Disk Mass Required to Form Planets via the Gravitational Instability

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We explore the viability of forming giant planets via the gravitational instability. This mechanism favors the production of giant planets at large radii, since a disk would need to be both gravitationally unstable (as characterized by the Toomre Q parameter) as well as possess the ability to radiate away the energy released by fragment formation (as characterized by the disk cooling time). To date, some of the best examples of planetary systems that we would expect the gravitational instability to produce are the planet Fomalhaut b, the outermost planet of HR 8799, and the potential protoplanet associated with HL Tau. Using a new technique to more accurately calculate cooling times, we can place upper and lower limits on the disk surface density—or, equivalently, the disk mass—required to form these observed planets. We find that the required mass interior to the planet's orbital radius is $\sim 0.1\ M_\odot$ for Fomalhaut b, the outermost planet of HR 8799, and the potential protoplanet associated with HL Tau. We also investigate how dust settling can reduce the cooling time, and the effects this has on the viability of the gravitational instability in the inner disk.

Two Modes of Planet Formation

1. Core + Gas Accretion
   - Efficient in inner disk, but slow in outer disk
   - Timescale $\propto$ (Distance)^3
   - $\sim$ 1 Myr to form Jupiter at 5 AU, but $\sim$ 1 Gyr at 50 AU!

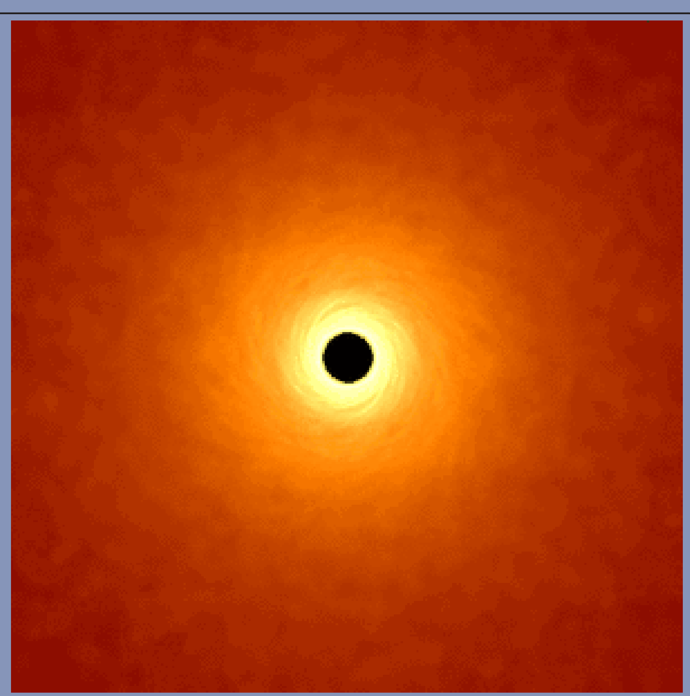
2. Direct Fragmentation
   - Very fast, but requires large surface density
   - Timescale $\propto$ Orbital Period
   - Works best in the outer disk
   - Are the required surface densities (disk masses) reasonable?

How Disks Fragment

Stable Disk

- Random motions + centrifugal forces keep disk stable
- Toomre $Q \gtrsim 1$

$$Q \equiv \frac{c_s \Omega}{\pi G \Sigma}$$



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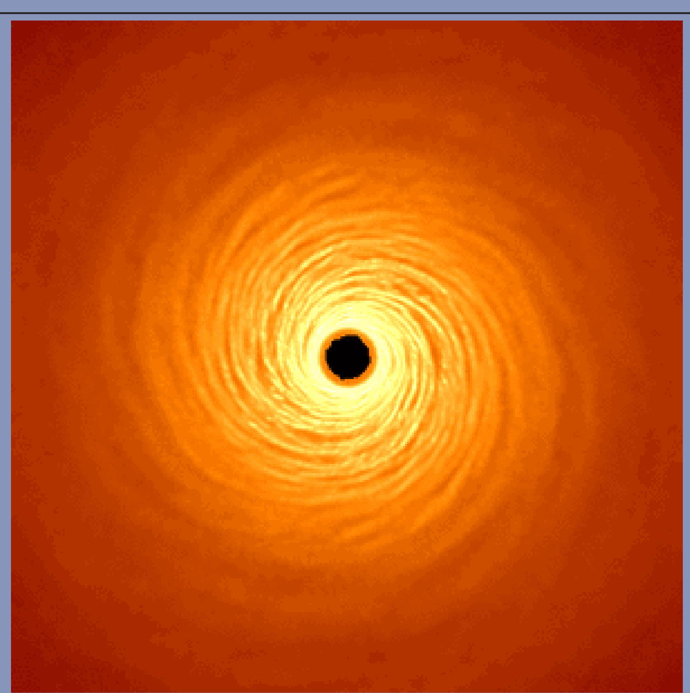
Spiral Arms Form

