$(\gamma - \alpha)(1 - 2q_P) - \alpha(1 - 2q_N)$ (28)
<i>MBN⁺</i>	$(\gamma - \alpha)(1 - 2q_P)$ (29)
<i>MBP⁻</i>	$\beta(1 - 2q_P) - (\gamma - \beta)(1 - 2q_N)$ (30)
<i>MBP⁺</i>	$-(\gamma - \beta)(1 - 2q_N)$ (31)

To illustrate how the relative availability for the two resources affects the relative shadow prices, consider a value of $q_P < 1/2 - (\gamma - \beta)/(2\alpha)$. We select this choice of q_P because the available prime time inventory is typically relatively low, and as we gradually increase the value of q_N , the optimal strategy shifts, according to Figure 2, from *PC* to *MBPN* to *MBN⁻* and finally to *MBN⁺*. We can do a similar analysis for the case $1/2 - (\gamma - \beta)/(2\alpha) < q_P < 1/2$. Selecting a value of q_P greater than or equal to a half does not result in a change in strategies as we increase the value of q_N , and so a similar analysis is not interesting in that case. Figure 3 depicts the relative shadow prices of the two resources when considering such an increase in q_N . (The solid dots in this and subsequent figures represent a shift in the strategy.)

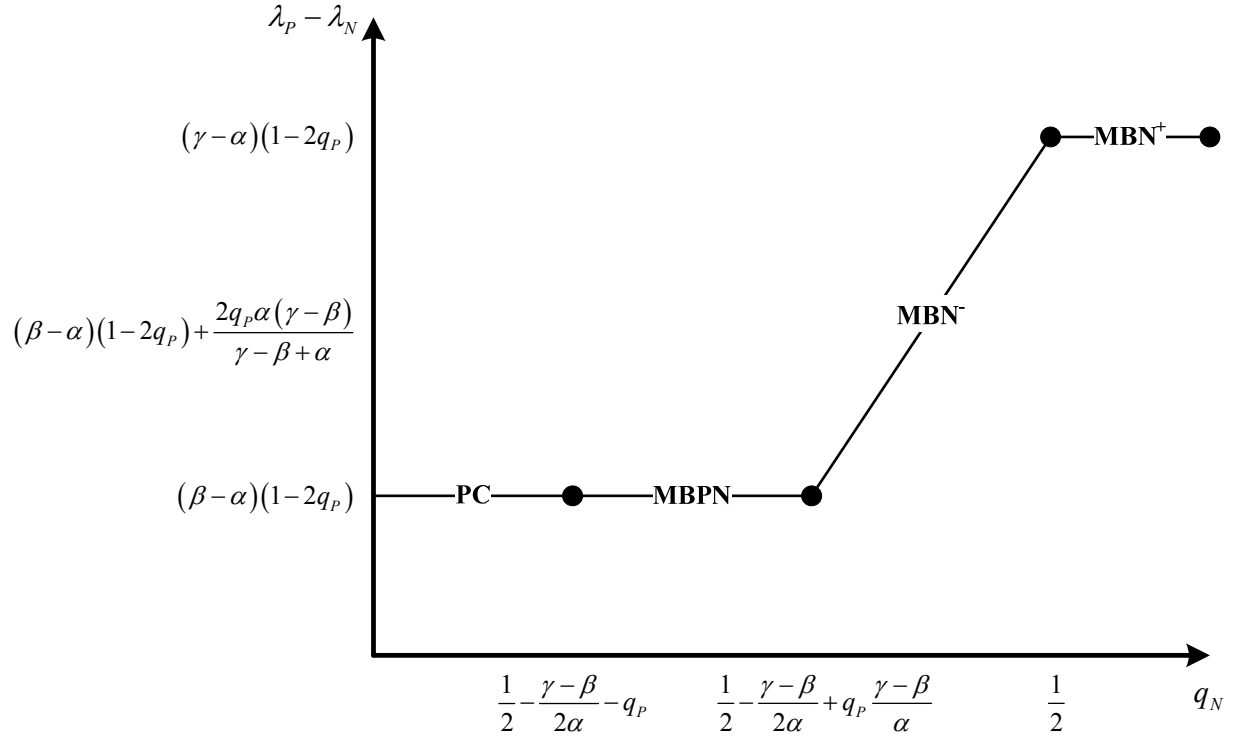


Figure 3. Relative shadow prices of the resources for any $q_p < \frac{1}{2} - \frac{\gamma - \beta}{2\alpha}$

The shadow price of the prime time resource is higher than that of the non-prime time resource since the broadcaster can charge higher prices from advertisers choosing to place advertisements on prime time. This higher price is proportional to the difference in the ratings of the prime and non-prime time products. Indeed, under the *PC* and *MBPN* regimes, the difference in the shadow prices is proportional to $(\beta - \alpha)$, which measures the difference in ratings between the pure components. Interestingly, the added segmentation of advertisers that is facilitated by bundling under *MBPN*, does not enhance the relative shadow price of the prime time resource. The reason for this result is that an additional unit of either resource is allocated in the same proportion towards the bundle, thus maintaining the relative desirability of the two resources to the broadcaster irrespective of whether full segmentation is feasible or not. Under *MBN⁺* regime, the difference in the shadow prices of the two resources is proportional to $(\gamma - \alpha)$, since this regime occurs under the extreme scarcity of the prime time resource, and each additional unit of the prime time resource is used only in the bundle, thus yielding the extra rating of γ rather than β .

A similar allocation of an extra prime time unit is optimal under *MBN⁻* also. However, since there are no unused units of the non-prime time resource under this regime, each additional unit of the prime time that is sold requires directing a non-prime time unit from being sold as an independent

component. As a result, the shadow price of the prime time resource under MBN^- is not as high as it is under MBN^+ . Under MBN^- , the difference $\lambda_P - \lambda_N$ is an increasing function of q_N , or alternatively, since q_P is fixed for this analysis, an increasing schedule of the relative scarcity of the prime-time resource, until it reaches its maximum value when $q_N = 1/2$, and the MBN^+ region is reached. Note also that a bigger value of q_P reduces the difference $\lambda_P - \lambda_N$ for all regimes. Hence, as the prime time becomes less scarce, its importance relative to the non-prime time resource declines.

3.4. Incentives for Improving the Programming Quality

We can now use the characterization of the optimal solution to assess the relative incentives of the broadcaster to increase the ratings of the time periods by improving the quality of the programming. We will evaluate those incentives by deriving the expression for the *Relative Incentive to Improve Ratings*, $RIIR = \frac{\partial \pi^*}{\partial \beta} - \frac{\partial \pi^*}{\partial \alpha}$, where π^* corresponds to the broadcaster's optimal revenue.

