# Markups, Aggregation, and Inventory Adjustment

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*In this paper I suggest a unified explanation for two puzzles in the inventory literature: first, estimates of inventory speeds of adjustment in aggregate data are very small relative to the apparent rapid reaction of stocks to unanticipated variations in sales. Second, estimates of inventory speeds of adjustment in firm-level data are significantly higher than in aggregate data. The paper develops a multisector model where inventories are held to avoid stockouts, and price markups vary along the business cycle. The omission of countercyclical markup variations from inventory targets introduces a downward bias in estimates of adjustment speeds obtained from partial adjustment models. When the cyclicality of markups differs across sectors, this downward bias is shown to be more severe with aggregate rather than firm-level data. Similar results apply not only to inventories, but also to labor and prices. Montercarlo simulations of a calibrated version of the model suggest that these biases are quantitatively significant.* (*JEL* E22, E32)

Finished goods inventories to expected sales ratios are countercyclical and persistent over the business cycle (Valerie A. Ramey and Kenneth D. West, 1999; Mark Bils and James A. Kahn, 2000). While this behavior suggests that inventories adjust slowly to changes in sales, traditional partial adjustment models of inventories have not been able to provide a satisfactory explanation for these facts. In particular, beginning with Martin S. Feldstein and Alan Auerbach (1976), researchers have found it difficult to reconcile the small estimated adjustment speeds of inventory stocks toward target with their apparent rapid reaction to unanticipated variations in sales.[1] For example, according to Feldstein and Auerbach's estimates, in a typical quarter firms eliminate less than 6 percent of the gap between current and desired inventories, while being able to correct *within* the quarter more than 95 percent of current sales surprises.[2]

Some recent papers have added a second dimension to this inventory adjustment puzzle. The stock-adjustment model has been traditionally estimated with aggregate or two-digit manufacturing data. In a comprehensive study of U.S. manufacturing firms, Scott Schuh (1996) has shown how inventory speeds of adjustment estimated using firm-level data are significantly

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Thanks to Mark Bils, Rui Castro, Aubhik Khan, Jim Kahn, David Orbison, Marla Ripoll, Tony Smith, Scott Schuh, and Amir Yaron for helpful conversations and suggestions. Thanks also to the audiences at the 2002 Society for Economic Dynamics Meeting in New York City, the 2004 AEA Meetings in San Diego, Carnegie Mellon University, the University of Pittsburgh, and The Wharton School at the University of Pennsylvania for helpful comments. The usual disclaimer applies.

[1] The conceptual framework underlying most of these empirical exercises is represented by Michael C. Lovell's (1961) reduced-form stock-adjustment model and Charles C. Holt et al.'s (1960) linear-quadratic model where firms solve an explicit dynamic optimization problem. Under some conditions (see Ramey and West, 1999) these two models give rise to observationally equivalent equations for inventories. In the linear-quadratic model the adjustment speed coefficient decreases with the slope of marginal cost. In turn, higher slopes of marginal cost induce firms to smooth production more relative to sales. However, the empirical literature (see, e.g., West, 1986; Jeffrey A. Miron and Stephen P. Zeldes, 1988) has not found significant evidence of production smoothing behavior.

[2] Similar results are reported by, among others, Louis J. Maccini and Robert J. Rossana (1984), Alan S. Blinder (1986a), and John C. Haltiwanger and Maccini (1989). Using monthly finished-goods inventory data from the Department of Commerce for the period 1967:01–1997:12, I obtain estimated speeds of adjustment equal to approximately 2 and 3 percent, for, respectively, durable and nondurable goods industries. The estimates of the sales surprise parameter imply that durable and nondurable goods firms correct, respectively, 95 and 86 percent of a sales surprise within a month.

larger than their counterparts estimated using aggregate data. Specifically, he finds that, according to the empirical specification of the model, the weighted average of adjustment speeds estimated using firm-level data is 67 to 105 percent larger than the one obtained using aggregated data constructed from the same panel of firms. Despite this evidence, the effect of aggregation across heterogeneous firms is not well understood yet.[3]

I suggest a unified explanation for Feldstein and Auerbach's and Schuh's inventory puzzles. The explanation relies on the idea that firms' inventory targets might change systematically over the business cycle for reasons other than variations in expected sales, as usually assumed in the literature. Following the empirical evidence presented by Bils and Kahn (2000), the specific mechanism leading to variations in inventory targets is countercyclical movements in price markups. The paper shows, qualitatively and quantitatively, how the omission of markups from standard partial adjustment equations results in downward-biased estimates of adjustment speeds. The bias is more severe in regressions that use aggregate rather than firm-level data. Moreover, while inventories are an illustrative special case of these mechanisms, their applicability extends to other aspects of firms' behavior, such as labor demand and price setting, of interest to macroeconomists.

To make these points, I consider a finished goods inventory model in the spirit of Bils and Kahn (2000). The key feature distinguishing this model from the linear-quadratic model of Holt et al. (1960) is that in the former, higher inventories contribute to increase firms' sales and revenues at a given price. Thus, in this model higher price markups induce firms to, *ceteris paribus,* hold more inventories. The economy is composed of two sectors with different cyclical volatilities of markups. The equilibrium law of motion for the model's sectoral and aggregate inventories can be written in the standard partial adjustment form with the inventory target being a function of expected sales and price markups. The model is calibrated and its simulated firm-level and aggregate inventory and sales data are then used to estimate partial adjustment equations in which price markups are omitted from the inventory target. The Montecarlo exercise reveals that this omission results in large downward biases in estimated speeds of adjustment of aggregate inventories relative to their "true" value implied by the model. For example, when sectors displaying the most volatile markup movements account on average for only 10 percent of aggregate inventories, the "true" speed of adjustment of inventories is 12 percent higher than the weighted average of firm-level estimates, and 327 percent higher than the estimate obtained from aggregate data (i.e., Schuh's puzzle). The average value of the latter implies that, in one quarter, firms eliminate only 30 percent of the gap between current and desired inventories. In contrast, the estimates of the effects of sales surprises on inventory investment suggest that firms are able to correct completely unanticipated sales shocks within the quarter (i.e., Feldstein and Auerbach's puzzle).

The intuition behind these results is as follows. If price markups are countercyclical, their omission from inventory targets leads standard partial adjustment models to overpredict the gap between desired and current stocks during periods of economic expansion, and to underpredict it in recessions. Therefore, these models tend to attribute the discrepancy between the desired and observed changes in inventories to the fact that firms are adjusting slowly. In contrast, in this model countercyclical variations in price markups induce firms to reduce their target inventories relative to sales in an expansion and increase them in a recession. According to this view, the desired stock of inventories should

[3] Other authors have also argued that estimated speeds of adjustment tend to be higher at lower levels of aggregation. Using data on German manufacturing firms, Helmut Seitz (1993) estimates average adjustment speeds at the firm-level that are more than twice as large as the one obtained by running the regression with aggregated data from the same panel. Blinder (1986a) finds that adjustment speeds estimated with aggregate data for the durable and nondurable goods sectors tend to be lower than the ones estimated with data from their constituent industries. Lovell (1993) shows how aggregation may bias adjustment speeds' estimates downward by means of a simulation approach. He uses a reduced-form model, however, where firms don't optimize and parameters are not calibrated. Moreover, he does not provide an explanation for this result. John A. Carlson and William C. Dunkelberg (1989), using firm-level data on small businesses in the United States, find that firms on average fully adjust their inventories to target within a quarter.

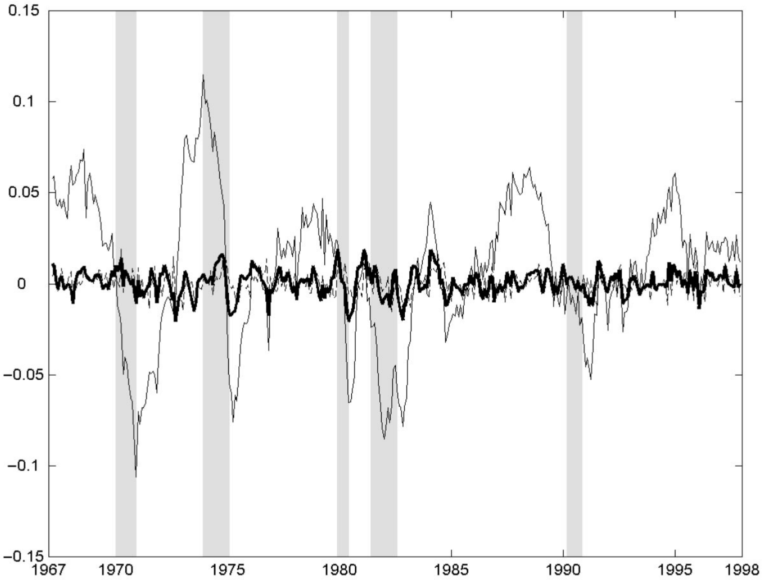


FIGURE 1. INVENTORY INVESTMENT AND INVENTORY GAPS

*Notes:* This figure represents three variables. The first one (– –) is inventory investment in finished goods in aggregate manufacturing, in the period 1967:1–1997:12. The second (—) is the gap between desired and actual inventories, where the former is estimated as function of a constant and expected sales only. The third one (**—**) is the gap between desired and actual inventories, where the former is obtained by interpolating the latter with the Hodrick-Prescott filter. The inventory and sales data used to construct the figure have been linearly detrended. Shaded areas denote recessions as defined by the NBER.

track the data more closely than what is usually predicted by estimated inventory targets.

These points can be illustrated with reference to Figure 1. The figure plots three variables: (i) investment in finished goods inventories in the U.S. manufacturing sector (dashed line);[4] (ii) the gap between the desired and current inventory stocks (solid line), where the former is estimated as a function of expected sales and a constant; and (iii) the gap between desired and current inventories (bold line), where the former is just a smoothed version of the latter; it is supposed to illustrate the effect of countercyclical variations in markups on desired inventory stocks.[5] In comparing the first and second variables, notice that while inventory investment is not closely correlated with the business cycle, the gap between desired and current inventories predicted by traditional partial adjustment regressions (second variable) increases systematically in expansions and decreases during recessions (the shaded areas in the figure represent recessions, as defined by the National Bureau of Economic Research [NBER]). Its associated speed of adjustment suggests that, in a quarter, firms eliminate less than 10 percent of the gap between desired and current inventories. The bold line in Figure 1 illustrates the effect of countercyclical variations in markups on desired inventories. As sales increase, markups fall, reducing firms' incentives to increase their desired inventories during expansions. There-

[4] The real finished goods inventory and sales data used to construct Figure 1 are from the Department of Commerce for the sample period 1967:1–1997:12. They are seasonally adjusted and expressed in chain-weighted 1996 dollars. Both inventories and sales have been linearly detrended.

[5] To construct this third series, I have Hodrick-Prescott filtered the ratio of inventories to expected sales to extract a smooth trend. The smoothing parameter is equal to 140 and is chosen so that the implied estimated speed of adjustment of inventories is equal to 0.47. This value corresponds to the "true" adjustment speed of aggregate inventories in the calibrated model of this paper. The resulting trend is then multiplied by expected sales in order to obtain the measure of desired inventory stocks.

fore, the gap between desired and current inventory stocks is always relatively small. The speed of adjustment associated with this measure of the inventory gap is such that it would take firms only one quarter to close about 85 percent of the gap between current and desired inventories.

