Basic Formulas for Growth Analysis.

I. Computing Annual Average Growth Per Capita

INIT = initial standard of living (examples of measurements include GNP per capita and real income per capita);

TERM(t) = standard of living t years after the initial period.

Suppose that t = 1, so that we are computing growth during a one year period. Then,

TERM(1) = INIT*(1 + g), where g is the annual average growth rate.

By simple algebraic manipulation, then

\[ g = \frac{TERM(1)}{INIT} - 1 \]

Clearly, the rate of growth can vary over time. However, if we want to compute an annual average growth rate, g, then we make this calculation assuming that growth each year is constant. Suppose that t = 2, so that we are computing annual average growth over a two year period. Then,

TERM(2) = TERM(1)*(1 + g), and

TERM(1) = INIT*(1 + g)

If you think about these two equations for some time, then it is clear that

TERM(2) = INIT*(1 + g)^2, and

\[ g = \left(\frac{TERM(2)}{INIT}\right)^{1/2} - 1 \]

Suppose that t = n > 2, so that we are computing annual average growth over an n year period. Then

TERM(n) = TERM(n-1)*(1+g)

....
TERM(2) = TERM(1)*(1 + g), and

TERM(1) = INIT*(1 + g)

If you think about these equations that emerge from this exercise, then it will be come clear that

TERM(n) = INIT*(1 + g)^n, and

g = \frac{(TERM(n)/INIT)^{(1/n)} - 1}{n}

PRACTICE

Suppose that INIT = 100, TERM(20) = 850. Compute the annual average growth rate.

Exponential growth means that in each year the growth increases. Using a simple spreadsheet or calculator and the numbers from the preceding example, generate an example of accelerating growth.

Suppose that INIT = 100 and g = 4%. Compute TERM after 30 years. If g = 5%, again COMPUTE after 30 years. What is the impact of a 1-percentage point difference in growth on terminal income?

Suppose that TERM = 1000 and g over a 50 year period 3 percent. What was INIT 50 years ago?

To really understand this growth formula, make up some of your own examples.

II. Total Factor Productivity (TFP)

The idea here is to draw a distinction between sustainable and unsustainable growth. For example, while Singapore grew at a very rapid annual average rate (roughly 6-7%) between 1960 and 1985, economists such University of Chicago’s Alwyn Young asked whether this performance was sustainable. Economists during the 1970s such as Columbia’s Padma Desai and University of Penn’s Donald Green and Herb Levine questioned whether the growth performance of the Former Soviet Economy (FSU) was sustainable. The idea is that even if growth is rapid, it is not sustainable when production is done in the same old fashion. Loosely speaking, evidence of production in the same old way is that economy just keeps employing more labor and capital. Growth is sustainable when it is achieved without adding a great deal of capital and labor.

To get at this issue, let
TFP = total factor productive, gK = growth of capital, gL = growth of labor, shK =
capital’s share and shL = labor’s share, and growth is an annual average figure over some
period of time.

I will explain just what this means during class lectures. Nevertheless, for reasons that I
will do my best to explain during lectures, we assume that

shK + shL = 1. The reason for this is that there are CONSTANT RETURNS TO SCALE
in production (i.e., if you double your capital stock and your employed labor force, your
overall output will increase; and, j-percent increase in the capital and labor force would
result in a j-percent increase in GNP).

Also, we assume that shK > 0, shL > 0.

Then growth is decomposed into growth driven by just adding more capital and labor (i.e.
doing things in the same old fashion) and growth driven by total factor productivity (i.e.,
doing things in a more clever fashion than before). Therefore,

\[ g = \text{growth from factor accumulation} + \text{growth from total factor productivity (TFP)}, \]

growth from factor accumulation = shK*gK + shL*gL.

Then

\[ g = shK*gK + shL*gL + \text{ TFP}, \]

\[ \text{TFP} = g - (shK*gK + shL*gL) \]

It really helps to do some examples to understand what is going on here. So, I attach the
following for both growth analysis and tfp (sustainability) analysis. I will go over these
problems and those above in tutorial.

Practice Problems for Growth and Growth Accounting

GROWTH

1. Note that (TERM) = (INIT)*(1 + g)^t is the growth equation, where TERM and INIT
denote terminal and initial GNP per capita, g denotes annual average growth, t denotes
the years between the terminal and initial period of time.

2. Suppose that TERM, INIT, and t are data. Derive the formula for g. Consider the
following examples. Suppose INIT = 100, t = 20 and TERM = 400. What is g? Suppose
INIT = 100, t = 20, and TERM = 500. What is g? Make up examples for yourself. You
should note that small differences in g over a long period (t = 20 for example) translate
into big differences in TERM.
3. Suppose that TERM, INIT and g are data. Derive the formula for t. Consider the following examples. Suppose INIT = 100, TERM = 400 and g = .05 (5%). What is t (i.e., how long does it take for the terminal GNP per capita level to be attained?). Suppose INIT = 100, TERM = 400 and g = .06. What is t? Make up some examples for yourself and note that small differences in g have a big impact on t!

4. Suppose that TERM, g and t are data. Derive the formula for INIT. If g = .03, t = 20 and TERM = 1000, what was INIT? Make up your own examples.

5. Consider the data from the following economy covering the fourth quarter of 1970 to the fourth quarter of 1974.

<table>
<thead>
<tr>
<th>Year</th>
<th>GNP per capita</th>
<th>Capital</th>
<th>Labor</th>
</tr>
</thead>
<tbody>
<tr>
<td>1970:IV</td>
<td>1,000</td>
<td>100</td>
<td>20</td>
</tr>
<tr>
<td>1971:IV</td>
<td>1,050</td>
<td>105</td>
<td>21</td>
</tr>
<tr>
<td>1972:IV</td>
<td>1,100</td>
<td>110</td>
<td>22</td>
</tr>
<tr>
<td>1973:IV</td>
<td>1,120</td>
<td>118</td>
<td>23</td>
</tr>
<tr>
<td>1974:IV</td>
<td>1,250</td>
<td>125</td>
<td>24</td>
</tr>
</tbody>
</table>

Compute the average annual growth of GNP, capital and labor over this 4 year period.

Suppose that labor’s share is .25 and capital’s share is .75. Compute TFP for the entire period.

Generate an alternative capital and labor series that would increase TFP.

Generate an alternative capital and labor series that would depress TFP.