$RIIR = \frac{\partial \pi^*}{\partial \beta} - \frac{\partial \pi^*}{\partial \alpha}$, where π^* corresponds to the broadcaster's optimal revenue.

In our analysis, we assume that $\frac{\partial \gamma}{\partial \alpha} = \frac{\partial \gamma}{\partial \beta}$. Specifically, improving the quality of programs on non-prime time has the same effect on the ratings of the bundle as an equivalent improvement in prime time programming. We refer to this as the *equal ratings-improvement effect* assumption. Given that we restrict attention to bundles with equal proportions of prime time and non-prime time resources, such an assumption seems reasonable. Substituting the optimal prices from Proposition 4 back into the objective function (10) yields the optimal equilibrium revenue, π^* .

Proposition 7 reports on the $RIIR$ values, that is, the added incentive of increasing the ratings of prime time over non-prime time, for the different regions.

Proposition 7. *The $RIIR$ values, $\frac{\partial \pi^*}{\partial \beta} - \frac{\partial \pi^*}{\partial \alpha}$, can be expressed as follows:*

STRATEGY	RIIR VALUES $\frac{\partial \pi^*}{\partial \beta} - \frac{\partial \pi^*}{\partial \alpha}$
PC	$(q_P - q_N)(1 - q_P - q_N) + 2q_P q_N$ (32)
MBPN	$\frac{((\gamma - \beta)^2 + \alpha^2)(1 - q_P - q_N)^2}{(\gamma - \beta + \alpha)^2} - 2\left(\frac{1}{2} - q_P\right)^2$ (33)
MBN ⁻	$(q_P - q_N)(1 - q_N - q_P)$ (34)

MBN^+	$-\left(\frac{1}{2}-q_P\right)^2$	(35)
MBP^-	$(q_P-q_N)(1-q_N-q_P)$	(36)
MBP^+	$\left(\frac{1}{2}-q_N\right)^2$	(37)

Inspecting the expressions derived in Proposition 7, it immediately follows that the *RIIR* values depend on their relative scarcity. In general, irrespective of which bundling strategy is optimal, the broadcaster has greater incentives to improve the ratings of the resource that is more abundant. The value of *RIIR* is an increasing function of q_P and a decreasing function of q_N , implying that the broadcaster benefits more from upgrading the quality of the more plentiful resource.

The results of Proposition 7 allow us to also assess the implications of different bundling strategies on the incentives to improve the quality of the programming. To control for the relative size effect reported above, in conducting this assessment, we consider the symmetric case: $q_N = q_P = q$. Given this symmetry, we can only compare the *PC* regime with the *MBPN* regime since the partial spectrum mixed bundling strategies (*MBP* and *MBN*) arise when the availabilities of the two resources is asymmetric.

Corollary 8. When $q_N = q_P = q$, the *RIIR* value, $\frac{\partial \pi^*}{\partial \beta} - \frac{\partial \pi^*}{\partial \alpha}$, can be derived as follows:

- i. for the *PC* Strategy (that is, when $q < \frac{1}{4} - \frac{\gamma - \beta}{4\alpha}$), $\frac{\partial \pi^*}{\partial \beta} - \frac{\partial \pi^*}{\partial \alpha} = 2q^2$
- ii. for the *MBPN* Strategy (that is, when $\frac{1}{4} - \frac{\gamma - \beta}{4\alpha} < q < 1/2$),

$$\frac{\partial \pi^*}{\partial \beta} - \frac{\partial \pi^*}{\partial \alpha} = \frac{(\alpha + \beta - \gamma)^2 (1 - 2q)^2}{2(\gamma - \beta + \alpha)^2}$$

Figure 4 graphically presents the results stated in the Corollary.

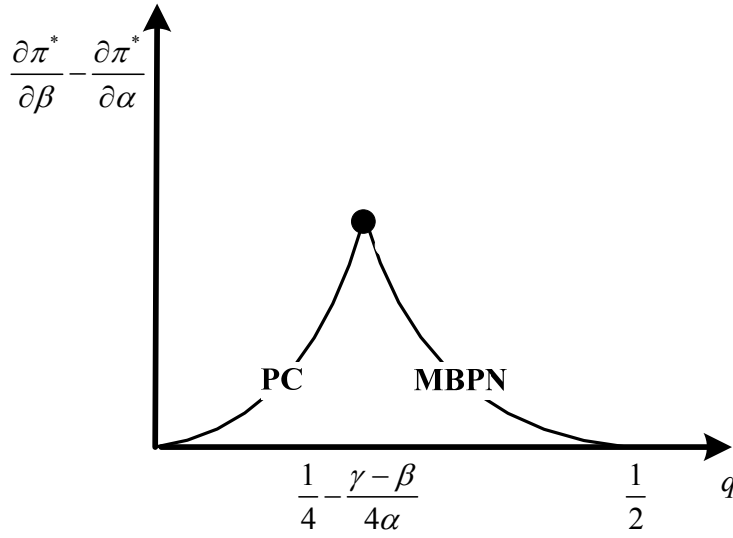


Figure 4. Relative incentive to improve ratings, *RIIR*

As illustrated in Figure 4, when the capacities of the prime time and non-prime time resources are comparable, it is always more advantageous to improve the ratings of the prime time product. However, whereas under the *PC* regime the relative benefit of enhancing the ratings of prime time over non-prime time programming is higher the larger the capacities are, the opposite is true for the *MBPN* region. The reason for the increase in the *RIIR* value in the *PC* region as q increases is the following. Given a value of q (recall that $q = q_N = q_P$ for this discussion), the price p_N is determined by the indifference relationship (7) of the customer with efficiency $t = 1 - 2q$, and equals $\alpha(1 - 2q)$. On the other hand, the price p_P is determined by the indifference relationship (6) of the customer with efficiency $t = 1 - q$, and equals $(\beta - \alpha)(1 - q) + p_N = \beta(1 - q) - \alpha q$. Improving the quality of the prime time programming by a unit results in an increase of $(1 - q)$ in p_P which is greater than the increase in p_N of $(1 - 2q)$ attributable to increasing the quality of non-prime programming. The difference between these two quantities, $(1 - q) - (1 - 2q) = q$, helps measure the difference in the change in optimal prime and non-prime product prices as the corresponding ratings increase, and is obviously an increasing function of q . This difference gets further pronounced because increasing the ratings of the non-prime product decreases p_P . Since the available quantities of both the resources are equal, the *RIIR* value, $\frac{\partial \pi^*}{\partial \beta} - \frac{\partial \pi^*}{\partial \alpha}$, increases as q increases.