The paper also shows that while aggregate inventory and sales data are just averages of firm-level data, the difference between the estimated and "true" speed of adjustment of aggregate inventories depends disproportionately on the properties of inventories in sectors with the most volatile markups. Therefore, using aggregate data tends to give rise to smaller estimates of adjustment speeds than what is obtained when firm-level data are used. The apparent fast adjustment of inventories to "unanticipated" sales shocks is explained by the fact that the latter are assumed to be such only to the econometrician, not to firms (Blinder, 1986b).

The implication of these results is that aggregate inventories are significantly more responsive to changes in target stocks than usually found in the literature. They are more responsive because countercyclical variations in price markups make target stocks less responsive to changes in expected sales. Therefore, the results of the paper are consistent with the basic feature of the data cited at the beginning: inventories do adjust slowly to variations in expected sales.

This is not the first paper arguing that traditionally specified inventory targets might be omitting important variables. In particular, several authors (see e.g., Maccini and Rossana, 1984; Miron and Zeldes, 1988; Martin S. Eichenbaum, 1989) have tested the production-smoothing model by allowing for both observable and unobservable cost shifters in the inventory target. The latter group of variables, including real wages, material prices, and interest rates, has usually been found to be insignificant for explaining inventory behavior (see Table 11 in Ramey and West, 1999, for a summary of the evidence). Unobservable and persistent cost shocks, instead, seem to improve significantly the model's fit. Notice, however, that in order to explain why finished-goods inventories to sales ratios *fall* systematically during economic expansions, one might need to argue that these unobservable costs are procyclical. This would then be inconsistent with the usual interpretation of these shocks as technology shocks (Eichenbaum, 1989).[6] Bils and Kahn (2000), instead, have recently provided empirical support in favor of the hypothesis that inventory targets are significantly affected by cyclical variations in price markups. Using data for six manufacturing industries, they estimate a representative-agent version of the model adopted here and show how allowing for countercyclical markups significantly improves the fit of the model. Differently from Bils and Kahn (2000), though, the issue of aggregation across firms plays a central role here in reconciling the relatively high estimates of inventory speeds of adjustment obtained using firm-level data with the relatively small estimates obtained using aggregate data.[7]

The remainder of the paper is organized as follows. Section I describes the model economy. Section II discusses, from a qualitative point of view, the effects of omitting cyclical markups from target stocks on the estimates of inventory speeds of adjustment. Section III contains the quantitative results of the paper. Section IV extends the analysis by considering first the adjustment speeds of labor and prices and then a version of the model where sectors differ in the slope of marginal cost rather than markups. Section V concludes.

## I. The Model Economy

The economy I examine extends Bils and Kahn's (2000) model of a representative firm to

[6] Aubhik Khan and Julia K. Thomas (2002) develop a general equilibrium model of inventories with lumpy adjustment at the firm level due to fixed costs. In their model, technology shocks generate fluctuations in output and inventory-sales ratios are countercyclical. They focus on input, however, rather than output inventories.

[7] A related literature, reviewed by Ricardo J. Caballero (1999), has instead explored the aggregate implications of infrequent and discrete microeconomic adjustment. In this literature, the slow adjustment of aggregates is due to heterogeneity in the timing of adjustment at the micro level. Since in this case firms either adjust or don't adjust, the real dichotomy is between lumpy microeconomic adjustment and smooth aggregate dynamics, rather than between fast micro adjustment and slow macro adjustment. Caballero and Edwardo Engel (2003) show how failure to account for microeconomic lumpiness might erroneously induce an econometrician to conclude that aggregates adjust more slowly than individual units.

an economy with heterogeneous sectors. Sectors are assumed to differ in terms of the cyclical properties of their markups.[8] The market structure is the simplest that captures the following two key elements: (i) firms have some degree of market power, so that it is meaningful to discuss the effects of changes in price markups; and (ii) firms are ex ante heterogeneous, which is necessary to analyze the effects of aggregation.

### A. *Setup*

To keep the model as simple as possible I work with a two-sector economy. Each sector, indexed by $k = 1, 2$, produces a continuum of varieties of a product. Each variety is produced by one, and only one, monopolistically competitive firm. Firms within a sector are otherwise homogeneous, as the key dimension of heterogeneity in the model is across sectors. There is a measure $\mu$ of firms in sector 1 and $1 - \mu$ in sector 2.

The objective of each firm is to maximize the present discounted value of its profits, expressed in units of an arbitrary numeraire good. Time is discrete and infinite and future profits are discounted at a rate $\beta < 1$, assumed to be constant relative to the numeraire.

*Sales and Inventories.*—The distinguishing feature of Bils and Kahn's (2000) inventory model is that, unlike the standard linear-quadratic model, inventories contribute to increased sales at a given price by reducing the likelihood of stock-outs.[9] This revenue role of inventories is important in order to analyze the effect of variations in price markups on a firm's decision to hold stocks. The building block of the model is represented by the relationship between a firm's finished goods inventories and its sales. Denote by $a^j_{kt}$ the sum of firm $j$'s beginning-of-the-period output inventories $i^j_{kt}$ and current production $y^j_{kt}$. Then, sales for firm $j$ in sector $k$ at time $t$ are given by

$$s^j_{kt} = \gamma_t \left( \frac{p^j_{kt}}{P_{kt}} \right)^{-\delta_{kt}} (a^j_{kt})^{\phi},\ 0 < \phi < 1,\ \delta_{kt} > 1. \tag{1}$$

The term $(a^j_{kt})^{\phi}$ in this equation captures the revenue-generating role of inventories. The parameter $\phi$ determines the extent to which a higher stock of goods contributes to generate higher sales at a given price. Period $t$ sales in all sectors are affected by an aggregate shock $\gamma_t$, observed at the beginning of the period. The latter evolves over time according to the process

$$\gamma_t = \gamma^{\rho}_{t-1} u_t,\ 0 < \rho < 1. \tag{2}$$

The random variable $u_t$ is identically and independently distributed over time according to some distribution with positive support. Without loss of generality, I normalize its unconditional mean to one.

In equation (1), a firm's sales in sector $k$ are also assumed to depend on this firm's price relative to a measure $P_{kt}$ of the price level in sector $k$. The only restriction imposed on $P_{kt}$ is that when all firms in sector $k$ charge the same price $p$, then $P_{kt} = p$.[10] The elasticity of demand faced by firms operating in sector $k$ is denoted by $\delta_{kt}$ and is allowed to change stochastically over time. Cyclical variations in the elasticity of demand give rise to cyclical variations in markups. As will become clear in the next section, a constant elasticity $\delta_{kt} = \delta$ in (1) implies that firms choose a constant markup of price over expected discounted marginal cost. To generate

[8] In Section IV B, I discuss the case where the slope of the marginal cost function differs across sectors.

[9] For a discussion of this point in relation to the linear-quadratic model instead, see Ramey and West (1999, page 885, footnote 11). Bils and Kahn's approach to modeling the role of inventories acknowledges that in reality firms might stock out even if their observed inventory stocks are not zero, because goods come in different colors, sizes, etc., and consumers have preferences about these characteristics. Therefore, having higher inventories decreases the chances of a mismatch between the available stock and the preferences of consumers. Kahn (1987, 1992) develops and tests a structural model of the stock-out avoidance motive for holding inventories.

[10] Notice that a firm's sales do not depend on the relative price of goods in the two sectors. The demand function (1) can be easily generalized to allow for a dependence of $s^j_{kt}$ on the ratio $P_{1t}/P_{2t}$. This generalization does not, however, significantly affect the quantitative properties of the model, so it is omitted for simplicity.

cyclical variations in markups in a simple way, I allow this elasticity to change over time with the aggregate state of the economy:[11]

(3)

$$\delta_{kt} = 1 + (\delta - 1)^{1-\eta}(\delta_{kt-1} - 1)^{\eta} u_t^{\pi_k}, \ \delta > 1.$$

When $\pi_k > 0$, positive sales shocks tend to make demand relatively more elastic and decrease firms' desired price markups during economic expansions.[12]

*Production.*—With the exception of Ramey (1991), researchers in the inventory literature have mostly postulated that marginal production cost is upward sloping. While estimates of this slope vary greatly across studies, some researchers (see, in particular, Jeffrey C. Fuhrer et al., 1995) have found strong evidence in support of this hypothesis.[13] Bils and Kahn (2000) have, instead, emphasized variations in measures of hourly wages as an important reason for the procyclicality of real marginal cost. According to their measured wages, though, the growth rate of marginal cost is too persistent at a monthly frequency to give rise to significant intertemporal substitution in production. Given these results and the partial equilibrium nature of this model, in this paper I allow for increasing marginal cost of production resulting from diminishing returns to labor, rather than cyclical variations in input prices. The latter are, therefore, held constant in terms of the numeraire good.[14] A firm's cost function takes the standard linear-quadratic form

$$c(y_{kt}^j) = \nu y_{kt}^j + \frac{\omega}{2}(y_{kt}^j)^2 \tag{4}$$

where $\omega$ denotes the slope of the marginal cost curve.

A firm $j$ that starts period $t$ with inventories $i_{kt}^j$, produces output $y_{kt}^j$, and sells $s_{kt}^j$ units of the good, begins period $t + 1$ with inventories equal to

$$i_{kt+1}^j = a_{kt}^j - s_{kt}^j \tag{5}$$

with $a_{kt}^j = i_{kt}^j + y_{kt}^j$. Since inventories cannot be negative it must be the case that $a_{kt}^j \geq s_{kt}^j$. To simplify the notation, in the rest of the paper I ignore this non-negativity constraint on inventories. In the simulations presented below this constraint never binds.

*Heterogeneity across Sectors.*—Markups in sectors one and two are assumed to be countercyclical ($\pi_1 > 0$ and $\pi_2 > 0$), but more so in sector one than in sector two ($\pi_1 > \pi_2$). The empirical evidence largely supports the assumption that the cyclical properties of markups vary across sectors.[15] Bils (1987) and Rotemberg and Woodford (1991) study two-digit-SIC manufacturing industries and report that price markups over marginal costs are countercyclical in almost all of these industries. Moreover, their results point to a wide dispersion across industries in the degree of cyclicality of markups (see especially Bils' Table 5 and Rotemberg and Woodford's Table 8). Evidence of differences across sectors in the cyclical properties of markups can also be found in more highly disaggre-

[11] As Julio J. Rotemberg and Michael Woodford (1999, page 1,119) observe, "the simplest and most familiar model of desired markup variations attributes them to changes in the elasticity of demand faced by the representative firm." Here I assume, for simplicity, that variations in the elasticity of demand are exogenous. Bils (1989) and Jordi Gali (1994) show how, when purchasers differ in their elasticity of demand, cyclical changes in the composition of demand can generate endogenous variations in its elasticity. Alternatively, it is possible to obtain countercyclical variations in markups through some degree of price stickiness combined with increasing marginal cost of production. This approach, which I pursued in a previous version of the paper, gives rise to qualitative and quantitative results that are similar to the ones presented here.

[12] The specification in equation (3) guarantees that $\delta_{kt} > 1$ at all times and that in the steady state, the elasticity $\delta_{kt}$ is the same across sectors: $\delta_{kt} = \delta$.

[13] The usual motivation for assuming decreasing returns to scale in production is the fixity of capital in the short run.

[14] I have experimented with versions of the model in which the cost function (4) is affected by a procyclical cost shock that can be interpreted as resulting from cyclical variations in real wages. As long as this shock is sufficiently persistent, though, the quantitative results of the paper are not affected.

