

## Learning Objectives

- Describe these concepts
- Present value
- Future value
- Interest
-Perform simple computations involving


## Definition

Cost of a resource is the decrease in wealth that results from committing this resource to a particular alternative (before benefits from the alternative are computed)

## Types of cost

- Past vs. future costs

Postponable cost

- Joint costs
- Replacement cost
- Direct and indirect
- Fixed and
incremental
- Long and short run
- Opportunity costs
- Sunk costs


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## Mathematics of Money

## Formulation

- If an amount of money ( $P$ ) were invested
- Value of money changes with time such that it grew at precisely the rate of
- Inflation causes future dollars to be worth less inflation ( $I$ ) for one time period, then
- Investment risk devalues future dollars - $F=P+P i=P(1+i)$ proportionately to the risk
- That is, $F$ is the equivalent future value of $P$
- Elements
- Future value ( $F$ )
- Present value ( $P$ )
- For $n$ time periods,
- Rate (I)
- Annuity ( $A$ ) - A sequence of uniform payments
- $F=P(1+\mathrm{i})+P(1+i) i=P(1+i)(1+i)$
- Generalizing, $F=P(1+i)^{n}$
- This is referred to as the future worth of a present amount


## Cash Flow Diagrams



## Cash Flow Series

$$
i=\%
$$



## Cash Flow Series

## Cash Flow Series

- Payments of $A$ made at regular intervals
- Computing $P$
-Compute

$$
\begin{aligned}
F & =A\left[1+(1+i)+(1+i)^{2}+\ldots+(1+i)^{N}\right] \\
& =A\left[\frac{(1+i)^{N}-1}{i}\right] \\
F & =A(F / A ; i \%, N)
\end{aligned}
$$

$F=A\left[\frac{(1+i)^{N}-1}{i}\right]$
and

$$
\begin{aligned}
& F=P(1+i)^{N} \\
& \therefore P=A\left[\frac{(1+i)^{N}-1}{i}\right]\left(\frac{1}{1+i}\right)^{N}=\left[\frac{(1+i)^{N}-1}{i(1+i)^{N}}\right]
\end{aligned}
$$

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## Other Formulations

- Present worth of a future amount $\quad P=\frac{F}{(1+t)^{n}}$

Annuity of a future amount $A=\frac{i}{(1+)^{-1}-1} F$

- Future worth of an annuity
$F=\frac{(1+i)^{n}-1}{i} A$
- Spreadsheet formula: @FV $(A, i, n)$
- Present worth of an annuity
$P=\frac{(1+i)^{n}-1}{i(1+i)^{n}} A$
- Spreadsheet formula: @PV(A,i,n)

Annuity from a present amount
$A=\frac{i(1+i)^{n}}{(1+i)^{n}-1} P$

- Spreadsheet formula: @PMT(P,i,n)


## Project Justification Techniques

- Net future value (NFV)
- The difference between the future value of your project and the future value of alternative investments
- NFV $=F-F^{*}=F-P\left(1+\digamma^{*}\right)$
- $I^{*}$ is the rate of return of an alternative investment
- Use money market rates, bond rates, return on equity

Internal Rate of Return

- Ratio of the value change and the present value $I R R=\frac{F-P}{P}$
This is useful when a minimum RoR is required


## Application of these Concepts

- Compare the financial implications of implementing the project to not implementing the project
- Case 1: P consists of
- The cost of implementing the project
- The PV of maintenance costs over the life of the project
- Case 2: Compute the PV of an annuity, where the annuity is the estimated cost of waiting, retry behavior, and other productivity decreasing factors
- Compare PVs


## Depreciation

- Definitions of Depreciation
- A System of Accounting which Aims to Distribute Cost or Other Basic Value of Tangible Capital Assets, Less Salvage Value, Over the Estimated Useful Life of a Unit in a Systematic and Rational Manner for the Purpose of Allocation (Paraphrased from AICPA)
- Loss in Service Value Not Restored by Maintenance
- Due to Normal Wear and Tear, Exposure and Decay, Technological Obsolescence, etc.
- Depreciation Does Not Involve Actual Cash

Outlays

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## Computing Depreciation Expense

- Original Cost of Equipment
- Estimated Service Life of Equipment

Estimated Net Salvage Value of the Equipment

- Remaining Value at the End of the Service Life
- Can Include the Cost of Removal
- Depreciation Method
- Retirement/Replacement
- Age-Life


## Retirement/Replacement Methods

- Not Widely Used Because Operating Expenses

Vary Widely from Year to Year
Retirement Accounting

- Carrys Original Cost until the Equipment is Retired
- Full Cost is Charged as an Operating Expense at Retirement
- Replacement Accounting
- Similar to Retirement Accounting
- Replacements and Retirements Without Replacements are Charged to Operating Expense