- Toomre $Q \lesssim 1$
- Spiral arms form
- Gas pressure supports spiral arms against further collapse
- Cooling time is long

$$t_{cool} \gtrsim 1/\Omega$$



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Spiral Arms Fragment

- Toomre $Q \lesssim 1$
- $t_{cool} \lesssim 1/\Omega$
- Pressure support is lost and spiral arms fragment
- Fragments may go on to form giant planetary embryos

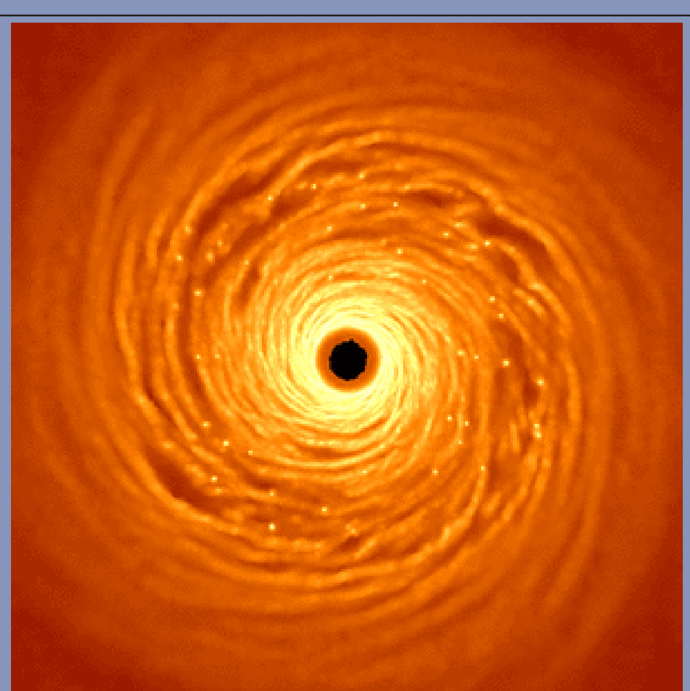
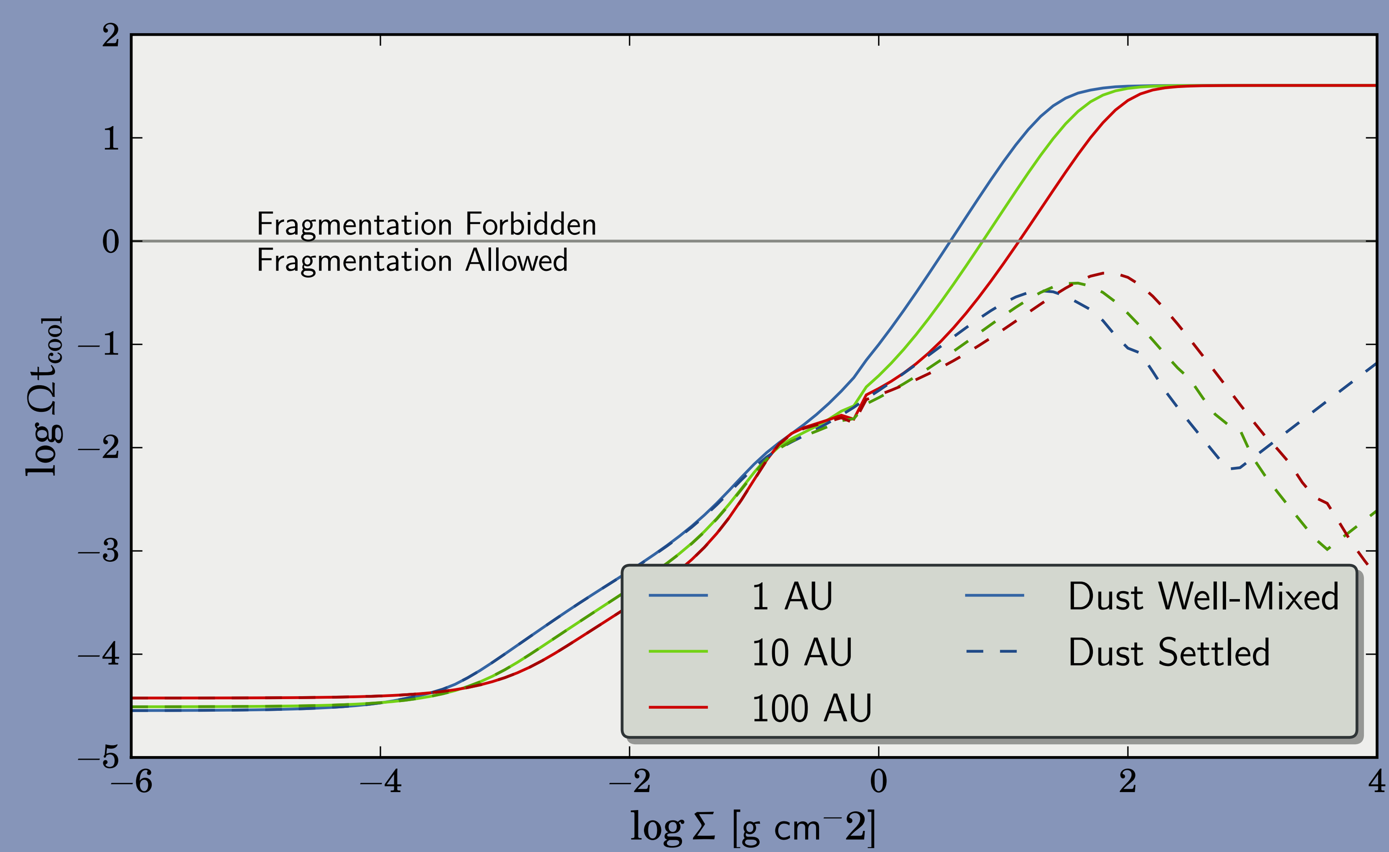


