We explore the viability of forming giant planets via the gravitational instability. This mechanism favors the production of giant planets at large radii, since a disk would need to be both gravitationally unstable (as characterized by the Toomre Q parameter) as well as possess the ability to radiate away the energy released by fragment formation (as characterized by the disk cooling time). To date, some of the best examples of planetary systems that we would expect the gravitational instability to produce are the planet Fomalhaut b, the outermost planet of HR 8799, and the potential protoplanet associated with HL Tau. Using a new technique to more accurately calculate cooling times, we can place upper and lower limits on the disk surface density—or, equivalently, the disk mass—required to form these observed planets. We find that the required mass interior to the planet’s orbital radius is \( \lesssim 0.1 \, M_\odot \). For Fomalhaut b, the outermost planet of HR 8799, and the potential protoplanet associated with HL Tau. We also investigate how dust settling can reduce the cooling time, and the effects this has on the viability of the gravitational instability in the inner disk.

### Two Modes of Planet Formation

1. Core + Gas Accretion
   - Efficient in inner disk, but slow in outer disk
   - Timescale \( \propto (\text{Distance})^3 \)
   - \( \sim 1 \, \text{Myr} \) to form Jupiter at 5 AU, but \( \sim 1 \, \text{Gyr} \) at 50 AU!

2. Direct Fragmentation
   - Very fast, but requires large surface density
   - Timescale \( \propto \) Orbital Period
   - Works best in the outer disk
   - Are the required surface densities (disk masses) reasonable?

### How Disks Fragment

**Stable Disk**
- Random motions + centrifugal forces keep disk stable
- Toomre Q \( \gtrsim 1 \)

\[ Q \equiv \frac{c_s \Omega}{\pi G \Sigma} \]

**Spiral Arms Form**
- Toomre Q \( \lesssim 1 \)
- Spiral arms form
- Gas pressure supports spiral arms against further collapse
- Cooling time is long

\[ t_{\text{cool}} \gtrsim 1/\Omega \]

**Spiral Arms Fragment**
- Toomre Q \( \lesssim 1 \)
- \( t_{\text{cool}} \lesssim 1/\Omega \)
- Pressure support is lost and spiral arms fragment
- Fragments may go on to form giant planetary embryos

### Cooling Time of a Perturbation

A plot of cooling time at 1 (blue), 10 (green), and 100 AU (red) with respect to surface density for a typical T-Tauri star \( (M_*, 0.5 \, M_\odot; R_*, 2 R_\odot; T_*, 4000 \, \text{K}) \). The solid lines show well-mixed gas and dust, while the dashed lines show the limiting case where dust has completely settled to the disk mid-plane. Note that for this star, the dust settled case imposes no upper limit on surface densities that can fragment. Thus, the only constraint on the viability of the gravitational instability is Toomre Q \( \lesssim 1 \).

### Disk Fragmentation Limits

**Dust Well-Mixed**
- \( \Sigma_{\text{max}} \) surface density
- \( \Sigma_{\text{min}} \) minimum surface density
- \( \Sigma_{\text{cool}} \) cooling time constraint

**Dust Settled**
- \( \Sigma_{\text{max}} \) surface density
- \( \Sigma_{\text{min}} \) minimum surface density
- \( \Sigma_{\text{cool}} \) cooling time constraint

**Fomalhaut**

\begin{align*}
\Sigma_{\text{max}} &= 20 \, \text{g cm}^{-2} \\
\Sigma_{\text{min}} &= 9 \, \text{g cm}^{-2} \\
\Sigma_{\text{cool}} &= 120 \, \text{g cm}^{-2}
\end{align*}

**HR 8799**

\begin{align*}
\Sigma_{\text{max}} &= 30 \, \text{g cm}^{-2} \\
\Sigma_{\text{min}} &= 15 \, \text{g cm}^{-2} \\
\Sigma_{\text{cool}} &= 100 \, \text{g cm}^{-2}
\end{align*}

**HL Tau**

\begin{align*}
\Sigma_{\text{max}} &= 21 \, \text{g cm}^{-2} \\
\Sigma_{\text{min}} &= 5 \, \text{g cm}^{-2} \\
\Sigma_{\text{cool}} &= 50 \, \text{g cm}^{-2}
\end{align*}

### Disk Masses

- Minimum mass estimate assumes \( \Sigma \propto r^{-1/2} \). For \( \Sigma \propto r^{-1} \), increase \( M_{\text{min}} \) by a factor of 1.5
- \( \Sigma_{\text{max}} \) and \( M_{\text{max}} \) are set by the locus Toomre Q = 1, and are thus independent of the effects of dust settling

<table>
<thead>
<tr>
<th>Object</th>
<th>( M_\text{min} ) (( M_\odot ))</th>
<th>( \Sigma_{\text{min}} ) (g cm(^{-2}))</th>
<th>( M_{\text{max}} ) (( M_\odot ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fomalhaut b</td>
<td>2.0</td>
<td>35</td>
<td>0.17</td>
</tr>
<tr>
<td>HR 8799 b</td>
<td>1.5</td>
<td>62</td>
<td>0.14</td>
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<tr>
<td>HL Tau b</td>
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<td>21</td>
<td>0.04</td>
</tr>
</tbody>
</table>

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