For larger resource capacities and full segmentation, the relative advantage of improving the prime time programming diminishes. To gain insight into why this effect manifests, first consider the extreme case when $q = \frac{1}{2}$ (where *MBPN* transitions to the *PB* strategy). Since the broadcaster offers only the bundle in this case, improving the ratings of the prime time product affects the *PB* revenue ($\gamma/4$) only through its impact on the ratings of the bundle. But by our “equal ratings-improvement effect”

assumption, improving the ratings of the non-prime time product has an identical impact on the ratings of the bundle. Therefore, the *RIRR* value is zero. Another way to look at this is that since a bundle needs both a prime and non-prime product, their impact is equivalent as far as the bundle is concerned. Now consider the *MBPN* region. As q goes to $\frac{1}{2}$, the broadcaster sells more of the bundle, and so (i) the amount of the prime time resource used in the bundle increases and (ii) the amount sold separately decreases. For reasons similar to those in the *PB* case above, the impact of improving prime time quality approximately equals the impact of improving non-prime time quality for the part used in the bundle, and the approximation becomes more exact as q approaches $\frac{1}{2}$. So, the impact on *RIRR* due to the bundle decreases as q approaches $\frac{1}{2}$. The impact on *RIRR* due to the individual component sale also reduces because the amount sold separately decreases. Therefore, the *RIRR* value decreases as q increases.

3.5. Value of Bundling

We now analyze the economic benefit of bundling from both the broadcaster's and the advertisers' perspective. For the broadcaster, bundling only makes sense if her revenue when she does consider bundling as an option is at least equal to the revenue when she does not. On the other hand, we say that advertisers (as a group) derive positive value from bundling if their total premium when the broadcaster considers the bundling option exceeds the total premium if the broadcaster does not. Table 1 gives the *value of bundling* from the *broadcaster's* viewpoint (VoB_B) and the *advertisers' viewpoint* (VoB_A) respectively. The last column gives the total (or, social) value of bundling $VoB = VoB_A + VoB_B$. We derive these values numerically for the symmetric case, $q = q_N = q_P$, with $\alpha = 1$, $\beta = 2$ and $\gamma = 2.5$. The tabulated values are expressed as a function of q . Note that the pricing strategy changes, both with and without bundling, as q is increased.

Table 1. Value of Bundling
(Uniform Distribution; $\alpha = 1$, $\beta = 2$, $\gamma = 2.5$)

q_P	q_N	VoB_B	VoB_A	VoB
0.10	0.10	0.0000	0.0000	0.0000
0.20	0.10	0.0017	-0.0092	-0.0075
0.20	0.20	0.0150	-0.0325	-0.0175
0.30	0.30	0.0717	-0.0442	0.0275
0.40	0.40	0.1117	0.0025	0.1142
0.50	0.50	0.1250	0.0625	0.1875

As we can see from the table, VoB_B is zero initially, and then increases as q goes to $\frac{1}{2}$. On the other hand, after remaining at zero initially, VoB_A becomes negative (due to the impact of full segmentation as a result of offering all three products) and then increases. As a result, the total value of bundling VoB is zero initially, becomes negative, and then increases. The reason is that when q is small,

bundling is not used and so all three values of bundling are zero. Then VoB becomes negative because the increase in VoB_B does not fully compensate for the decrease in VoB_A , and the net impact is negative.

4. Extensions

In this section, we generalize our results by relaxing our model assumptions. First, rather than assuming that the advertiser efficiencies are uniformly distributed, we consider several other density functions, and investigate the robustness of our results. Next, we allow for bundles to be comprised of arbitrary number of units of the prime and the non-prime resources, and determine how the optimal composition of the bundle changes as the problem parameters change.

4.1. General Density Functions

The results that we have presented so far assume that the efficiency random variable has a uniform density function. A natural inquiry might be to check the sensitivity of our results to changes in the density function. For example, if the density distribution was left skewed or right skewed, how would the optimal strategies, product prices, and the total revenue change for the same level of resources? Or, if the density function was strictly concave and symmetric about an efficiency of one-half, how would the results compare with the uniform distribution?

We use the family of (standard) Beta distributions to model the density function of the efficiencies. The Beta distribution has two shape parameters, which we denote by a and b . We use the Beta distribution because it has the domain $[0, 1]$ which equals our assumed efficiency range, and changing the parameter values generates the different shapes that are interesting from our perspective. Figure 5 gives the parametric settings and the four different shapes that we will investigate. Given the complexity of deriving the analytic solution for these more general density functions, we complement our analytical results with numerical computations. We assume that $\alpha = 1$, $\beta = 2$, and $\gamma = 2.5$. Since $\alpha = 1$ and $\beta = 2$, γ must lie in the open interval $(2, 3)$ and so a value of 2.5 for γ denotes “medium” incentive to bundle, thus not favoring either a PB or a PC strategy.

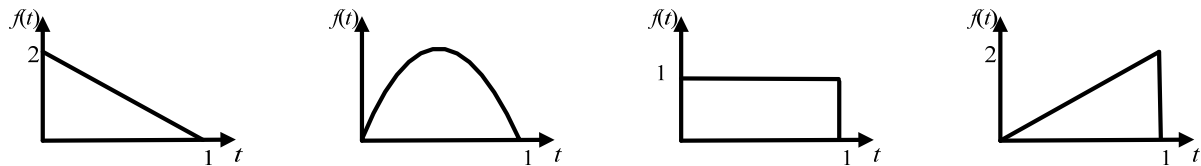


Figure 5 (a): $a=1, b=2$ **Figure 5 (b):** $a=2, b=2$ **Figure 5 (c):** $a=1, b=1$ **Figure 5 (d):** $a=2, b=1$

Figure 5. Density functions

We refer to the advertisers having the efficiency distribution in Figure 5 (a) as *parsimonious* advertisers because a large majority of them have a low willingness to pay. Similarly, we refer to

advertisers in Figures 5 (b), 5 (c), and 5 (d) as *centric* advertisers, *uniform* advertisers and *high-spenders* respectively (we have shown the uniform distribution in this figure for consistency with our later tables and figures).

Figure 6 presents the broadcaster's optimal strategies (determined numerically) for each of the four different types of advertisers as the availability of the two resources changes. Even though there are differences across the different distribution types that reflect the distributions' unique characteristics, the general structure of the optimal strategies is similar.