[15] One reason why the cyclical properties of price markups vary across sectors is represented by differences in levels of market power. The latter can be introduced in the model by assuming that the steady state elasticity of demand differs across sectors. This generalization is ignored for simplicity.

gated data.[16] For example, Binder (1995) analyzes business cycles across four-digit-SIC manufacturing industries and concludes (page 27) that in light of his results, "findings of a uniform cyclical variation of markups in producer goods manufacturing industries may have to be reconsidered." Ian Domowitz et al. (1987) consider 57 four-digit-SIC manufacturing industries from 1958 to 1981 and report a wide dispersion in the yearly standard deviation of markups across industries.

### B. *Firms' Optimization and Equilibrium*

When making its pricing and production choices, firm $j$ in sector $k$ takes as given the stochastic process for the price index $\{P_{kt}\}$, the elasticity $\{\delta_{kt}\}$, and the demand shocks $\{\gamma_t\}$. Its optimization problem in sequence form is

$$\max_{\{p^j_{kt},a^j_{kt}\}} E_0\left[\sum_{t=0}^{\infty} \beta^t \left(p^j_{kt}s^j_{kt} - \nu(a^j_{kt} - a^j_{kt-1} + s^j_{kt-1}) - \frac{\omega}{2}(a^j_{kt} - a^j_{kt-1} + s^j_{kt-1})^2\right)\right]$$

$$\text{subject to: } s^j_{kt} = \gamma_t\left(\frac{p^j_{kt}}{P_{kt}}\right)^{-\delta_{kt}}(a^j_{kt})^{\phi},\ i_0 \geq 0,\ \gamma_0 > 0.$$

The first order conditions with respect to $a^j_{kt}$ and $p^j_{kt}$ are respectively

$$(6)\quad \nu + \omega(a^j_{kt} - a^j_{kt-1} + s^j_{kt-1}) = p^j_{kt}\phi\left(\frac{s^j_{kt}}{a^j_{kt}}\right) + \beta\left(1 - \phi\frac{s^j_{kt}}{a^j_{kt}}\right) \times E_t[\nu + \omega(a^j_{kt+1} - a^j_{kt} + s^j_{kt})]$$

$$(7)\quad (\delta_{kt} - 1)p^j_{kt} = \delta_{kt}\beta E_t[\nu + \omega(a^j_{kt+1} - a^j_{kt} + s^j_{kt})].$$

The first-order conditions (6) and (7) have a straightforward interpretation. Consider (6) first. The marginal cost of increasing $a^j_{kt}$ is represented by the left-hand side of (6). Its marginal benefit is represented by the right-hand side of this equation and is composed of two terms. First, an extra unit of $a^j_{kt}$ contributes to increase current sales at the price $p^j_{kt}$; second, to the extent that it does not contribute to current sales, it increases future inventories. An extra unit of the good held in inventory at the beginning of period $t + 1$ allows the firm to save marginal cost of production in that period.

Consider now the first-order condition with respect to $p^j_{kt}$. A marginally higher price causes a loss of current revenue, represented by the left-hand side of equation (7). Given $a^j_{kt}$, this reduction in current sales translates into a higher stock of inventories at the beginning of $t + 1$, which allows the firm to save on production costs in that period. This marginal benefit of a higher price is represented by the right-hand side of equation (7) and at the margin must compensate the firm exactly for the corresponding loss of current revenue.

Define firm $j$'s price markup as the ratio between its price and expected discounted marginal cost minus one:

$$(8)\quad M_{kt} \equiv \frac{p^j_{kt}}{\beta E_t[\nu + \omega(a^j_{kt+1} - a^j_{kt} + s^j_{kt})]} - 1 = \frac{1}{\delta_{kt} - 1}$$

where the second equality follows from equation (7).

In the following I focus on a symmetric equilibrium in which all firms in a given sector make the same production and pricing decisions. Thus, in equilibrium $p^j_{kt}$ equals $P_{kt}$ for all firms $j$ in sector $k$. I denote by $A_{kt}$, $I_{kt+1}$, $S_{kt}$ and $Y_{kt}$ the equilibrium values of $a^j_{kt}$, $i^j_{kt+1}$, $s^j_{kt}$ and $y^j_{kt}$.

[16] It might be necessary to consider this more disaggregated evidence because Schuh (1996) suggests that industry affiliation, as measured by two-digit-SIC codes, explains a relatively small fraction of the cross-sectional variance of his firm-level estimates of adjustment speeds.

The model does not admit a closed-form solution, except when marginal costs are constant over time ($\omega = 0$).[17] I obtain a linear approximation to the solution of the model by linearizing the first order conditions (6) and (7) around the non-stochastic steady state and then applying the method of undetermined coefficients (see, e.g., Lawrence Christiano, 2002).[18] The linearized decision rule for the level of the stock of goods available for sale takes the form

$$(9) \qquad A_{kt} = \xi_0 + \xi_1 A_{kt-1} + \xi_2 M_{kt} + \xi_3 \gamma_t + \xi_4 \gamma_{t-1}, \qquad k = 1, 2$$

where the $\xi$ coefficients are nonlinear functions of the structural parameters of the model.

Notice that the $\xi$ coefficients are the same for firms of the two sectors. This follows from certainty equivalence because in this model the only source of sectoral heterogeneity is represented by the forcing process $u_t^{\pi_k}$ in (3). This property of the linearized solution implies that it is possible to aggregate the firm-level decision rules (9) into a decision rule for the aggregate stock of goods available for sale, $A_t$:

$$A_t = \xi_0 + \xi_1 A_{t-1} + \xi_2 M_t + \xi_3 \gamma_t + \xi_4 \gamma_{t-1}$$

where $A_t$ and the average markup $M_t$ are defined as follows:

$$A_t = \mu A_{1t} + (1 - \mu) A_{2t}$$

$$M_t = \mu M_{1t} + (1 - \mu) M_{2t}.$$

This aggregation result will prove useful in providing intuition about the effects of aggregation on the estimated speeds of adjustment of aggregate inventories.

## II. Mechanisms

In this section I discuss the qualitative implications of cyclical variations in target inventories for the adjustment speeds estimated using standard partial adjustment models. In the first subsection I show how the linearized decision rule for inventories implied by the model can be written in a standard partial adjustment form, with the markup variable $M_{kt}$ included in the target equation for inventories. The second and third subsections derive the implications of omitting this markup variable for the estimates of inventory speeds of adjustment. In particular, the second subsection considers an individual sector in isolation, while the third one analyzes the effect of aggregation across sectors with different cyclical properties of markups. The fourth subsection discusses Feldstein and Auerbach's inventory adjustment puzzle.

### A. *A Correctly Specified Partial Adjustment Equation*

In this section I show how the equilibrium law of motion for inventories in a given sector and for the economy as a whole can be rewritten in the traditional partial adjustment form. To do so, consider the linearized version of equation (1):

$$(10) \qquad S_{kt} = S\gamma_t + \phi \frac{S}{A} (A_{kt} - A)$$

and use it to replace the unobservable shocks $\gamma_t$ and $\gamma_{t-1}$ in equation (9). Then, use the identity $A_{kt} = I_{kt+1} + S_{kt}$ to obtain a version of equation (9) that depends on the inventory

[17] In this case it is easy to show that the solution where inventories are always strictly positive is

$$A_{kt} = \left[ \frac{\gamma_t \phi \beta}{(\delta_{kt} - 1)(1 - \beta)} \right]^{1/(1-\phi)} \qquad \text{and}$$

$$P_{kt} = \frac{\beta c}{1 - \delta_{kt}^{-1}}$$

provided that demand is sufficiently inelastic, i.e., $\delta_{kt} \leq 1 + (1 - \beta)^{-1} \phi \beta$.

[18] Linearization provides an accurate approximation to the decision rules of the model despite the fact that the marginal benefit of increasing inventories is not linear, as in the standard linear-quadratic model, but rather convex in the inventory stock. This is because the calibration of the model is such that the steady state value of $A_{kt}$ is quite far from zero and the magnitude of the shocks to the economy is relatively small. Appendix A contains expressions for the coefficients of the linearized decision rules as function of the structural parameters of the model.

stock $I_{kt+1}$, rather than the stock available for sale $A_{kt}$:

$$(11) \quad I_{kt+1} = \zeta_0 + \zeta_1 I_{kt} + \zeta_2 M_{kt} + \zeta_3 S_{kt}$$

where the $\zeta$ coefficients depend on the $\xi$ ones and are reported in Appendix B.[19] Now, denote by $S^e_{kt}$ the expectation of period $t$ sales conditional on the information available in the previous periods. Then, add and subtract $\zeta_3 S^e_{kt}$ from equation (11) and rewrite the latter in the following partial adjustment form:[20]

$$(12) \quad I_{kt+1} - I_{kt} = \theta(I^*_{kt+1} - I_{kt}) + \varphi(S_{kt} - S^e_{kt})$$

where the inventory target is defined as:

$$(13) \quad I^*_{kt+1} \equiv \alpha + \lambda S^e_{kt} + \iota M_{kt}.$$

The parameters in (12) and (13) are a function of the structural parameters of the model and are defined as follows:

$$\theta \equiv 1 - \zeta_1,\ \varphi \equiv \zeta_3,\ \alpha \equiv (1 - \zeta_1)^{-1}\zeta_0$$

$$\lambda \equiv (1 - \zeta_1)^{-1}\zeta_3,\ \iota \equiv (1 - \zeta_1)^{-1}\zeta_2.$$

The parameter $\theta$ in equation (12) is usually referred to as the "speed of adjustment" of inventory stocks toward their target $I^*_{kt+1}$, because it denotes the fraction of the gap between current and desired inventories that firms fill in a period. In optimizing models, such as the linear-quadratic model and the model of this paper, $\theta$ depends on the curvature of the marginal cost function, with $\theta = 1$ corresponding to the case of constant returns to scale.[21] The term representing the discrepancy between actual and expected sales in (12) captures the effect of "unanticipated" changes in sales on end-of-the-period inventory stocks.

Equation (12) together with the target (13) represent the reduced-form representation of the law of motion for the inventory stock in sector $k$ implied by the (linearized) version of Bils and Kahn's (2000) model adopted here. Moreover, given that the $\zeta$ coefficients are the same across sectors, these two equations also hold when sectoral inventory stocks, sales, and markups are replaced by their aggregate counterparts, defined as:[22]

$$I_t = \mu I_{1t} + (1 - \mu) I_{2t}$$

$$S_t = \mu S_{1t} + (1 - \mu) S_{2t}$$

$$S^e_t = \mu S^e_{1t} + (1 - \mu) S^e_{2t}.$$

Thus, the parameter $\theta$ also denotes the "true" adjustment speed of *aggregate* inventories.

It is interesting to notice that the partial adjustment equation (12) has the same form as the standard stock-adjustment equation estimated in the empirical literature (see, for example, Ramey and West, 1999). However, the latter usually specifies the inventory target $I^*_{kt+1}$ only as a function of expected sales and possibly some measure of wages and interest rates. Bils and Kahn's model implies that the inventory target should also depend on the markup variable $M_{kt}$. Failure to incorporate a time-varying price markup in the inventory target might result in a downward bias in the estimate of the adjustment speed parameter $\theta$. This point is developed in the following section in regard to one sector in isolation.

### B. *Partial Adjustment Equations and Cyclical Markups*

Consider first sector $k$ in isolation. Suppose that an econometrician would try to estimate equation (12), but did not include the markup variable $M_{kt}$ in the target equation (13). For-

[19] Notice that $I_{kt+1}$ does not depend on $S_{kt-1}$.

[20] For the purpose of the Montecarlo experiment, I will specify the expectation variable $S^e_{kt}$ as an affine function of lagged sales $S_{kt-1}$ (see Section III C).