## Age-Life Methods

- Designed to Provide More Consistent

Expense Accounts from Year to Year

- Straight-Line Depreciation
- Depreciation Charge is Computed for Each Retirement Period
- Draw a Straight Line Between Original Cost and Salvage Value, and Allocate the Difference over Service Life

Depreciation Charge $=\frac{\text { Original Cost }- \text { Salvage Value }}{\text { Service Life }}$

## Age-Life Methods

- Accelerated depreciation allows higher depreciation early in the equipment life than straight line method
- Sum-of-the-Year's-digits

Depreciation Expense $=\frac{\# \text { Years Remaining at Beginning of Year }}{\text { Total of the Digits of the Year's Life }} \times$ (Original Cost - Salvage Value)

- Subtract from current value (undepreciated value)
- Repeat next year
- Double declining balance
- Double the depreciation rate of straight line
- Subtract from current value
- Depreciate remaining balance by straight line


## Deprecation Concepts



Comparison of Depreciation Approaches


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## Comments on Depreciation

## Review

- Money has time value
-Size of depreciation charge depends on
- Service Life
- Salvage value

Estimating both parameters in advance is difficult

- Service life must take technological and usage factors into account
- Actual salvage value depends on costs and prices at the time of decommissioning

This value is called interest

- Simple interest - Interest is not earned/paid on interest
- Compound interest - Interest is computed on principle as well as accrued interest


## Types of Cash Flows

- Single
- Uniform series

Linear gradient series
Geometric gradient series

- Irregular series


## Single

- Studied earlier
- $F=P(i ; N)$
$\stackrel{F}{ }=\mathrm{P}(1+\mathrm{i})^{\mathrm{N}} \quad i=\%$


Pdollars
deposited


## Applications

Sinking Fund- Interest bearing account into which money is paid on a regular basis, usually for replacing a fixed asset - Capital recovery

## Linear Gradient

## Linear Gradient

$$
\begin{aligned}
P & =0+G /(1+i)^{2}+2 G /(1+i)^{3} \\
& =\cdots+{ }^{(N-1) G} /(1+i)^{N} \\
& =G\left[\frac{(1+i)^{N}-i N-1}{i^{2}(1+i)^{N}}\right]=G(P / G ; i ; N)
\end{aligned}
$$

## Linear Gradient

What is the equivalent uniform

## Geometric Gradient Series

- Payments change by a constant percentage over time, where $g$ is the percent change
-This is also called compound growth
$\Rightarrow$ Thus, $A_{n}=A_{1}(1+g)^{n-1}$
$F=\frac{G}{i}\left[\frac{(i+1)^{N}-1}{i}-N\right]=G(F / G ; i ; N)$


## Irregular Payments

- Bring each payment to the present (or future)
-Add the values at the same point in time


## Effective Interest Rates

Nominal interest rate - the rate that is used in computations

- Effective interest rate - interest rate that is actually experienced
-Equivalence

$$
i_{a}=(1+r / M)^{M}-1
$$

- Where $r$ is the nominal rate and $M$ is the number of compounding periods


## Comparing Projects

## Net Present Value

- Net Present Value (NPV)

Rate of Return Payback period

- Data needed
- First cost
- Annual costs
- Annual receipts
- Salvage value
- Compute the present value of these streams

| NPV Example |
| :--- | :--- | :--- |
| $\qquad$Item Switch A Switch B <br> First cost $\$ 10,000$ $\$ 15,000$ <br>  5 years 10 years <br> Sife $\$ 2,000$ $\$ 0$ <br> Ann. <br> Receipts $\$ 5,000$ $\$ 7,000$ <br> Annual <br> Costs $\$ 2,200$ $\$ 4,000$ |


| Which is Better? <br> WConsider $1=8 \%$ <br> Item Switch $A$ Switch B <br> PV of rcpts $\$ 5 K(P / A ; 8 ; 10)$ $\$ 7 K(P / A ; 8 ; 10)$ <br> PV of salvage $\$ 2 K(P / F ; 8 ; 10)$ $\$ 0$ <br> Cost $-\$ 2.2 K(P / A ; 8 ; 10)$ $-\$ 4 K(P / A ; 8 ; 10)$ <br> First cost $-\$ 10,000$ $-\$ 15,000$ <br> Replacement $-(10 K-2 K)$ <br> $(P / F ; 8 ; 5)$  |
| :--- |


| Results |  |
| :--- | :--- | :--- |
| Item Switch A <br> PV of rcpts $\$ 33,551$ <br> PV of salvage $\$ 926$ <br> Cost $-\$ 14,762$ <br> First cost $-\$ 10,000$ <br> Replacement $-\$ 5,445$ <br> TOTAL $\$ 4,270$ |  |

## Rate of Return

- Internal Rate of Return
- What is the interest rate at which the PV of the cash inflow equals the PV of the cash outflow?
- Compare this to the Minimum Attractive Rate of Return (MARR)
$\diamond$ External RR
- What is the interest rate that equates the future worth of investments to the accumulation of reinvested returns?