Comparing the parsimonious advertisers and the high-spenders cases (Figures 6 (a) and 6 (d)) we observe that the *PC* region is smaller for parsimonious advertisers. This difference is a consequence of parsimonious advertisers being concentrated near the low end of the efficiency scale. In order to extract greater revenue from them, the broadcaster offers full spectrum mixed bundling even when the availabilities of the two resources are low (and the relative availabilities are about the same). For the high-spenders case, the broadcaster uses the *PC* strategy for a greater range of resource availabilities because of the concavity assumption about the bundle ratings.

As we mentioned earlier for the uniform advertisers case, unconstrained optimization corresponds to both q_N and q_P values being at least a half. For the parsimonious advertisers case, we can use Proposition 1 to show that the unconstrained region begins at $q_N = q_P = 4/9$. Figure 6 (a) reflects this observation. For the high-spenders case, again using Proposition 1, we can show that the unconstrained region begins at $q_N = q_P = 2/3$. Just like for the uniform advertisers case, these values of $4/9$ and $2/3$ do not seem to depend on the value of γ . Thus, the pure bundle is not offered for the high-spenders case when the sum of the resource availabilities is at most one, as we have assumed in this paper. Schmalensee (1984) has previously observed that mixed bundling reduces the heterogeneity in the customers, and therefore allows better price discrimination. A natural measure of heterogeneity is variance, and the distributions for both parsimonious advertisers and high-spenders have the same variance. Yet, for parsimonious advertisers, pure bundling is the optimal strategy for a larger region defined by q_N and q_P , and for high-spenders, mixed bundling is the optimal strategy for a larger region. This comparison of Figures 6 (a) and 6 (d) thus demonstrates that the *skewness* of the efficiency distribution, besides its heterogeneity (as measured by variance), seems to affect the benefits of mixed bundling.

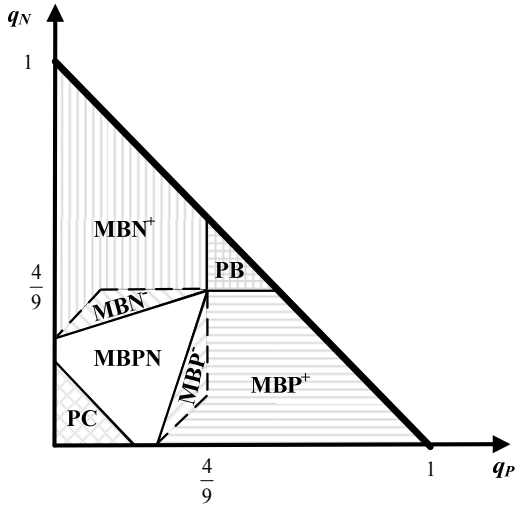


Figure 6 (a): Parsimonious advertisers

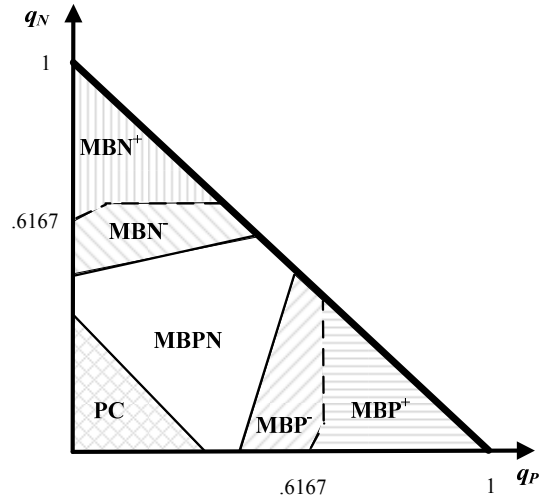


Figure 6 (b): Centric advertisers

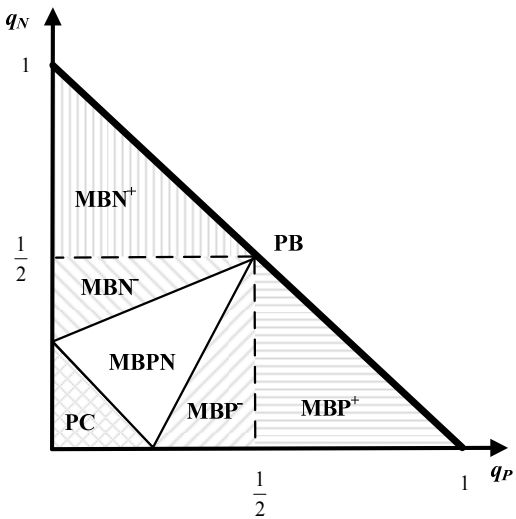


Figure 6 (c): Uniform advertisers

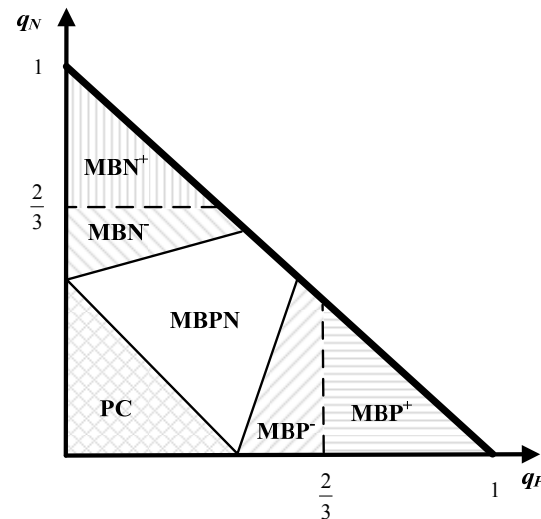


Figure 6 (d): High-spenders

Figure 6. Strategies for the different advertiser types

Tables 2 and 3 show how the solution and the optimal strategy change as the distribution changes for the same values of resource availabilities. The T^* values are the lowest (keeping the resource availability constant) for parsimonious advertisers, and increase as we progressively go through centric and uniform advertisers; they are the highest for high-spenders. Thus, the broadcaster does not have to resort to bundling when the majority of the customers have a high willingness to pay. This is also borne out in Table 3, where the *MBPN* strategy appears more frequently for the parsimonious advertisers case. As expected, the broadcaster charges the highest prices for the bundle in the high-spenders case.