[21] As shown in Appendix B, the parameter $\zeta_1$ depends linearly on $\xi_1$ and $\xi_4$ and is equal to zero when $\omega = 0$.

[22] Aggregate sales and inventories are constructed as weighted sums of the corresponding firm-level data using sale prices in the model's steady state as weights. Since firms in all sectors charge the same price in the steady state, this implies that aggregate inventories and sales can be obtained as simple sums of firm-level variables.

mally, he would estimate the following equation

$$\mathbf{Z}_k = \mathbf{X}_k \boldsymbol{\chi} + \boldsymbol{\varepsilon}_k \tag{15}$$

where the $t$-th row of $\mathbf{X}_k$ is the vector $\mathbf{X}_{kt} = [1, -I_{kt}, S^e_{kt}, S_{kt} - S^e_{kt}]$; the $t$-th element of $\mathbf{Z}_k$ is $Z_{kt} = I_{kt+1} - I_{kt}$, and $\boldsymbol{\varepsilon}_k$ is an error term. The vector $\boldsymbol{\chi} = [\alpha\theta, \theta, \lambda\theta, \varphi]$ denotes the parameters to be estimated.[23] The ordinary least squares estimator of $\boldsymbol{\chi}$ obtained using sector $k$'s data is denoted by $\hat{\boldsymbol{\chi}}_k$ and is given by

$$\hat{\boldsymbol{\chi}}_k = (\mathbf{X}_k'\mathbf{X}_k)^{-1}\mathbf{X}_k'\mathbf{Z}_k. \tag{16}$$

If price markups change over the business cycle, $\mathbf{Z}_k$ does not evolve according to (15), but rather according to equation (12). The latter can be rewritten as

$$\mathbf{Z}_k = \mathbf{X}_k \boldsymbol{\chi} + \mathbf{M}_k \zeta_2 \tag{17}$$

where $\mathbf{M}_k$ denotes the vector whose $t$-th element is $M_{kt}$. Replacing (17) into (16) yields:

$$\hat{\boldsymbol{\chi}}_k = \boldsymbol{\chi} + \hat{\mathbf{q}}_k \zeta_2 \tag{18}$$

where $\hat{\mathbf{q}}_k$ represents the $4 \times 1$ vector of estimated coefficients in a regression of $M_k$ on $X_k$:

$$\hat{\mathbf{q}}_k \equiv (\mathbf{X}_k'\mathbf{X}_k)^{-1}\mathbf{X}_k'\mathbf{M}_k.$$

Denote by $\hat{q}_{k2}$ the second element of $\hat{\mathbf{q}}_k$, i.e., the estimator of the coefficient on $-I_{kt}$. Then, the estimator of $\theta$ obtained using sector $k$'s data is

$$\hat{\theta}_k = \theta + \hat{q}_{k2}\zeta_2. \tag{19}$$

The parameter $\zeta_2$ is a positive number as higher markups, *ceteris paribus,* result in higher desired stocks of inventories. Thus, $\hat{\theta}_k$ is a downward-biased estimator of $\theta$ if $E[\hat{q}_{k2}] < 0$. In turn, $\hat{q}_{k2}$ has the same sign as the partial correlation coefficient between $\mathbf{M}_k$ and $-\mathbf{I}_k$, after controlling for sales and expected sales in sector $k$. Since the model predicts that larger markups, for given sales, are associated with higher inventory stocks, the sign of $E[\hat{q}_{k2}]$ is likely to be negative. Thus, omitting the markup variable $\mathbf{M}_k$ from standard partial adjustment regressions will induce a downward bias in the estimates of inventory speeds of adjustment.

### C. *Cyclical Markups and Aggregation*

In this section, I consider the estimate of $\theta$ obtained using aggregate data. Let $\mathbf{X}$ and $\mathbf{Z}$ denote the aggregate counterparts of $\mathbf{X}_k$ and $\mathbf{Z}_k$. Also, $\mathbf{M}$ represents the vector whose $t$-th element is the average markup $M_t$.[24] Then, the ordinary least squares estimator of $\boldsymbol{\chi}$ obtained using aggregate data is

$$\hat{\boldsymbol{\chi}} = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{Z}. \tag{20}$$

Following the same steps as in the previous section to replace $Z$ with the weighted average of inventory investment in the two sectors, it is easy to show that

$$\hat{\theta} = \theta + \hat{q}_2\zeta_2 \tag{21}$$

where $\hat{q}_2$ denotes the estimator of the coefficient on $-I_t$ in a regression of $M_t$ on aggregate sales $S_t$ and expected sales $S^e_t$. Equations (21) and (19) together imply that the expected difference between the weighted average of the estimators of $\theta$ obtained using firm-level data and the one obtained with aggregate data is given by

$$\begin{aligned} &E[\mu\hat{\theta}_1 + (1-\mu)\hat{\theta}_2 - \hat{\theta}] \\ &\quad = \zeta_2 E[\mu\hat{q}_{12} + (1-\mu)\hat{q}_{22} - \hat{q}_2]. \end{aligned} \tag{22}$$

To obtain some intuition about this expression, consider the extreme case where markups in sector two do not depend on the state of the economy, and the volatility of markups in sector one is sufficiently large ($\pi_2 = 0$ and $\pi_1$ "large"). In this circumstance, $\pi_2 = 0$ implies that $\hat{q}_{22} = 0$. Moreover, a sufficiently large $\pi_1$ implies that the dynamics of aggregate invento-

[23] Of course, from $\boldsymbol{\chi}$ it is possible to identify separately $\theta$, $\alpha$, $\lambda$ and $\varphi$.

[24] Since markups in sector two are constant by assumption, $\mathbf{M}_2$ is just a constant vector.

ries, conditional on aggregate sales, is dominated by variations in sector one inventories, so that $\hat{q}_2 \approx \hat{q}_{12}$.[25] It follows that (22) simplifies to

$$(23) \qquad E[\mu\hat{\theta}_1 + (1-\mu)\hat{\theta}_2 - \hat{\theta}] \approx -\zeta_2(1-\mu)E[\hat{q}_{12}].$$

The term on the right-hand side of this equation is positive because $\zeta_2$ is positive and, as argued in the previous section, $E[\hat{q}_{12}]$ tends to be negative.

To analyze the general case where markups in sector two are allowed to vary over time and also to provide a quantitative estimate of the expression in (22), it is necessary to calibrate and simulate the model. This task is undertaken in Section III.

### D. *Feldstein and Auerbach's Puzzle*

In order to address Feldstein and Auerbach's original inventory "puzzle," it is not sufficient to show that the estimated speed of adjustment of aggregate inventories is "small." It needs to be "small" relative to the apparent ease with which firms adjust their inventories in response to sales surprises. Formally, this means that the estimates of the sales surprise coefficient $\varphi$ must also be either negative and relatively small or positive. This would suggest that firms react to unexpectedly high sales by increasing within period production and either drawing down or even increasing their inventory stocks.

In the Montecarlo experiments that follow, the key to generating a relatively quick adjustment of inventories to sales surprises is the fact that the sales shock $u_t$ is assumed to be observed by the firm prior to making its period $t$ production decisions. Thus, any discrepancy between current and expected sales is corrected within the same period since the variable $S_t - S_t^e$ represents a surprise only to the econometrician, not to the firm. This argument was first advanced by Blinder (1986b), who also provided some empirical evidence to support it.

The "true" coefficient on the sales surprise variable in the correctly specified partial adjustment equation is just $\zeta_3$, i.e., the coefficient on current sales in the law of motion (11) for inventories. In turn, the sign and magnitude of $\zeta_3$ is determined by two opposite forces. First, increasing marginal costs of production and countercyclical variations in markups tend to make inventories countercyclical, so that higher sales would lead to lower inventory stocks. In Bils and Kahn's model, however, inventories contribute to increase a firm's revenue at a given price by reducing stockouts. This second effect tends to make inventories procyclical relative to sales. In aggregate data inventory investment is moderately procyclical, which is consistent with a positive, though relatively small, value of $\zeta_3$.

It follows that, if the estimated value of $\zeta_3$ is not significantly biased by the omission of the markup variable $M_t$ from the regression, an econometrician would estimate small positive (or possibly negative) values for this parameter. Interpreting the estimated parameter as the effect of sales surprises on end-of-the-period inventories would then lead to the erroneous conclusion that firms respond quickly to unexpected sales, while possibly adjusting slowly to bridge the gap between current and target inventories.

## III. Quantitative Results

The following two subsections, respectively, describe the calibration of the model of Section I and verify that it can account for the aggregate moments of sales, production, and inventories that have been emphasized in the inventory literature (see, for example, Blinder and Maccini, 1991). The third subsection reports results on the Montecarlo experiment where artificial data on inventories and sales are first generated from the calibrated version of the model and then used to estimate the speed of adjustment parameter $\theta$ and the "sales-surprise" parameter $\varphi$.

### A. *Calibration*

Calibration of the model requires choosing values for the following parameters: $\beta$, $\sigma$, $\rho$, $\phi$, $\delta$, $\eta$, $\omega$, $\nu$, $\mu$, $\pi_1$, $\pi_2$, as well as a choice for the distribution of the forcing variable $u_t$. Table

[25] To see this, notice that $\hat{q}_{12}$ represents the coefficient on $-I_{1t}$ in a regression of $M_{1t}$ on $-I_{1t}$, $S_{1t}$, and $S_{1t}^e$. Moreover, up to a constant term, $M_t = M_{1t}$ and, since there is only one aggregate demand shock in the model, the correlation between $S_t$ and $S_{1t}$ is very close to one. If $\pi_1$ is large and $\pi_2 = 0$, after controlling for variations in $S_t$, $I_{1t}$ and $I_t$ tend to move together. This is equivalent to $\hat{q}_{12} \approx \hat{q}_2$.

TABLE 1—BENCHMARK CALIBRATION

| Parameter | $\beta$ | $\sigma$ | $\rho$ | $\phi$ | $\delta$ | $\eta$ | $\omega$ | $\nu$ | $\mu$ | $\pi_1$ | $\pi_2$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Value | 0.98 | 0.015 | 0.95 | 0.51 | 16.38 | 0.97 | 0.05 | 0.92 | 0.10 | 0.76 | 0.063 |

*Note:* This table reports the parameters used in the benchmark calibration of the model with heterogeneous markups and homogeneous cost.

1 summarizes the benchmark values of the model's parameters.

The model is calibrated at a monthly frequency. The discount factor $\beta$ is set equal to 0.98, implying a real (in terms of the numeraire good) interest rate of about 2 percent per month. This includes storage and goods' depreciation costs for the firm. The distribution of the random variable $u_t$ is taken to be lognormal with mean one and standard deviation $\sigma$. The autocorrelation parameter $\rho$ and the standard deviation $\sigma$ are set by estimating the following autoregressive process for the logarithm of linearly detrended sales in the manufacturing sector:

$$\ln S_{t+1} = \rho \ln S_t + v_{t+1}, \qquad \text{std}(v_{t+1}) = \sigma.$$

Estimating this equation for the period 1967:01–1997:12 with U.S. manufacturing data yields point estimates of $\rho = 0.94$ for nondurable sectors and $\rho = 0.96$ for durables. I, thus, set $\rho = 0.95$. The estimate of the monthly standard deviation of the shock is approximately 0.02 for durables and 0.01 for nondurables. Here, I choose a value $\sigma = 0.015$.