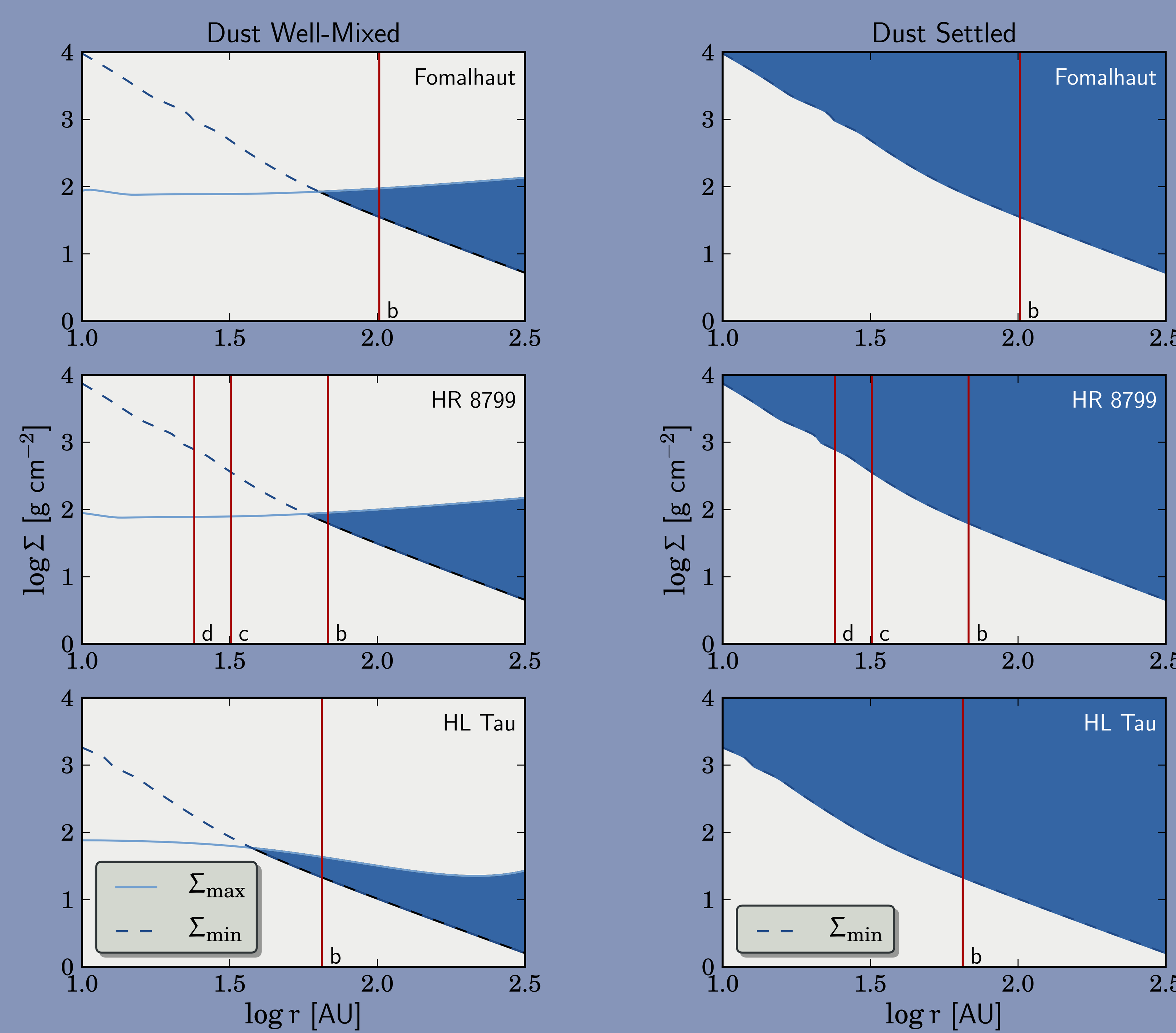
Image credit: `http://faculty.ucr.edu/~krice/`
Used for illustration purposes only.