Table 2. Threshold values and Price of the Bundle for Different Beta Distributions
 $(\alpha = 1, \beta = 2, \gamma = 2.5)$

Parameter	q_P	q_N	Types of Advertisers			
			Parsimonious	Centric	Uniform	High-spenders
T*	0.10	0.10	0.8352	0.9996	1.0000	1.0000
	0.10	0.20	0.7224	0.9231	0.9667	1.0000
	0.10	0.30	0.6838	0.8421	0.9000	1.0000
	0.10	0.40	0.6838	0.8042	0.9000	0.9659
	0.10	0.70	0.6838	0.8042	0.9000	0.9487
	0.20	0.10	0.7224	0.9231	0.9667	1.0000
	0.20	0.20	0.6344	0.8421	0.9000	1.0000
	0.30	0.10	0.6838	0.8421	0.9000	1.0000
	0.30	0.30	0.4939	0.7215	0.7667	0.9264
	0.40	0.10	0.6838	0.8042	0.9000	0.9659
	0.40	0.40	0.3791	0.6208	0.6333	0.8427
	0.50	0.50	0.3333	0.5276	0.5000	0.7517
0.70	0.10	0.6838	0.8042	0.9000	0.9487	
T**	0.10	0.10	0.6838	0.8042	0.9000	0.9487
	0.10	0.20	0.6838	0.8042	0.9000	0.9487
	0.10	0.30	0.6838	0.8042	0.9000	0.9487
	0.10	0.40	0.6838	0.8042	0.9000	0.9487
	0.10	0.70	0.6838	0.8042	0.9000	0.9487
	0.20	0.10	0.5528	0.7129	0.8000	0.8944
	0.20	0.20	0.5528	0.7129	0.8000	0.8944
	0.30	0.10	0.4523	0.6367	0.7000	0.8367
	0.30	0.30	0.4523	0.6367	0.7000	0.8367
	0.40	0.10	0.3675	0.5671	0.6000	0.7746
	0.40	0.40	0.3675	0.5671	0.6000	0.7746
	0.50	0.50	0.3333	0.5000	0.5000	0.7071
0.70	0.10	0.3419	0.4215	0.5000	0.5773	
T***	0.10	0.10	0.5843	0.7129	0.8000	0.8944
	0.10	0.20	0.5279	0.6490	0.7333	0.8367
	0.10	0.30	0.4523	0.6132	0.7000	0.7746
	0.10	0.40	0.3675	0.5671	0.6000	0.7530
	0.10	0.70	0.3419	0.4215	0.5000	0.5774
	0.20	0.10	0.5279	0.6490	0.7333	0.8367
	0.20	0.20	0.4839	0.6132	0.7000	0.7746
	0.30	0.10	0.4523	0.6132	0.7000	0.7746
	0.30	0.30	0.4136	0.5599	0.6333	0.7361
	0.40	0.10	0.3675	0.5671	0.6000	0.7530
	0.40	0.40	0.3562	0.5149	0.5667	0.6999
	0.50	0.50	0.3333	0.4724	0.5000	0.6595
0.70	0.10	0.3419	0.4215	0.5000	0.5774	
P_B	0.10	0.10	1.6856	2.0169	2.2000	2.3431
	0.10	0.20	1.5728	1.9147	2.1167	2.2853
	0.10	0.30	1.4779	1.8385	2.0500	2.2233
	0.10	0.40	1.3932	1.7734	1.9500	2.1847
	0.10	0.70	1.3675	1.6278	1.8500	2.0004
	0.20	0.10	1.4418	1.8234	2.0167	2.2311
	0.20	0.20	1.3539	1.7471	1.9500	2.1690
	0.30	0.10	1.2464	1.6710	1.8500	2.1113
	0.30	0.30	1.1128	1.5574	1.7167	2.0359
	0.40	0.10	1.0770	1.5362	1.6500	2.0106
	0.40	0.40	0.9133	1.3924	1.4833	1.8958
	0.50	0.50	0.8333	1.2362	1.2500	1.7425
0.70	0.10	1.0257	1.2452	1.4500	1.6291	

Table 3. Strategies for Different Types of Advertisers
 $(\alpha = 1, \beta = 2, \gamma = 2.5)$

q_P	q_N	Parsimonious	Centric	Uniform	High-spenders
0.10	0.10	MBPN	PC	PC	PC
0.10	0.20	MBPN	MBPN	MBPN	PC
0.10	0.30	MBN ⁻	MBPN	MBN ⁻	PC
0.10	0.40	MBN ⁻	MBN ⁻	MBN ⁻	MBPN
0.10	0.70	MBN ⁺	MBN ⁺	MBN ⁺	MBN ⁺
0.20	0.10	MBPN	MBPN	MBPN	PC
0.20	0.20	MBPN	MBPN	MBPN	PC
0.30	0.10	MBP ⁻	MBPN	MBP ⁻	PC
0.30	0.30	MBPN	MBPN	MBPN	MBPN
0.40	0.10	MBP ⁻	MBP ⁻	MBP ⁻	MBPN
0.40	0.40	MBPN	MBPN	MBPN	MBPN
0.50	0.50	PB	MBPN	PB	MBPN
0.70	0.10	MBP ⁺	MBP ⁺	MBP ⁺	MBP ⁺

Across the different resource availability combinations in Table 2, the T^{***} values are the lowest for parsimonious advertisers indicating that the broadcaster has to “dig deeper” into the market, when many of the advertisers have low willingness to pay. A further analysis of the T^{***} values shows that the location of the marginal advertiser, under all possible resource combinations, that the broadcaster chooses not to serve also varies. It is 0.5 for the uniform advertisers, and for centric customers, this value is 0.4215, which is achieved for both MBN^+ and MBP^+ cases. The reason for this change is that a lack of advertisers with high willingness to pay at the very top end in the centric advertisers case lowers T^* and the increase in the willingness to pay in the middle of the distribution lowers T^{***} (for the MBN^+ strategy) and T^{**} (for the MBP^+ strategy). The minimum T^{***} value (again, over all combinations of the resource availabilities) for parsimonious advertisers is $1/3$, and 0.5773 (this data point does not appear in Table 2 as not all resource availabilities are included in the table) for high spenders. Despite the fact that the minimum T^{***} is the highest for high-spenders and the lowest for parsimonious advertisers, the maximum proportion of the market served (again, over all combinations of the resource availabilities) by the broadcaster is the highest ($2/3$) for high spenders and the lowest ($4/9$) for parsimonious advertisers. (It is 0.5 for uniform and 0.6167 for centric advertisers, respectively.) These values make sense because the broadcaster serves a greater proportion of the market if the majority of advertisers have a higher willingness to pay.

4.2. Bundling with Unequal Resource Proportions

As we mentioned earlier, our model can be modified to allow for an unequal proportion of the two resources in the bundle. In this context, there are two different scenarios, differentiated by whether or not this proportion is fixed or can be chosen optimally. In each case, the solution approach differs slightly

depending on whether or not we can redefine the non-prime product when we offer a bundle with unequal proportions of the prime and non-prime products.