## Example using IRR

| Year | Switch 1 | Switch 2 | Difference |
| :--- | :--- | :--- | :--- |
| 1 | $-10,000$ | $-15,000$ | $-5,000$ |
| 2 | 2,800 | 3,000 | 200 |
| 3 | 2,800 | 3,000 | 200 |
|  |  |  |  |
| 5 | $-8,000$ |  | 8,000 |
|  |  |  |  |
| 10 | 4,800 | 3,000 | $200-2 \mathrm{~K}$ |

## Example using IRR

- PV for the difference in cash flows: - 5 K $+200(P / A, i, 10)+8 K(P / F, i, 5)-2 K$ (P/F,i,10)
What is the value of $i$ for which this equals zero?
- $i=12.1 \%$
- This is greater than MARR (10\%), so the larger investment (switch B) is justified


## Risk in Projects

-An investment project where the cash flows are not known with certainty
-Project risk is variability in Net Preset Worth (or IRR)

- Risk is the consequence of uncertainty, and usually implies a potential for loss


## Analytical Approaches

-Sensitivity analysis
-Break-even analysis
-Scenario analysis

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## Sensitivity Analysis

-An approach that can be used when some assumptions are questionable
-Useful in all branches of analysis, not just engineering economics

## Basic Approach

- Vary model parameters
- Observe the outcome of the model
- Determine at what parameter values the difference in the model outcome becomes significant
-Assess whether this critical parameter value is possible or likely


## Example

- Postal Service is considering purchasing


## Example

 a 4,000 lb forklift truck- Assume
-Fuel alternatives
- 7 year life
- $10 \%$ interest
- Gasoline
- One shift $=8$ hrs operation
- 200 to 260 shifts per year


## Example

今Fuels

- Gasoline: 11.1 gal per shift, $\$ 1.20 / \mathrm{gal}$
- LPG: 11 gal/shift, \$1.02/gal
- Diesel: 7.2 gal/shift, $\$ 1.13 / \mathrm{gal}$
- Electric: $31.25 \mathrm{kWh}, \$ 0.05 / \mathrm{kWh}$


## Example

\begin{tabular}{|l|r|r|r|r|}
\hline \& \multicolumn{1}{l|}{ Gas` } \& \multicolumn{1}{l|}{ LPG } \& \multicolumn{1}{l|}{ Diesel } \& \multicolumn{1}{l|}{ Elec. } <br>
\hline Initial \& $\$ 20,107$ \& $\$ 21,200$ \& $\$ 22,263$ \& $\$ 29,739$ <br>
\hline Salv. \& $\$ 2000$ \& $\$ 2000$ \& $\$ 2200$ \& $\$ 3000$ <br>

\hline Fuel/shift \& 11.1 gal \& 11 gal \& 7.2 gal \& | 31.25 |
| ---: |
| kWh | <br>

\hline Fuel cost \& $\$ 1.20$ \& $\$ 1.02$ \& $\$ 1.13$ \& $\$ 0.05$ <br>
\hline Fuel/shift \& $\$ 13.32$ \& $\$ 11.22$ \& $\$ 8.14$ \& $\$ 1.56$ <br>
\hline Maint./yr \& $\$ 1000$ \& $\$ 1000$ \& $\$ 1000$ \& $\$ 500$ <br>
\hline Var./shift \& $\$ 7$ \& $\$ 7$ \& $\$ 7$ \& $\$ 4.5$ <br>
\hline
\end{tabular}

## Computations

## Computations

- Find annual costs for each alternative:
- Annual operating costs
- Gasoline: \$20,170(A/P,10\%,7)-
- Gasoline: $\$ 1000+(13.32+7) \mathrm{M}$
- LPG:\$1000 + (11.22+7)M
- LPG: \$21,200(A/P,10\%,7)-
- Diesel: $\$ 1000+(8.14+7) \mathrm{M}$
\$2000(A/F,10\%,7) = \$4,144
- Diesel: \$22,263(A/P,10\%,7)\$2200(A/F,10\%,7) = \$4,341
- Electric: \$29,739(A/P,10\%,7)\$3000(A/F,10\%,7) = \$5,792
- Electric: \$500 + (1.56+4.5)M


## Computations

- Total Equivalent Annual Costs
- Gasoline: \$4919+\$20.32M
- LPG:\$5144+\$18.22M
- Diesel:\$5341+\$15.14M
- Electric: \$6292+\$5.06M