In order to calibrate the parameters $\delta$ and $\phi$, it is convenient to use a steady-state version of the first order conditions (6) and (7). Equation (8) implies that the steady-state markup of price over discounted marginal cost in the two sectors is

$$M = \frac{1}{\delta - 1}.$$

I set the latter equal to 0.065, so that the corresponding price elasticity of demand is $\delta = 16.38$. This choice for the average markup is consistent with the available empirical estimates of price markups (see, e.g., Morrison, 1992; Norrbin, 1993).[26]

To calibrate the elasticity of sales with respect to goods available for sale, use the steady-state expression for the ratio $A/S$, and solve it in terms of the parameter $\phi$:

$$\phi = \frac{A}{S} \frac{(\delta - 1)(1 - \beta)}{\beta}.$$

The value for $\delta$ derived above, together with a ratio $A/S = 1.5$, implies that $\phi = 0.51$. The choice of $A/S$ yields a steady-state inventory-sales ratio of 0.5. This is consistent with the data from the manufacturing sector where the average ratio of nominal inventories to nominal sales is equal to 0.46 for durables and to 0.57 for nondurables in the period 1967:01–1997:12.[27]

The parameter $\omega$ that determines the slope of marginal cost is set equal to 0.05. The scale parameter $\nu$ is chosen to normalize steady-state marginal costs $\nu + \omega S$ to one in both sectors. Given this normalization, $\omega$ can be directly compared with the estimates of the slope of marginal cost obtained by Bils and Kahn (2000, Table 6). In only two out of the six sectors they study, the estimated slope of marginal cost is above 0.10, and in two of the remaining four the slope is slightly negative, indicating increasing returns to scale.

The elasticity $\delta_{kt}$ is assumed to comove with aggregate demand over the business cycle. The parameters $\pi_1$ and $\pi_2$ determine the extent to which innovations to aggregate sales affect $\delta_{kt}$ and, consequently, the markup variable $M_{kt}$. The parameter $\eta$, instead, determines the

[26] Estimates of markup ratios in the U.S. manufacturing industry tend to be higher when value-added, rather than gross-output, data are used (see, e.g., Hall, 1988; Norrbin, 1993). The benchmark value of $M$ selected in this paper reflects estimates of markups based on gross output data. The quantitative results of Sections III A, III B, and IV, however, depend on the different cyclical properties of price markups across sectors, rather than on their steady-state values. They are therefore robust to significant variations in $M$.

[27] The nominal data used to compute this ratio are published by the U.S. Census Bureau.

first-order autocorrelation of $\delta_{kt}$ and $M_{kt}$.[28] A value of $\eta = \rho$ corresponds to the case where $S_{kt}$ and $M_{kt}$ are almost linearly related. In this specific circumstance, the omission of $M_{kt}$ from the partial adjustment equation (12) would not lead to any bias in the estimate of the adjustment speed parameter $\theta$. Even small differences between $\eta$ and $\rho$ imply, however, that the omission of $M_{kt}$ from (12) can lead to relatively large downward biases in the estimates of $\theta$. Here, I choose a value of $\eta$ equal to 0.97.[29]

The parameter $\mu$ determines the relative size of the two sectors and, thus, determines the extent by which aggregate data reflect relatively more the behavior of sector-one or sector-two firms. For completeness, in sections III C and IV I report the Montecarlo results for different values of this parameter in (0,1). In order to impose more discipline on this quantitative experiment, though, it is convenient to set a benchmark value for $\mu$. Specifically, to do so, I choose the parameters $\mu$, $\pi_1$ and $\pi_2$, jointly, in order to replicate some of the estimates of firm-level speeds of adjustment obtained by Schuh (1996). In particular, I set $\mu = 0.10$, $\pi_1 = 0.76$, and $\pi_2 = 0.063$. The values for $\pi_k$ imply that the average speeds of adjustment for sector one and sector two firms are respectively 0.13 and 0.45. These figures are consistent with the results reported by Schuh (1996, Table 3). He finds that for 10 percent of the divisions in his sample, the estimated inventory speeds of adjustment were equal to or less than 0.13, while for the next 80 percent of firms they were between 0.13 and 1.03, with a median value of 0.40. The choice of $\pi_1$ gives rise to cyclical markup variations that are empirically plausible: when sector-one sales are 1 percent above their steady state, the markup of price over marginal cost for sector-one firms is about 0.1 percent below its steady-state value of 1.065. For comparison, Rotemberg and Woodford (1999) present estimates of this elasticity as high as 0.4.

Before undertaking the Montecarlo experiment, it is useful to verify that the calibrated version of the model is consistent with the observed business cycle dynamics of aggregate inventories, sales, and output. The next section analyzes the cyclical implications of the model for these variables.

### B. *Business Cycle Implications of the Model*

In order to better understand the working of the model, it is useful to consider a graph depicting the aggregate variables produced by the model economy. Figure 2 presents aggregate sales, production, and the inventory stock in percentage deviation from their steady state values over 360 months. Aggregate production and sales move quite closely together and are basically indistinguishable in the figure. The stock of inventories moves procyclically, but it is smooth enough for the inventory-sales ratio to move countercyclically.

Tables 2 and 3 present some key statistics about these artificially generated aggregate variables and compare them to the corresponding ones for the durables and nondurables industries of the U.S. manufacturing sector. Table 2 shows that the benchmark version of the model is consistent with the main features of aggregate inventories, production, and sales data. In particular, the model correctly predicts that the variance of production is roughly the same as the variance of sales and that inventory investment is procyclical. It also correctly predicts the volatility of the stock-sales ratios $I_t/S_t$ and $A_t/S_t$ and their countercyclical nature, due to the positive slope of marginal cost and countercyclical variations in price markups. Table 3 displays their autocorrelation structure in the data and in the model at a one-, three- and six-month lag. Table 3 captures the persistency of these ratios, even if it tends to predict a slightly faster convergence toward their long-run values than what is observed in the data. Taken together, these two tables suggest that the model considered here, calibrated with reasonable degrees of cyclical variations in markups and marginal cost, does a good job in accounting for the main stylized facts about finished goods inventories, production, and sales.

[28] Notice that equations (3) and (8) jointly imply that the markup variable $M_{kt}$ follows the law of motion:

$$M_{kt} = M^{1-\eta} M_{kt-1}^{\eta} u_t^{-\pi_k}.$$

[29] The qualitative results of sections II B and II C on the estimated adjustment speeds of inventories are robust to different choices of the parameter $\eta$, both above and below $\rho$. The quantitative results tend to vary, but the difference between estimates of firm-level and aggregate speeds of adjustment is always close to the one reported in section III C.

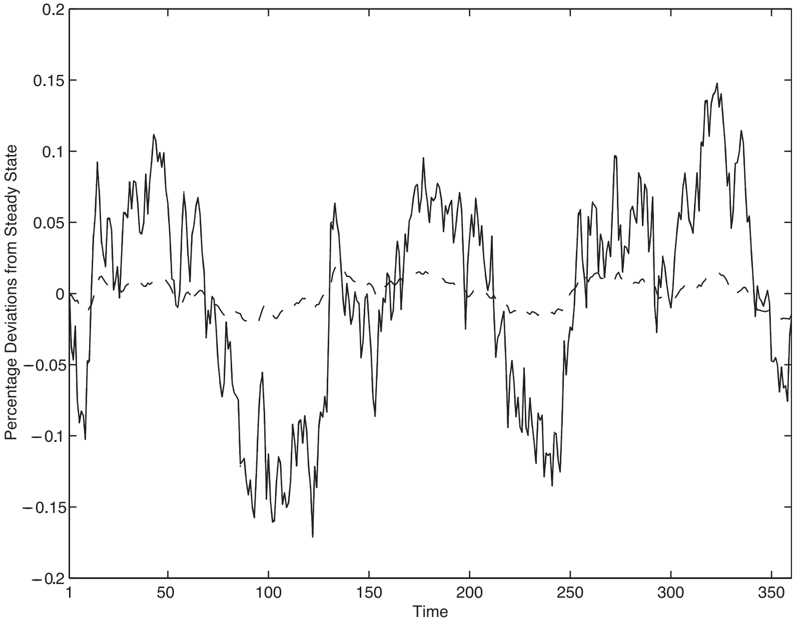


FIGURE 2. AGGREGATE SALES, OUTPUT, AND INVENTORIES OVER TIME

*Notes:* This figure represents time series for aggregate sales (—), aggregate output (– · ), and the aggregate inventory stock (– –), expressed as percentage deviations from their steady state. Notice that the output and sales series overlap almost perfectly. Data have been generated by simulating the benchmark version of the model for 360 periods.

TABLE 2—AGGREGATE STATISTICS

| | $\frac{\text{var}(Y)}{\text{var}(S)}$ | $c(\Delta I, S)$ | $c(A/S, Y)$ | $c(I/S, Y)$ | cv($A/S$) | cv($I/S$) |
|---|---|---|---|---|---|---|
| Benchmark model | 1.01 | 0.19* | −0.99* | −0.99* | 0.02 | 0.06 |
| | (0.00) | (0.06) | (0.00) | (0.00) | (0.00) | (0.01) |
| U.S. manufacturing | | | | | | |
| Durables | 1.02 | 0.21* | −0.86* | −0.87* | 0.04 | 0.09 |
| Nondurables | 1.00 | 0.06 | −0.71* | −0.69* | 0.02 | 0.05 |

*Notes:* $Y$: output, $S$: sales, $I$: beginning-of-the-period inventory stock, $\Delta I$: inventory investment, $A = I + Y$. cv: coefficient of variation, $c$: correlation, var: variance. The statistics for the U.S. economy have been computed using chain-weighted data on finished-goods inventories and sales from the Department of Commerce for the period 1967:01–1997:12. The $Y$ and $S$ series have been linearly detrended. Model statistics have been obtained by simulating the benchmark economy for 372 periods for 1,000 times and then computing averages. The standard deviation across simulations is reported in parenthesis. An asterisk indicates that the correlation is significant at the 1-percent level.

### C. *Inventory Adjustment Speeds: Results from Simulated Data*

Given the calibration of the model of Section III A, the "true" speed of adjustment of both firm-level and aggregate inventories implied by the model's parameters is $\theta = 0.47$. This section presents the estimates of $\theta$ obtained using artificial firm-level and aggregate data generated by the calibrated version of the model. The estimated equation has the partial adjustment form (12). Instead of including a markup variable as in (13), however, these regressions specify the inventory target in the

TABLE 3—AUTOCORRELATION PROPERTIES OF INVENTORY-SALES RATIOS AT DIFFERENT LAGS

| | $c(A/S, (A/S)_{-k})$ | | | $c(I/S, (I/S)_{-k})$ | | |
|---|---|---|---|---|---|---|
| | $k = 1$ | $k = 3$ | $k = 6$ | $k = 1$ | $k = 3$ | $k = 6$ |
| Benchmark model | 0.93* | 0.82* | 0.69* | 0.92* | 0.80* | 0.67* |
| | (0.02) | (0.06) | (0.10) | (0.02) | (0.06) | (0.10) |
| U.S. manufacturing | | | | | | |
| Durables | 0.96* | 0.90* | 0.76* | 0.97* | 0.90* | 0.77* |
| Nondurables | 0.96* | 0.85* | 0.67* | 0.96* | 0.86* | 0.69* |

*Notes:* $S$: sales, $I$: beginning-of-the-period inventory stock, $A = I + Y$, where $Y$ is output. $c$: correlation. The statistics for the U.S. economy have been computed using chain-weighted data on finished-goods inventories and sales from the Department of Commerce for the period 1967:01–1997:12. Model statistics have been obtained by simulating the benchmark economy for 372 periods for 1,000 times and then computing averages. The standard deviation across simulations is reported in parentheses. An asterisk indicates that the correlation is significant at the 1-percent level.

traditional form adopted in the empirical literature:

$$I^{*}_{kt+1} \equiv \alpha + \lambda S^{e}_{kt}. \tag{24}$$

Both the regressions that use firm-level data and the ones that use aggregate data, therefore, suffer from an omitted variable problem. The purpose of the Montercarlo exercise is to assess the quantitative significance of the difference between the firm-level estimates of $\theta$ and the aggregate estimate.