First, consider the case when the proportion is fixed and a redefinition of the non-prime product is possible. In this case, we can simply recalibrate the units of measurement of the non-prime product. For instance, since the non-prime resource tends to be more plentiful than the prime time resource, one unit of the non-prime time resource can be calibrated to a supra-unitary multiple of the prime time unit. For example, if the non-prime time resource is ten times more plentiful, a unit of the non-prime time product can consist of ten minutes, while a unit of the prime time product can consist of only one minute. Hence as long as the proportion of the two resources in the bundle has to remain fixed and the non-prime product can be redefined, the analysis we have done so far immediately carries over with one caveat: if the fixed proportion times α turns out to be greater than β , then we switch the names of the prime and non-prime products. We make this exchange in the terminology to satisfy our assumption that the prime time ratings exceed the non-prime time ratings.

When the proportion is fixed, but the units of the non-prime time cannot be recalibrated, the analysis is slightly different but the conclusions remain qualitatively similar to what we saw in Section 3.

The interesting case arises when the proportion is a decision variable. Here, we focus on the situation where we cannot redefine the non-prime product. When this proportion can be chosen optimally as a function of the parameters of the model $(q_P, q_N, \alpha, \beta, \gamma)$, the characterization of the regions depicted in Figure 2 is likely to be more difficult since each combination of capacity levels (q_P, q_N) leads to a different optimal proportion of the resources used in the bundle. Due to the analytical complexity of solving the problem with variable proportions, we illustrate the solution via numerical calculations.

Let the *bundle composition parameter*, θ , with $\theta > 0$, denote the number of units of the non-prime resource in the bundle with one unit of the prime resource. If the non-prime resource has high availability, we expect the optimal value of θ to be at least one. When we introduce the bundle composition parameter θ as a decision variable, we need to make three changes in the *ROMB_U* model. First, since the bundle needs θ units of the non-prime resource (with one unit of the prime resource), the non-prime resource constraint (13) changes to $\theta \left(1 - \frac{p_B - p_P}{\gamma - \beta} \right) + \frac{p_P - p_N}{\beta - \alpha} - \frac{p_N}{\alpha} \leq q_N$. Second, to avoid the price arbitrage opportunity, so that bundle is “survivable” (Schmalensee, 1984), we need the constraint that the price of the bundle be no more than the sum of the prices of the components it comprises, that is, $p_B \leq p_P + \theta p_N$. Finally, we have to assume a functional form for the ratings of the

bundle, γ , as a function of α , β and θ . We use the specification $\gamma = \sqrt{\beta + \theta\alpha}$, and investigate two cases: $\alpha = 1$ and $\beta = 1.5$, and $\alpha = 1$ and $\beta = 2$. Such a specification for γ guarantees that the ratings of the bundle increase with the ratings of the two resources sold separately, and with θ , the number of units of the non-prime resource used in the bundle. Additionally, the functional form of γ maintains the concavity property we assumed in our original model with fixed proportions.

Figures 7 (a) – 7 (c) depict how θ changes with the problem parameters. In each case, the graphs do not include the *PC* region as the bundle is not offered in this region and so the value of θ is irrelevant. Figure 7 (a) shows how the optimal θ changes with $q = q_N = q_P$. The smaller values of q in this graph correspond to the *MBPN* regime. As q increases, the optimal θ value increases because by consuming more of the non-prime resource as part of the bundle, the broadcaster can charge higher prices for the bundle while at the same time avoiding the downward pressure that selling the non-prime product by itself imposes on the price of the prime product. Indeed, as q increases, the optimal strategy shifts from *MBPN* to *MBP⁻*. When the β/α increases to two, θ jumps up because, relatively speaking, the prime product becomes more attractive and commands a higher price, and therefore the amount of the prime sold by itself increases. Consequently, the amount of the prime resource sold as part of the bundle, and hence the amount of bundle sold by itself decreases, and θ increases.

Figure 7 (b) shows how θ changes as q_N increases keeping q_P constant at 0.1. Similarly, Figure (c) depicts how θ changes as q_P increases keeping q_N constant at 0.1. In both cases, increasing the resource availability tends to increase the value of θ , but in slightly different ways. As q_N increases, the strategy changes from *PC* to *MBPN* and then to *MBN⁻*. However, as q_P increases, the strategy changes from *PC* to *MBPN* to *MBP⁻* and then finally to *MBP⁺*. Because θ is a decision variable, we always end up using all of the non-prime resource, but that is not so for the prime resource. Also, once we reach the *MBP⁻* region, the value of θ no longer changes since the prime resource is not fully utilized in the bundle. Therefore, in Figure 7 (c), we see that θ plateaus; the kink that just precedes the plateau is the point at which the strategy switches from *MBPN* to *MBP⁻* region.