Cooling Time of a Perturbation



A plot of cooling time at 1 (blue), 10 (green), and 100 AU (red) with respect to surface density for a typical T-Tauri star ($M_\star = 0.5\ M_\odot$, $R_\star = 2\ R_\odot$, $T_\star = 4000$ K). The solid lines show well-mixed gas and dust, while the dashed lines show the limiting case where dust has completely settled to the disk mid-plane. Note that for this star, the dust settled case imposes no upper limit on surface densities that can fragment. Thus, the only constraint on the viability of the gravitational instability is Toomre $Q \lesssim 1$.

Disk Fragmentation Limits



Surface density limits for disk fragmentation. The solid line denotes Σ_{max}, which is the maximum surface density for the cooling time constraint. The dashed line denotes the minimum surface density for fragmentation, Σ_{min}, which is the locus Toomre $Q = 1$. Disk fragmentation is only allowed in the region $\Sigma_{max} > \Sigma_{min}$, which is shaded blue. The locations of known planets are plotted as vertical lines.

Disk Masses

Object	$M_\star$ ($M_\odot$)	Σ_{min} (g cm^{-2})	M_{min} ($M_\odot$)
Fomalhaut b	2.0	35	0.17
HR 8799 b	1.5	62	0.14
HL Tau b	0.3	21	0.04

- Minimum mass estimate assumes $\Sigma \propto r^{-1/2}$. For $\Sigma \propto r^{-1}$, increase M_{min} by a factor of 1.5
- Σ_{min} and M_{min} are set by the locus Toomre $Q = 1$, and are thus independent of the effects of dust settling

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