To estimate these partial adjustment equations, it is necessary to specify the sales expectation variable $S^{e}_{kt}$. The latter is taken to be an affine function of lagged sales:[30]

$$S^{e}_{kt} = S(1 - \rho) + \rho S_{kt-1}. \tag{25}$$

The model is simulated 1,000 times, and for each set of data the firm-level and aggregate partial adjustment equations with the inventory target as in (24) are estimated by ordinary least squares. The length of the data series in each simulation is 100 periods, which corresponds approximately to the one in Schuh (1996). The results are reported in Table 4 for different values of the parameter $\mu$, determining the relative size of the two sectors.

The second column of Table 4 shows the average (across simulations) estimate of $\theta$ obtained using aggregate data. The third and fourth columns show the average speed of adjustment estimated for sector one and sector two firms. The fifth column represents the weighted (by $\mu$) sum of firm-level estimates of adjustment speeds. Finally, the last column shows the amount by which the aggregate adjustment speed computed using firm-level data exceeds the one computed using aggregate data.

Table 4 shows several interesting results. First, it confirms the qualitative predictions developed in Section II: countercyclical changes in markups tend to bias the inventory adjustment speeds estimated from partial adjustment equations downward. The bias gets larger as markups become more cyclical, as can be inferred from comparing columns (3) and (4). Moreover, countercyclical changes in markups result in an econometric aggregation bias, in the sense that the adjustment speeds estimated from aggregate data are significantly smaller than the weighted average of firm-level speeds of adjustment. In interpreting these results, it is useful to keep in mind that if the markup variables $M_{kt}$ were included in the partial adjustment regressions, the estimated speeds of adjustment, at both the firm and aggregate levels, would always be equal to 0.47.

Second, the quantitative effects of time-varying markups on inventory adjustment speeds are quite large. When sector one firms account on

[30] Equation (25) is obtained by manipulating equation (10) and noticing that when the calibrated $\phi$ is relatively small, $S_{kt}$ is approximately given by

$$S_{kt} \approx S(1 - \rho) + \rho S_{kt-1} + S(u_t - 1).$$

The parameters of this equation are, for simplicity, specified a priori, rather than estimated, but the results that follow do not change when sales expectations are estimated instead.

TABLE 4—ESTIMATES OF AGGREGATE AND FIRM-LEVEL ADJUSTMENT SPEEDS FOR INVENTORIES (BENCHMARK MODEL)

| | True $\theta$ | Aggregate data $E[\hat{\theta}]$ | Firm-level data $E[\hat{\theta}_1]$ | $E[\hat{\theta}_2]$ | $E[\mu\hat{\theta}_1 + (1-\mu)\hat{\theta}_2]$ | Bias $E[\mu\hat{\theta}_1 + (1-\mu)\hat{\theta}_2 - \hat{\theta}]$ |
|---|---|---|---|---|---|---|
| $\mu = 0.10$ | 0.47 | 0.11 | 0.13 | 0.45 | 0.42 | 0.31 |
| | | (0.11) | (0.04) | (0.05) | (0.05) | (0.09) |
| $\mu = 0.30$ | 0.47 | 0.07 | 0.13 | 0.45 | 0.36 | 0.29 |
| | | (0.02) | (0.04) | (0.05) | (0.04) | (0.03) |
| $\mu = 0.60$ | 0.47 | 0.13 | 0.13 | 0.45 | 0.26 | 0.13 |
| | | (0.04) | (0.04) | (0.05) | (0.04) | (0.01) |
| $\mu = 0.90$ | 0.47 | 0.13 | 0.13 | 0.45 | 0.16 | 0.03 |
| | | (0.04) | (0.04) | (0.05) | (0.04) | (0.00) |

*Notes:* $\theta$: "true" speed of adjustment of inventory stocks implied by the benchmark model. The estimates of $\theta$ have been obtained by simulating the benchmark model and estimating a partial adjustment equation for inventories for 1,000 times. The sample size of each regression is 100. $E[\hat{\theta}]$: average estimate of $\theta$ obtained using aggregate data. $E[\hat{\theta}_k]$: average estimate of $\theta$ obtained using data on a sector $k$'s firm, $k = 1, 2$. The standard deviation of the estimates across simulations is reported in parentheses.

average for only 10 percent of aggregate inventories and sales, the weighted average of estimated firm-level speeds of adjustment is almost four times higher than the aggregate estimate. This aggregation bias is large for a wide range of values of $\mu$ and declines as the share of sector-one firms in the economy increases. The decline is due to two forces. On the one hand, mechanically, a higher $\mu$ gives rise to a lower weighted average of firm-level speeds of adjustment (column [5] in Table 4). On the other hand, a higher $\mu$ has ambiguous effects on $\hat{q}_2$ in equation (21).[31] The numerical results suggest that if $\mu$ is sufficiently large, $E[\hat{\theta}]$ increases with $\mu$.

In the spirit of the stock-adjustment model it is instructive to compute the number of months $T$ that are required to close 95 percent of the gap between current and target inventories, according to the "true" $\theta$ and the average estimate of $\theta$ obtained with aggregate data.[32] The "true" speed of adjustment of aggregate inventories, $\theta = 0.47$, suggests that, approximately, $T = 5$ months. Using the average speed of adjustment estimated with aggregate data generated by the benchmark version of the model (0.11) yields, instead, $T = 25$. Thus, failure to control for changes in markups would induce one to conclude erroneously that the aggregate economy takes more than 2 years, rather than only 5 months, to bridge 95 percent of the gap between target and desired inventories. Notice, however, that as observed in the introduction, this result does not imply that aggregate inventory stocks are more responsive to variations in expected sales than previously found. They are simply more responsive to a target that tends to move countercyclically relative to sales due to variations in markups.

Last, notice that the results of Table 4 are broadly consistent with the ones reported by Schuh (1996). In particular, the benchmark calibration of the model (i.e., $\mu = 0.10$) implies that the weighted average of firm-level adjustment speeds is 0.42, while Schuh (1996, Table 5) reports a value of 0.45 for a balanced panel of divisions in the M3 Longitudinal Research Database. He also estimates an adjustment speed of 0.27 based on aggregate data constructed from that same balanced panel. The latter figure is somewhat higher than 0.11, the average estimate of $\theta$ based on aggregate data obtained here.

Table 5 presents the estimates of the sales surprise parameter $\varphi$, whose "true" value is 0.10. The estimate of $\varphi$ obtained using aggregate data from the benchmark version of the model is on average equal to 0.04. As $\mu$ increases,

[31] Notice that the partial regression coefficient $\hat{q}_2$ tends to increase with the covariance between $M_t$ and $-I_t$, and to decrease with the variance of $I_t$, after netting out from these two variables the effects of $S_t$ and $S_t^e$. A higher $\mu$ makes the covariance and variance larger, with ambiguous effects on $\hat{q}_2$.

[32] If $x$ is the relevant adjustment speed, then

$$T = \frac{\ln 0.05}{\ln(1-x)}.$$

TABLE 5—ESTIMATES OF THE SALES SURPRISE PARAMETER (BENCHMARK MODEL)

| | True $\varphi$ | Aggregate data $E[\hat{\varphi}]$ | Firm-level data $E[\hat{\varphi}_1]$ | $E[\hat{\varphi}_2]$ |
|---|---|---|---|---|
| $\mu = 0.10$ | 0.10 | 0.04 | −0.32 | 0.07 |
| | | (0.00) | (0.01) | (0.00) |
| $\mu = 0.30$ | 0.10 | −0.03 | −0.32 | 0.07 |
| | | (0.00) | (0.01) | (0.00) |
| $\mu = 0.60$ | 0.10 | −0.15 | −0.32 | 0.07 |
| | | (0.01) | (0.01) | (0.00) |
| $\mu = 0.90$ | 0.10 | −0.27 | −0.32 | 0.07 |
| | | (0.01) | (0.01) | (0.00) |

*Notes:* $\varphi$: "true" sales-surprise parameter implied by the benchmark model in the partial adjustment equation for inventories. The estimates of $\varphi$ have been obtained by simulating the benchmark model and estimating a partial adjustment equation for inventories for 1,000 times. The sample size of each regression is 100. $E[\hat{\varphi}]$: average estimate of $\varphi$ obtained using aggregate data. $E[\hat{\varphi}_k]$: average estimate of $\varphi$ obtained using data on a sector $k$'s firm, $k$ = 1, 2. The standard deviation of the estimates across simulations is reported in parentheses.

the average values of $\hat{\varphi}$ become negative, but they remain relatively small in absolute value. This result could lead to the incorrect conclusion that in the aggregate firms respond extremely fast to sales surprises by increasing production and even, in the benchmark economy, end-of-the-period inventory stocks. This interpretation would then lead to the puzzle originally identified by Feldstein and Auerbach (1976), because the estimates of $\theta$ in Table 4 suggest that it takes a few months for firms to correct the imbalance between current and target inventories.

## IV. Extensions

In this section I consider two extensions of the analysis.[33] First, I ask whether the mechanism emphasized in Section II tends to affect the estimated speeds of adjustment of other variables of the model in the same way as it biases the estimates for inventories. Second, I consider an extension of the model where the slope of marginal cost is different across sectors and ask whether this kind of heterogeneity can also give rise to the aggregation bias results of Section III C.

### A. *Speed of Adjustment of Labor and Prices*

This model focuses on the adjustment speed of finished goods inventories. It is interesting to ask, though, whether cyclical changes in markups will also generate an econometric aggregation bias of the kind described in this paper in the estimated adjustment speeds of other variables of interest to macroeconomists. Extending the analysis to variables other than inventories also represents a further consistency check for the mechanism emphasized in this paper. In fact, for many macroeconomic variables, such as aggregate employment and prices, speeds of adjustment estimated using aggregate data tend to be relatively small (see, e.g., Robert H. Topel, 1982; Oliver J. Blanchard, 1987). It also appears that considering more disaggregated units, such as firm and sectors, leads to higher estimates of adjustment speeds for these variables than what is obtained with aggregate data (see Caballero and Engel, 2003; Blanchard, 1987).[34]

The following two sections extend the analysis of Sections II and III to consider the adjustment speeds of labor demand and prices. In what follows, I show that the omission of markups from partial adjustment regressions for prices and the labor input might lead to downward-biased estimates of adjustment speeds, particularly when using aggregate data.

*Labor Demand.*—In this simple model, the demand for labor as a function of output is, in linearized form, given by

$$L_{kt+1} = L + \frac{L}{S}(1 + \omega S)(A_{kt+1} - I_{kt+1} - Y) \tag{26}$$

where $L$ denotes the steady state value of labor,

[33] I thank an anonymous referee for suggesting these extensions.

[34] Caballero and Engel (2003) show how failure to account for $(S, s)$-type of adjustment policies used by firms when estimating partial adjustment models at the firm level results in an upward bias in the estimates of adjustment speeds for employment and prices. In their model, aggregation across firms tends to mitigate this bias.