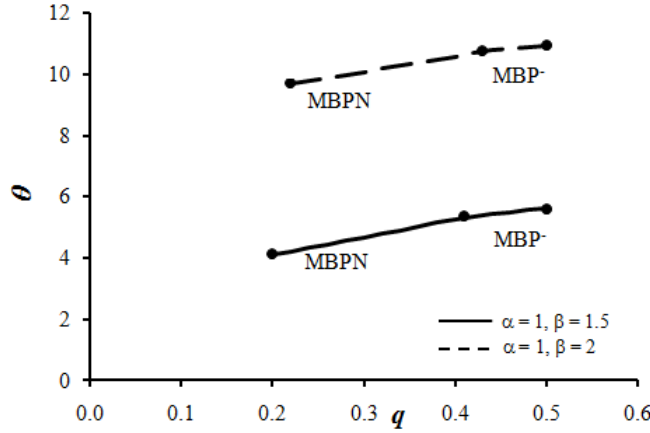


Figure 7 (a). θ as a function of $q = q_N = q_P$

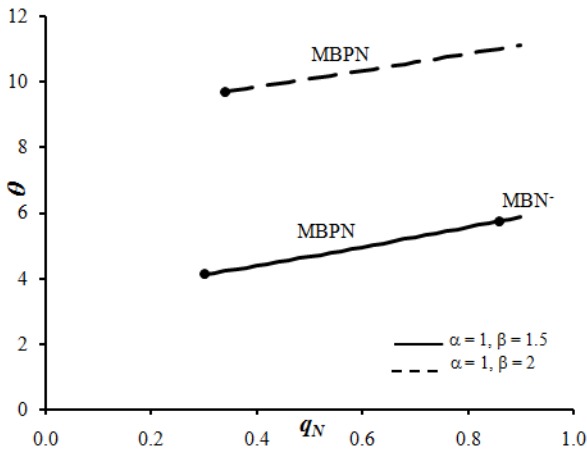


Figure 7 (b). θ as a function of $q_N, q_P = 0.10$

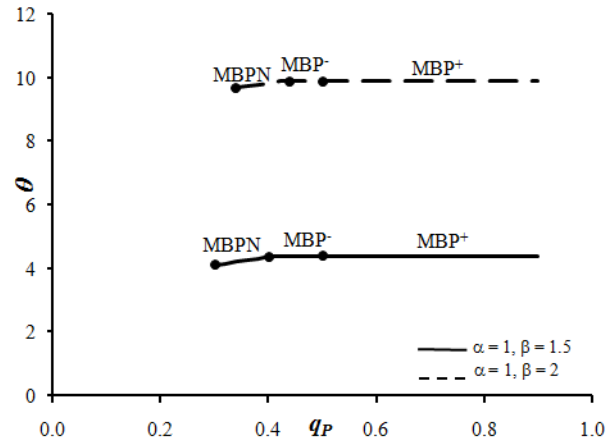


Figure 7 (c). θ as a function of $q_P, q_N = 0.10$

Figure 7. Change in θ with resource availability

5. Conclusions

In this paper, we have examined bundling strategies when the bundle's constituents satisfy a universal preference ordering and have limited availability. While this research is motivated by television advertising, where a preference ordering of the products exists naturally, several other situations (e.g. billboard and internet advertising) also exhibit this characteristic. Our results show that the relative availabilities of the resources strongly influence the broadcaster's optimal strategy of implementing full spectrum mixed bundling (offering the bundle and each of the components), or partial spectrum mixed bundling (offering the bundle with one of the components), or not using bundling at all. Clearly, the resource availabilities also influence their marginal value to the broadcaster; we determine how much more valuable it is to increase the availability of one resource over the other. We also investigate the relative benefits of improving the quality of prime versus non-prime time programming. The robustness of the managerial guidance provided by this analytical work is substantiated by our numerical testing.

Our research points towards several promising research directions. First, we have assumed a monopolistic scenario with only one broadcaster. Introducing competition, where advertisers desiring to place commercials during, say, prime time have a choice of multiple networks, would both add complexity to and enhance realism of the model. Incorporating the advertisers' objectives such as recent work in targeted advertising (Chen and Iyer, 2002; Iyer et al., 2005; Gal-Or and Gal-Or, 2005; Gal-Or et al., 2006), or combative advertising (Chen et al., 2009) into this competitive bundling framework promises to be interesting. Second, we have assumed that the resource capacities are limited and that their marginal costs are zero (or, equivalently, that the resource availabilities are limited and the resource costs are sunk). It might be worth investigating how the results change if this marginal cost assumption does not hold. Third, it might be useful to investigate the optimal bundling strategies in the presence of multiple resource classes (for example, in internet advertising, the number of clicks needed from the home page to reach the advertisement). Fourth, our model is deterministic along the advertisers' willingness to pay; introducing stochastic elements with respect to this dimension (Venkatesh and Mahajan, 1993; Ansari et al., 1996) might also be worthwhile.

Finally, we have assumed concavity of the rating function. This need not always be the case. Continuing with the television advertising situation, if there are multiple decision makers who have different viewing preferences, the advertiser may derive super-additive benefits from advertising during prime time and during prime time. For example, Mattel might advertise during non-prime time to target children and during prime time to target the parent. To sell a big ticket item such as an automobile or a large kitchen appliance, both spouses (who may have different viewing habits) may need to be targeted, and so a company like Maytag may see advertisements during prime time and non-prime time as complementing each other. Due to joint decision making and complementarity of components, therefore, Mattel's or Maytag's willingness to pay for the bundle of advertisements may be greater than the sum of their willingness to pay for the components of the bundle. In this case, the price arbitrage constraint may be always binding. Moreover, assuming that there is no secondary market that allows an intermediary to buy the components and assemble the bundle for sale and that the broadcaster can impose a restriction rationing each advertiser to buy at most one product, the price arbitrage constraint (2) may not be economically valid. These and other nuances associated with bundling, as well as the challenge in modeling and analyzing bundling situations and its inter-disciplinary appeal, will in all likelihood guarantee that bundling will continue to be a fertile research area.

Appendix A

Proof of Proposition 1 and Corollary 2: Consider the total revenue gained by the monopolist:

$$\pi = p_B [1 - F(T^*)] + p_P [F(T^*) - F(T^{**})] + p_N [F(T^{**}) - F(T^{***})].$$

We will derive first the conditions for the concavity of the revenue function. Let $K_1 = 2f(T^*) + T^*f'(T^*)$, $K_2 = 2f(T^{**}) + T^{**}f'(T^{**})$, and $K_3 = 2f(T^{***}) + T^{***}f'(T^{***})$, with $0 \leq T^*, T^{**}, T^{***} \leq 1$. The Hessian matrix H associated with the revenue function is

$$H = \begin{bmatrix} -\frac{K_1}{\gamma - \beta} & \frac{K_1}{\gamma - \beta} & 0 \\ \frac{K_1}{\gamma - \beta} & -\frac{K_1}{\gamma - \beta} - \frac{K_2}{\beta - \alpha} & \frac{K_2}{\beta - \alpha} \\ 0 & \frac{K_2}{\beta - \alpha} & -\frac{K_2}{\beta - \alpha} - \frac{K_3}{\alpha} \end{bmatrix}.$$

Let $x = [x_1 \ x_2 \ x_3]$ be a three dimensional real-valued vector. The product $x^T H x$ is equal to:

$$x^T H x = -\frac{K_1}{\gamma - \beta} (x_1 - x_2)^2 - \frac{K_2}{\beta - \alpha} (x_2 - x_3)^2 - \frac{K_3}{\alpha} x_3^2.$$

Hence, the Hessian matrix is negative semi-definite, and therefore the revenue function is concave and admits to a local maximum, as long as K_1 , K_2 and K_3 are non-negative. The non-negativity assumption on K_1 , K_2 and K_3 is satisfied by a large class of probability density functions bounded on the $[0, 1]$ domain, including the Beta distribution, of which the uniform distribution is a special case.

