Directions: Construct a spreadsheet, in the form of the template provided below, by Tues March 21.

The purpose of this assignment is to assess some empirical predictions of the Solow model for per worker incomes (relative to the U.S.) across a group of 10 countries of your choosing. It will be a good idea to include the country, or collection of countries, you are evaluating as part of your empirical research project, as you may wish to include the calculations you make for this country as part of your project (e.g., as background information). The collection of countries you select should contain representatives from the continents of Africa, Asia, Europe, North America, and South America.

Throughout, we will use the following notation. Per worker income is total income $Y$ divided by the number of workers $L$: $y = Y/L$. In terms of the data in the Penn World Tables, $y$ is REAL GDP CHAIN PER WORKER. Relative to the U.S., per worker income for country $j$ is given by

$$\hat{y}_j = y_j / y_{US}$$

Likewise, relative savings is given by $\hat{s}_j$, and the relative depreciation factor is given by $\hat{x}_j$, where $x_j = n_j + \delta + g$, and throughout, we will assume $\delta + g = 0.075$.

We will also assume that for each country, capital’s share of labor $\alpha = 1/3$, so that $\alpha/(1-\alpha) = 1/2$.

We will use $\theta$ to refer to the ratio of savings to the depreciation factor for country $j$:

$$\theta_j = \frac{s_j}{x_j}$$

Therefore,

$$\hat{\theta}_j = \frac{\theta_j}{\theta_{US}} = \frac{\hat{s}_j}{\hat{x}_j}$$

Finally, we will define the human capital for country $j$ as $h_j = e^{0.1u_j}$, where $u_j$ is average educational attainment in years for country $j$. Therefore,

$$\hat{h}_j = e^{0.1(u_j - u(U.S))}$$

For all countries, data on $s$, $n$, and $u$ may be obtained from Table C.2 of the text (pp. 216-219).
For each of the ten countries in your sample, provide the following information:

<table>
<thead>
<tr>
<th>Country</th>
<th>Date</th>
<th>( y )</th>
<th>( y^* )</th>
<th>( s )</th>
<th>( s^* )</th>
<th>( x )</th>
<th>( x^* )</th>
<th>( \theta )</th>
<th>( \theta^* )</th>
<th>( \theta^{1/2} )</th>
<th>( \theta^{1/2^*} )</th>
<th>( u )</th>
<th>( h )</th>
<th>( \hat{h} )</th>
<th>( \theta^{1/2^*} \hat{h} )</th>
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</tbody>
</table>

Next, construct two scatter-plot diagrams. The first should feature \( \theta^{1/2} \) (on the vertical axis) plotted against \( y_{2000} \) (on the horizontal axis). This is an “actual versus predicted” plot of the version of the model that assumes technology and human capital is equal across all countries. The second should feature \( \theta^{1/2^*} \hat{h} \) (on the vertical axis) plotted against \( y_{2000} \) (on the horizontal axis). This is an “actual versus predicted” plot of the version of the model that assumes technology is equal across all countries.

Copy and paste your diagrams into a word document, and include in the document a discussion of the results of your two diagrams. In particular: do your diagrams reflect favorably on the model’s characterization of the data? Are some countries better characterized than others? Are there common features across the countries for which the models predictions are particularly good (and likewise, bad)? Etc.
Next, augment the spreadsheet created above by relaxing the assumption that the level of technology across countries is equal. Do so as follows.

For each country in the sample of ten countries you selected, and in addition for the U.S., begin by constructing \( \{i_t\}, t = 1960, 1961, \ldots, 2000 \), where \( i_t = I_t / L_t \) is investment per worker. In the terminology of the Penn-World Tables, this is the product of INVESTMENT SHARE OF CGDP and REAL GDP CHAIN PER WORKER \( (y) \), divided by 100.

Next, calculate \( k_{1960} = \theta y_{1960} \), where \( \theta \) was calculated as described above. Combine \( k_{1960} \) with \( \{i_t\} \) to obtain \( \{k_t\} \), using

\[
k_{t+1} = i_t + 0.94 * k_t
\]

Next, combine \( \{y_t\} \) and \( \{k_t\} \) to obtain \( \{A_t\} \), using

\[
A_t = \left( \frac{y_t}{k_t} \right)^{1/2} * y_t * e^{-0.1u_t}
\]

using as reported for each country in Table C.2 of the text (pp. 216-219).

At this point, you have constructed 11 time series of \( A_t \). Convert these into 10 time series of \( \hat{A}_{11t} = A_{11t} / A_{US,1} \). Use these series for two purposes.

First, for EACH series, calculate the average value of \( \hat{A} \) over the sample period, multiply by the value of \( \hat{\theta}^{1/2} h \) calculated in Part I, and interpret the result as providing the predicted value of \( \hat{y} \) given the relaxation of the assumption that the level of technology across countries is equal. Use these predictions to construct an “actual versus prediction” scatter-plot under this scenario (with actual \( \hat{y} \)’s on the horizontal axis and predicted values on the vertical axis). Compare your results to those obtained previously: has the empirical performance of the model improved?

Second, for TWO series, produce time-series plots of \( \hat{A} \). Are there any evident trends? Cyclical patterns? Etc.?

Copy and paste your diagrams into a word document, and include your discussions of the diagrams in that document.

FEEL FREE TO INCORPORATE YOU WORK ON THESE EXERCISES INTO YOUR EMPirical RESEARCH PROJECT!!