TABLE 6—ESTIMATES OF AGGREGATE AND FIRM-LEVEL ADJUSTMENT SPEEDS FOR LABOR (BENCHMARK MODEL)

| | True $\theta$ | Aggregate data $E[\hat{\theta}]$ | Firm-level data $E[\hat{\theta}_1]$ | $E[\hat{\theta}_1]$ | $E[\mu\hat{\theta}_1 + (1-\mu)\hat{\theta}_2]$ | Bias $E[\mu\hat{\theta}_1 + (1-\mu)\hat{\theta}_2 - \hat{\theta}]$ |
|---|---|---|---|---|---|---|
| $\mu = 0.10$ | 0.47 | 0.46 | 0.35 | 0.46 | 0.45 | −0.01 |
| | | (0.00) | (0.01) | (0.00) | (0.00) | (0.00) |
| $\mu = 0.30$ | 0.47 | 0.38 | 0.35 | 0.46 | 0.43 | 0.04 |
| | | (0.02) | (0.01) | (0.00) | (0.00) | (0.02) |
| $\mu = 0.60$ | 0.47 | 0.39 | 0.35 | 0.46 | 0.40 | 0.01 |
| | | (0.01) | (0.01) | (0.00) | (0.00) | (0.00) |
| $\mu = 0.90$ | 0.47 | 0.36 | 0.35 | 0.46 | 0.36 | 0.00 |
| | | (0.01) | (0.01) | (0.00) | (0.00) | (0.00) |

*Notes:* $\theta$: "true" speed of adjustment of labor implied by the model. The estimates of $\theta$ have been obtained by simulating the benchmark model and estimating a partial adjustment equation for labor for 1,000 times. The sample size of each regression is 100. $E[\hat{\theta}]$: average estimate of $\theta$ obtained using aggregate data. $E[\hat{\theta}_k]$: average estimate of $\theta$ obtained using data on a sector $k$'s firm, $k = 1, 2$. The standard deviation of the estimates across simulations is reported in parentheses.

and $A_{kt+1} - I_{kt+1}$ is simply output at $t + 1$.[35] Using equations (9) and (11) to replace $A_{kt+1}$ and $I_{kt+1}$ into equation (26), it is possible to rewrite (26) in partial adjustment form:

$$(27) \qquad L_{kt+1} - L_{kt} = \theta(L^*_{kt+1} - L_{kt})$$

where the labor target $L^*_{kt+1}$ is defined as[36]

$$(28) \qquad L^*_{kt+1} \equiv \alpha^l + \lambda^l S_{kt+1} + \chi^l S_{kt} + \iota^l(M_{kt+1} - M_{kt}).$$

The parameters in equation (27) are functions of the structural parameters of the model and are reported in Appendix B. It is easy to show that the "true" speed of adjustment of labor toward its target is equal to $\theta$, i.e., the "true" speed of adjustment of inventories toward their target. This is not a coincidence. Firms adjust their inventory stocks by changing production and, in this model, labor is approximately a linear function of production. Therefore, the speed of adjustment of labor and output coincides with the speed of adjustment of inventories. The target equation for labor depends on sales and markups in two consecutive periods.[37]

The omission of markups from the target for labor (28) generates similar biases to the ones discussed in Section II in relation to inventories. In Table 6, I report the estimates of $\theta$ generated by a version of equation (27) that ignores variations in price markups.[38] As the table shows, countercyclical changes in markups tend to generate relatively small estimates of adjustment speeds for labor demand in sector one (column [3]). Moreover, for higher values of $\mu$, the adjustment speeds estimated from aggregate data (column [2]) are generally lower than the weighted average of their firm-level counterparts (column [5]). As a reference for comparison, if markups were constant in both sectors, the estimated speeds of adjustment in all columns of Table

[35] This expression for labor demand is obtained by treating the cost function (4) as a linear-quadratic approximation of the cost function implied by a production function of the type $Y = L^{\varrho}$, for some $\varrho < 1$.

[36] Notice that, for simplicity, I have not distinguished between expected and unexpected variations in sales and markups.

[37] Notice that while the beginning-of-the-period inventory stock $I_{kt+1}$ does not appear explicitly in equation (28), its effect on $L^*_{kt+1}$ is captured, in part, by $S_{kt}$. It is possible to show, in fact, that the parameter $\chi^l$ is negative: higher period $t$ sales lead to higher end-of-the-period inventory stocks $I_{kt+1}$, which generates a lower target for labor in period $t + 1$. This dependence has been directly explored by Topel (1982), John C. Haltiwanger and Maccini (1989), and Ramey (1989), among others, with mixed evidence. Significant effects have been found by Haltiwanger and Maccini (1989, page 342), who estimate that higher initial inventories lead to lower hours per worker and more layoffs, especially in the nondurables sector.

[38] The description of the different columns of this table matches exactly the one for Table 4, so it is omitted.

6 would be equal to 0.47, and the aggregation bias would disappear.

From a quantitative point of view, the results concerning the aggregation bias for labor are not as large as those for inventories. Therefore, inventories seem to adjust more slowly than labor to a target that depends exclusively on sales. Empirically, the relationship between the relative speeds of adjustment of finished goods inventories and labor over the business cycle has been explored by several authors.[39] The general motivation of these papers is the observation that firms may adjust to a cyclical decline in demand for their product by choosing some combination of lower labor and higher inventories. It seems that this literature has not reached a consensus on this issue, with some authors (see, e.g., Haltiwanger and Maccini, 1989) arguing that the labor input adjusts faster than inventory stocks to demand shocks, and others (e.g., Eichenbaum, 1984) arriving at the opposite conclusion. The goal of this section is not to provide new evidence on this controversy, as both the estimated partial adjustment regressions for inventories and labor are affected by the omission of markups. Instead, this section points out how the omission of markups from partial adjustment equations for labor might also lead to a downward bias in estimated speeds of adjustment of this input.

*Prices.*—In this model, cyclical variations in the elasticity of demand induce firms to reduce their price relative to expected future marginal cost in an expansion and increase it in a recession. Failure to account for these cyclical changes in the elasticity of demand when estimating partial adjustment equations for prices tends to give rise to the same type of results already emphasized in the previous section with regard to labor.

The linearized first-order condition for prices is given by

$$(29)\quad P_{kt+1} = \beta(1 - (1 + M)\omega S) + \beta M_{kt} + \omega\beta(1 + M)E_t[Y_{kt+1}].$$

The second term on the right-hand side of this equation captures the effect of changes in the elasticity of demand on prices, while the third term reflects the effect of variations in marginal costs of production. It is easy, but lengthy, to show that equation (29) can be rewritten in partial adjustment form as:

$$P_{kt+1} - P_{kt} = \theta(P^*_{kt+1} - P_{kt})$$

where the price target is defined as[40]

$$P^*_{kt+1} \equiv \alpha^p + \lambda^p S_{kt+1} + \chi^p S_{kt} + \iota^p M_{kt+1} + \tau^p M_{kt}.$$

As in the previous section, the "true" speed of adjustment of prices toward a target that depends on sales and markups is equal to the "true" speed of adjustment $\theta$ of inventories and labor. This is not surprising, as both marginal cost of production—on which prices are based—and labor depend linearly on output.

For the same reason illustrated above for the labor input, if countercyclical markups are omitted from the partial adjustment regression, the estimated speed of adjustment of $P_{kt+1}$ toward target will tend to be smaller than $\theta$. Table 7 reports the average estimates for sectors one and two, as well as the speed of adjustment of the aggregate price index $P_t$, where $P_t$ is defined as $P_t \equiv P_{1t}^{\mu} P_{2t}^{1-\mu}$. In this case, the quantitative results are quite large. In particular, if $\mu$ is high enough, the speed of adjustment of the aggregate price index tends to be smaller than

[39] See the references in footnote 37. In the data, for both durable and nondurable goods sectors, aggregate hours tend to lead the aggregate inventory stock over the business cycle. In particular, for nondurable (durable) goods, the maximum correlation between the Index of Aggregate Weekly Hours (published by the Bureau of Labor Statistics) in month $t$ and the inventory stock at the beginning of month $t + k$ is equal to 0.37 (0.47) and occurs at $k = 6$ ($k = 12$). In the benchmark version of the model considered here the correlation between $L_t$ and $I_{t+k}$ for $k = 6$ is 0.49, which is close to the data. In the model, however, the maximum correlation between $L_t$ and $I_{t+k}$ occurs at $k = 1$ and is equal to 0.79.

[40] The parameters of this equation are a function of the structural parameters of the model. The exact expressions are available from the author upon request.

TABLE 7—ESTIMATES OF AGGREGATE AND FIRM-LEVEL ADJUSTMENT SPEEDS FOR PRICES (BENCHMARK MODEL)

| | True $\theta$ | Aggregate data $E[\hat{\theta}]$ | Firm-level data $E[\hat{\theta}_1]$ | $E[\hat{\theta}_2]$ | $E[\mu\hat{\theta}_1 + (1-\mu)\hat{\theta}_2]$ | Bias $E[\mu\hat{\theta}_1 + (1-\mu)\hat{\theta}_2 - \hat{\theta}]$ |
|---|---|---|---|---|---|---|
| $\mu = 0.10$ | 0.47 | 0.20 | 0.01 | 0.38 | 0.34 | 0.14 |
| | | (0.05) | (0.02) | (0.03) | (0.03) | (0.03) |
| $\mu = 0.30$ | 0.47 | 0.01 | 0.01 | 0.38 | 0.26 | 0.25 |
| | | (0.01) | (0.02) | (0.03) | (0.03) | (0.03) |
| $\mu = 0.60$ | 0.47 | 0.03 | 0.01 | 0.38 | 0.15 | 0.12 |
| | | (0.05) | (0.02) | (0.03) | (0.02) | (0.03) |
| $\mu = 0.90$ | 0.47 | 0.01 | 0.01 | 0.38 | 0.04 | 0.03 |
| | | (0.03) | (0.02) | (0.03) | (0.02) | (0.01) |

*Notes:* $\theta$: "true" speed of adjustment of prices implied by the model. The estimates of $\theta$ have been obtained by simulating the benchmark model and estimating a partial adjustment equation for prices for 1,000 times. The sample size of each regression is 100. $E[\hat{\theta}]$: average estimate of $\theta$ obtained using aggregate data. $E[\hat{\theta}_k]$: average estimate of $\theta$ obtained using data on a sector $k$'s firm, $k = 1, 2$. The standard deviation of the estimates across simulations is reported in parentheses.

the weighted average of the firm-level adjustment speeds.

This result is consistent with the evidence presented by Blanchard (1987). He first estimated separate price adjustment equations for seven two-digit manufacturing industries. He then aggregated the sectoral data to construct aggregate price and wage indices, and used them to estimate an aggregate price adjustment equation. The sectoral speeds of price adjustment to a wage shock found by Blanchard were significantly faster than the aggregate one, with the former exceeding the latter by about 30 percent on average.[41]

## B. *Heterogeneous Slopes of Marginal Cost*

This section discusses an alternative interpretation of the evidence that inventory speeds of adjustment estimated using aggregate data tend to be smaller than the ones obtained using micro data. Instead of focusing on the omission of the markup variable $M_{kt}$ from standard partial adjustment regressions, I ask whether the same evidence might be explained by differences in the slopes of marginal cost across sectors. This is a natural alternative explanation, as production technologies or the cyclical properties of input prices might differ across sectors. It turns out that the answer to this question is positive. This alternative interpretation has different implications, however, for the use of aggregate models of inventories with respect to the one offered thus far.