We will now use the first order KKT conditions to show that this local maximum is, in fact, unique, and therefore global. Noticing, for example, that

$$\begin{aligned} \frac{\partial \pi}{\partial p_B} &= [1 - F(T^*)] + p_B \left(-\frac{dF(T^*)}{dp_B} \right) + p_P \left(-\frac{dF(T^*)}{dp_B} \right) \\ &= [1 - F(T^*)] - \frac{p_B - p_P}{\gamma - \beta} f(T^*) \\ &= [1 - F(T^*)] - T^* f(T^*), \end{aligned}$$

we can complete the rest of the first order conditions associated with problem *ROMB*:

$$\begin{aligned} 1 - F(T^*) - T^* f(T^*) &= 0 \\ F(T^*) - F(T^{**}) + T^* f(T^*) - T^{**} f(T^{**}) &= 0 \\ F(T^{**}) - F(T^{***}) + T^{**} f(T^{**}) - T^{***} f(T^{***}) &= 0. \end{aligned}$$

Let the *hazard rate* function $h(t)$ be defined as $h(t) = f(t) / [1 - F(t)]$. From the first equation, we obtain the value of threshold T^* to be equal to the fixed point of the inverse of the hazard rate function, that is, $T^* = [1 - F(T^*)] / f(T^*)$. Substituting into the remaining equations, we obtain similar results for

T^{**} and T^{***} . Brouwer's fixed point theorem guarantees the existence of at least one such point; however due to the monotonicity of the hazard rate function, we can see that the solution must be unique, that is, $T^* = T^{**} = T^{***}$.

Since all thresholds are equal, it follows that the monopolist will only offer the bundle, cover the segment $1 - F(T^*)$, and collect total revenues $\pi = \gamma T^* [1 - F(T^*)] = \gamma [1 - F(T^*)]^2 / f(T^*)$. Using Markov's inequality, we can find an upper bound on the profit value as follows:

$$\Pr(t > a) \leq \frac{\mathbf{E}[T]}{a} \Leftrightarrow \gamma T^* \Pr(t > T^*) \leq \gamma T^* \frac{\mathbf{E}[T]}{T^*} \Leftrightarrow \pi \leq \gamma \mathbf{E}[T].$$

Proof of Proposition 3: The $ROMB_U$ model is a quadratic optimization program with linear constraints; therefore the first-order KKT conditions are both necessary and sufficient. The proof is straightforward once the first order conditions are expressed under all capacity scenarios: no binding capacity constraints; one binding capacity constraint (two cases), and both capacity constraints are binding. For brevity, we present only the optimal thresholds that result after optimizing $ROMB_U$ under the assumption that both capacity constraints are binding:

$$\begin{aligned} T^* &= \frac{1}{2} + \frac{\alpha}{\gamma - \beta + \alpha} (1 - q_P - q_N) \\ T^{**} &= 1 - q_P \\ T^{***} &= \frac{1}{2} + \frac{\gamma - \beta}{\gamma - \beta + \alpha} (1 - q_P - q_N). \end{aligned}$$

In order to maintain consistency, the thresholds must be ordered on the $[0, 1]$ line segment, that is $0 \leq T^{***} \leq T^{**} \leq T^* \leq 1$. For example, the last inequality is equivalent to the following:

$$\begin{aligned} T^* \leq 1 &\Leftrightarrow \frac{\alpha}{\gamma - \beta + \alpha} (1 - q_P - q_N) \leq \frac{1}{2} \\ &\Leftrightarrow 1 - q_P - q_N \leq \frac{\gamma - \beta + \alpha}{2\alpha} \\ &\Leftrightarrow q_P + q_N \geq \frac{1}{2} - \frac{\gamma - \beta}{2\alpha}. \end{aligned}$$

The bundle is not offered when equality holds, therefore condition (i) from the proposition follows naturally. Conditions (ii), (iii) and (iv) are derived similarly from the remaining inequalities involving the thresholds, and observing that either MBN^+ or MBP^+ strategies imply only one capacity constraint binding. Finally, using Proposition 1 with $F(x) = x$ and $f(x) = 1$, we obtain $T^* = T^{**} = T^{***} = \frac{1}{2}$ and the substitution into both capacity constraints yields condition (v).

Proof of Proposition 4: Just like the proof of Proposition 3, we use the fact that the first order conditions are both necessary and sufficient. Additionally, the invariant boundary conditions (5) – (7) establish the

relationships between thresholds and prices. For brevity, under the case of both capacity constraints binding, we derive the optimal price for the non-prime time product:

$$\left. \begin{aligned} \alpha T^{***} - p_N = 0 &\Leftrightarrow p_N = \alpha T^{***} \\ T^{***} &= \frac{1}{2} + \frac{\gamma - \beta}{\gamma - \beta + \alpha} (1 - q_P - q_N) \end{aligned} \right\} \Rightarrow p_N = \frac{\alpha}{2} + \frac{\alpha(\gamma - \beta)}{\gamma - \beta + \alpha} (1 - q_P - q_N).$$

Similarly we obtain the remaining prices as

$$\begin{aligned} p_P &= \frac{\beta}{2} + \frac{\beta(\gamma - \beta) + \alpha(\beta - \alpha)}{\gamma - \beta + \alpha} \left(\frac{1}{2} - q_P \right) + \frac{\alpha(\gamma - \beta)}{\gamma - \beta + \alpha} \left(\frac{1}{2} - q_N \right) \\ p_B &= \frac{\gamma}{2} + \frac{\beta(\gamma - \beta) + \alpha(\gamma - \alpha)}{\gamma - \beta + \alpha} \left(\frac{1}{2} - q_P \right) + \frac{2\alpha(\gamma - \beta)}{\gamma - \beta + \alpha} \left(\frac{1}{2} - q_N \right). \end{aligned}$$

The other cases follow the same argument and are derived in an identical fashion.

Proof of Proposition 5 and Corollary 6: The proof relies on the first order conditions and solving for the Lagrange multipliers. Corollary 6 follows immediately by taking the difference between the prime and the non-prime time multiplier.

Proof of Proposition 7 and Corollary 8: Using Proposition 3 and substituting the optimal prices in the objective function of $ROMB_U$ and taking the partial derivatives, we obtain, for example, for the PC case, the following:

$$\left. \begin{aligned} \pi^* &= \beta q_P (1 - q_P) + \alpha q_N (1 - q_N - 2q_P) \\ \frac{\partial \pi^*}{\partial \alpha} &= q_N (1 - q_N - 2q_P) \\ \frac{\partial \pi^*}{\partial \beta} &= q_P (1 - q_P) \end{aligned} \right\} \Rightarrow RIIR = (q_P - q_N)(1 - q_P - q_N) + 2q_P q_N.$$

The other cases are derived in a similar fashion. Corollary 8 follows from Proposition 7 by substituting $q = q_N = q_P$.

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