RBC Exercise

David N. DeJong
University of Pittsburgh

Spring 2008

The Model

$$\max_{c_t, l_t} \qquad U = E_0 \sum_{t=0}^{\infty} \beta^t \frac{\left(c_t^{\varphi} l_t^{1-\varphi}\right)^{1-\phi}}{1-\phi} \qquad ,$$

subject to

$$\begin{aligned}
y_t &= z_t k_t^{\alpha} n_t^{1-\alpha}, \\
1 &= n_t + l_t, \\
y_t &= c_t + i_t, \\
k_{t+1} &= i_t + (1-\delta) k_t, \\
z_t &= z_0 e^{\omega_t}, \quad \omega_t = \rho \omega_{t-1} + \varepsilon_t.
\end{aligned}$$

Note: exogenous growth rate $g = 0$, as in the text.

The Nonlinear System

$$
\begin{aligned}
\left(\frac{1-\varphi}{\varphi}\right)\frac{c_t}{l_t} &= (1-\alpha)z_t\left(\frac{k_t}{n_t}\right)^{\alpha} \\
c_t^{\kappa} l_t^{\lambda} &= \beta E_t\left\{c_{t+1}^{\kappa} l_{t+1}^{\lambda}\left[\alpha z_{t+1}\left(\frac{n_{t+1}}{k_{t+1}}\right)^{1-\alpha} + (1-\delta)\right]\right\} \\
y_t &= z_t k_t^{\alpha} n_t^{1-\alpha} \\
y_t &= c_t + i_t \\
k_{t+1} &= i_t + (1-\delta)k_t \\
1 &= n_t + l_t \\
\log z_t &= (1-\rho)\log(\bar{z}) + \rho \log z_{t-1} + \varepsilon_t,
\end{aligned}
$$

GAUSS Code for the System

```
z[1] = ln((1-ps)/ps) + c - l - ln(1-alp) - a - alp*klag + alp*n;
z[2] = cfac*clag + lfac*llag - ln(bet) - cfac*c - lfac*l
      - ln( alp*exp(a)*exp( (alp-1)*klag )*exp( (1-alp)*n )
      + (1-del) );
z[3] = y - a - alp*klag - (1 - alp)*n;
z[4] = y - ln( exp(c) + exp(i) );
z[5] = k - ln( exp(i) + (1-del)*exp(klag) );
z[6] = - ln ( exp(l) + exp(n) ) - ln(1)
z[7] = a - rh*alag;
```

Steady States

$$\bar{z} = 1, \quad \frac{\bar{y}}{\bar{n}} = \eta, \quad \frac{\bar{c}}{\bar{n}} = \eta - \delta\theta, \quad \frac{\bar{i}}{\bar{n}} = \delta\theta,$$

$$\bar{n} = \frac{1}{1 + \left(\frac{1}{1-\alpha}\right)\left(\frac{1-\varphi}{\varphi}\right)\left[1 - \delta\theta^{1-\alpha}\right]},$$

$$\bar{l} = 1 - \bar{n}, \quad \frac{\bar{k}}{\bar{n}} = \theta,$$

$$\begin{aligned} \theta &= \left(\frac{\alpha}{1/\beta - 1 + \delta}\right)^{\frac{1}{1-\alpha}} \\ \eta &= \theta^{\alpha}. \end{aligned}$$

GAUSS Code for Steady States

```
_1_alp = 1-alp;
ar = 1/bet - 1;
th = (alp/(ar+del))^(1/_1_alp);
et = th^alp;
nbar1 = 1 + (1/_1_alp)*((1-ps)/ps)*(1-del*(th^_1_alp));
nbar = 1/nbar1;
lbar = 1-nbar;
cbar = (et - del*th)*nbar;
kbar = th*nbar;
ybar = et*nbar;
ibar = ybar - cbar;
abar = 1;
```

Results

Parameters:
// alpha, beta, delta, rho, sigeps, phi, psi:
let p[7,1] = 0.24 0.99 0.025 0.78, 0.0067, 1.5, 0.35;

F:

0.0000	0.0000	0.0000	0.0000	0.0000	0.0820	1.3015
0.0000	0.0000	0.0000	0.0000	0.0000	0.3925	0.2766
0.0000	0.0000	0.0000	0.0000	0.0000	-1.4239	6.2728
0.0000	0.0000	0.0000	0.0000	0.0000	-0.2079	0.6862
0.0000	0.0000	0.0000	0.0000	0.0000	0.1026	-0.3387
0.0000	0.0000	0.0000	0.0000	0.0000	0.9394	0.1568
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.7800

G': 1.6686 0.3546 8.0420 0.8798 -0.4343 0.2010 1.0000

Results, cont.

σ_j	$\frac{\sigma_j}{\sigma_y}$	$\varphi(1)$	$\varphi_{j,y}(0)$	$\varphi_{j,y}(1)$	
0.0184	1.0000	0.7947	1.0000	0.7947	y
0.0085	0.4623	0.9601	0.7782	0.6244	c
0.0799	4.3402	0.7367	0.9459	0.7486	i
0.0087	0.4707	0.7290	0.9107	0.7198	n
0.0043	0.2323	0.7290	-0.9107	-0.7198	l
0.0160	0.8680	0.9923	0.5592	0.4523	k
0.0107	0.5812	0.7800	0.9979	0.7924	a

GAUSS Code for RBC Statistics


```
vcvx = inv(eye(nvars^2)-fmat.*.fmat)*vec(qmat);
vcvx = reshape(vcvx,nvars,nvars);
stdx = sqrt(diag(vcvx));
relstdx = stdx./stdx[1];            @ Note: model-specific!
normalizes by s.e. of y @
cov1x = fmat*vcvx;
cov2x = fmat*cov1x;
corrden = stdx*stdx';
corrx = vcvx./corrden;
auto1x = cov1x./corrden;
auto2x = cov2x./corrden;
resps = zeros(nvars,nresp);
```

GAUSS Code for RBC Statistics, cont.

```
resps[.,1] = gmat*p[5]; @ Note: model-specific! std dev of tech
shock is p[5] @
iii = 1; do while iii < nresp;
    iii = iii+1;
    resps[.,iii] = fmat*resps[.,iii-1];
endo;
//resps = resps./stdx;
retp(stdx~relstdx,corrx,auto1x,auto2x,resps);
```

Impulse Response Functions

From left to right: y, c, i, n, l, k, a