Consider a simple extension of the model developed in Section I that allows for heterogeneity across sectors in the parameter $\omega$ governing the slope of the marginal cost function. Assume that sector-one firms are characterized by steeper marginal cost functions than sector two firms: $\omega_1 > \omega_2$. For simplicity, suppose also that markups in the two sectors are constant over the business cycle ($\pi_k = 0$, for $k = 1, 2$). In this case the firm-level partial adjustment regressions (15) are correctly specified and therefore the least square estimators $\hat{\theta}_k$ of firm-level adjustment speeds are always equal to the "true" sectoral adjustment speeds, $\theta_k \equiv 1 - \zeta_{k1}$, for $k = 1, 2$.[42] Moreover, $\theta_1 < \theta_2$, because the slope of marginal cost is higher in sector one than in sector two.

Notice that in this economy the $\zeta$ coefficients in the equilibrium law of motion for inventories (see equation [12]) are sector-specific. Therefore, the firm-level partial adjustment equations *cannot* be aggregated into an aggregate partial adjustment equation of the form

[41] Blanchard (1987) explained these results by suggesting that individual price setters adjust their prices quickly, but that interactions among them leads to slow adjustment at the aggregate level.

[42] The parameter $\zeta_{k1}$ generalizes the parameter $\zeta_1$ in equation (11) to the case where $\omega_1 \neq \omega_2$. It can be obtained by replacing $\omega$ with $\omega_k$ in the expressions in Appendix B. In this section, markups are assumed to be constant in order to highlight the effects of heterogeneous slopes of marginal costs on the estimates of aggregate adjustment speeds. Introducing cyclical variations in markups across sectors does not significantly affect the quantitative results of Table 8.

$$(30) \qquad I_{t+1} - I_t = \theta(\alpha + \lambda S_t^e - I_t) + \varphi(S_t - S_t^e) + \varepsilon_t.$$

This means that, while it is still possible to interpret the firm-level equations (11) in terms of a traditional partial adjustment model with a constant adjustment speed, no such interpretation can be made in regard to the equilibrium equation for aggregate inventories, obtained by summing equations (11) across firms.[43]

It is, of course, still possible to try to estimate the parameter $\theta$ in equation (30). As shown by Henri Theil (1954), the estimator $\hat{\theta}$ is a weighted average of *all* the firm-level parameters of the model, not only the adjustment speeds $\theta_k$, but also the parameters $\lambda_k$ and $\varphi_k$. In particular, it is easy to show that the difference between the weighted average of firm-level speeds of adjustment and the expectation of $\hat{\theta}$ is given by

$$(31) \qquad E[\mu\hat{\theta}_1 + (1-\mu)\hat{\theta}_2 - \hat{\theta}] = \mu(1 - E[\hat{x}_\theta])(\theta_1 - \theta_2) + E[\hat{x}_\lambda]\mu(\lambda_2\theta_2 - \lambda_1\theta_1) + E[\hat{x}_\varphi]\mu(\varphi_2 - \varphi_1)$$

where $\hat{x}_\theta$, $\hat{x}_\lambda$ and $\hat{x}_\varphi$ denote the coefficient on aggregate inventories $I_t$ in a regression of (respectively) $I_{1t}$, $S_{1t}^e$, and $(S_{1t} - S_{1t}^e)$ on the three aggregate variables $I_t$, $S_t^e$, and $S_t$. The econometric aggregation bias (the left-hand side of [31]) can then be attributed, using Theil's language, to the effect of "corresponding" and "noncorresponding" microparameters. The former is represented by the first term on the right-hand side of equation (31), while the latter is represented by the second and third terms. While the bias due to noncorresponding microparameters is difficult to interpret, the bias due to corresponding microparameters is relatively straightforward. As an example, according to equation (31), the weighted average of firm-level speeds of adjustment tends to exceed $E[\hat{\theta}]$ if, *ceteris paribus,* $E[\hat{x}_\theta] > 1$. This condition is more likely to be verified when, after conditioning on aggregate sales and sales expectations, the inventory stock in sector one is more volatile than the average inventory stock. A sufficient condition for this is that sector-one firms smooth production through accumulation and decumulation of inventories more than sector-two firms, i.e., $\omega_1 > \omega_2$.

To make the calibrated version of the model comparable to the one in Section I, I set $\omega_1$ and $\omega_2$ so that the speeds of adjustment in the two sectors are the same as the ones estimated for the benchmark model and reported in Table 4. In order to obtain $\theta_1 = 0.13$ and $\theta_2 = 0.45$, the slopes of marginal cost in the two sectors have to be such that $\omega_1 = 1.05$ and $\omega_2 = 0.057$. The other parameters of the model are the same as in Section III.A. The aggregate statistics implied by this version of the model are remarkably similar to those of the benchmark economy (Tables 2 and 3) and are omitted for simplicity. The results of the Montecarlo experiment for this version of the model are reported in Table 8. This table also reports Theil's decomposition of the bias on the left-hand side of equation (31) into the effects of corresponding and noncorresponding microparameters. They are represented by the columns (C) and (NC) in Table 8.

Two observations are in order. First, notice that in Table 8 the average estimate of $\theta$ obtained with aggregate data is lower than the weighted average of the firm-level speeds of adjustment. The size of the bias is large, even if smaller than the one reported in Table 4 for the benchmark version of the model.[44] This result is very sensitive, however, to small variations in the slope parameter $\omega_2$. For example, if $\mu = 0.10$, when $\omega_2$ goes from 0.057 to 0.051 (0.059), the adjustment speed $\theta_2$ goes from 0.45 to 0.46 (0.45), but the average value of $\hat{\theta}$ increases (decreases) dramatically from 0.27 to 0.66

[43] It is still possible to organize the aggregate data in partial adjustment form with time-varying "parameters." In this case the "true" adjustment speed at the aggregate level would be a weighted average of $\theta_1$ and $\theta_2$ with time-varying weights represented respectively by the shares (at each point in time) of sector one and two's inventories in the aggregate. Schuh (1996) pursues this point further and shows how a model of this sort can fit the data better than the estimated aggregate model (30).

[44] The estimates of the "sales surprise" parameter $\varphi$ are close to the ones in Table 5.

TABLE 8—ESTIMATES OF AGGREGATE AND FIRM-LEVEL ADJUSTMENT SPEEDS FOR INVENTORIES (HETEROGENEOUS COST MODEL)*

| | Aggregate | Firm-level data | | | Bias | | |
|---|---|---|---|---|---|---|---|
| | data $\hat{\theta}$ | $\hat{\theta}_1$ | $\hat{\theta}_2$ | $\mu\hat{\theta}_1 + (1-\mu)\hat{\theta}_2$ | $\mu\hat{\theta}_1 + (1-\mu)\hat{\theta}_2 - \hat{\theta}$ | C | NC |
| $\mu = 0.10$ | 0.27 | 0.13 | 0.45 | 0.42 | 0.15 | 0.49 | −0.34 |
| | (0.10) | (0.00) | (0.00) | (0.00) | (0.10) | (0.30) | (0.21) |
| $\mu = 0.30$ | 0.26 | 0.13 | 0.45 | 0.35 | 0.09 | 0.33 | −0.24 |
| | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) |
| $\mu = 0.60$ | 0.22 | 0.13 | 0.45 | 0.26 | 0.04 | 0.16 | −0.12 |
| | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) |
| $\mu = 0.90$ | 0.16 | 0.13 | 0.45 | 0.16 | 0.00 | 0.03 | −0.03 |
| | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) |

*Notes:* Estimates have been obtained by simulating the heterogeneous cost model and estimating a partial adjustment equation for inventories for 1,000 times. The sample size of each regression is 100. $E[\hat{\theta}]$: average estimate of the adjustment speed of inventories obtained using aggregate data. $E[\hat{\theta}_k]$: average estimate of $\theta_k$ obtained using data on a sector $k$'s firm, $k = 1, 2$. Notice that for each simulation, $\hat{\theta}_k = \theta_k$ because the firm-level equations are correctly specified. The standard deviation of the estimates across simulations is reported in parentheses.

(0.13). Second, Theil's decomposition reveals that the bias on the left-hand side of (31), while close to the empirical evidence contained in Schuh (1996), is the sum of a large positive bias due to corresponding microparameters and a large negative bias due to noncorresponding microparameters. Therefore, while the bias due to corresponding microparameters is indeed positive as suggested above, the overall bias tends to be of the correct magnitude only because of the compensating effect of noncorresponding microparameters on $\hat{\theta}$.

More generally, the results of Table 8 suggest that the relatively small estimates of aggregate inventories' speeds of adjustment are consistent with two alternative stories. According to the first one, the target equation for inventories in standard partial adjustment regressions is misspecified because it fails to control for variations in price markups. Heterogeneity in the cyclical variations of price markups gives rise to relatively small estimates of aggregate speeds of adjustment and, on average, larger estimates of firm-level speeds of adjustment. According to the second story, the explanation for small estimated speeds of adjustment at the aggregate level is not misspecification of target stocks but structural heterogeneity across firms. The latter implies that there is no "true" aggregate adjustment speed parameter and the average value of $\hat{\theta}$ is a complex combination of all structural parameters of the model.

Given the lack of significant evidence of production-smoothing behavior in the data and the fact that the properties of the production-smoothing model are relatively well understood, this paper has emphasized more the explanation based on heterogeneous markups. Future empirical work will have to differentiate among these explanations by direct measurement of marginal costs.[45] Despite their differences, however, these two stories share one underlying theme: a small subset of sectors in the economy might exert a disproportionate impact on the estimates of the parameters of aggregate partial-adjustment models.

## V. Concluding Comments

In this paper I suggest a common explanation for two puzzles in the inventory literature. The explanation reconciles the small estimates of adjustment speeds of aggregate inventory stocks with the apparent rapid response of firms to aggregate sales surprises. It also reconciles these small estimates with the larger ones ob-

[45] In this regard, it is interesting to note that this kind of identification problem between variations in price markups and intertemporal substitution in production is also present in Bils and Kahn (2000). Since they measure marginal costs directly, they are able to resolve this issue. Interestingly, they conclude that cyclical variations in price markups, rather than intertemporal substitution in production, explains the behavior of inventories for the six manufacturing industries they consider.

tained with firm-level data. I show, qualitatively and quantitatively, how these results can emerge if countercyclical variations in price markups are omitted from standard partial adjustment regressions and firms belong to sectors with different cyclical variations in markups. The first condition explains the downward bias of estimated speeds of adjustment with respect to their "true" value. The second one explains why this bias is more severe when aggregate, instead of firm-level, data are used.

These arguments rationalize the relatively small estimates of adjustment speeds of inventories to their target found in the literature. They do not imply, however, that either aggregate or firm-level inventories adjust faster to changes in *sales* over the business cycle than previously thought. Countercyclical variations in markups and procyclical variations in marginal cost give rise to the sluggish movement of inventories relative to sales over the business cycle.

It is important to notice that the approach to aggregation developed here differs from other studies that have addressed the dichotomy between fast, but smooth, adjustment of microeconomic units to shocks and gradual adjustment at the aggregate level.[46] In these studies, staggering of decisions and production-chain interactions among fast-adjusting micro units leads to gradual adjustment at the aggregate level. In this paper, instead, interaction among firms within a sector is of a very limited nature and adjustment decisions are taken simultaneously. The advantage of this simple formulation is to allow for aggregation across firms, so that the laws of motion of aggregate and firm-level variables have the same form. The upshot is that, despite this similar structure, estimated speeds of adjustment of aggregate and firm-level variables might differ significantly if some key variables are omitted from the empirical specification of partial adjustment equations.

As far as the inventory literature is concerned, this study confirms the importance of using information contained in prices to improve the fit of traditional inventory models (Bils and Kahn, 2000). To implement this idea, it is necessary to allow inventories to affect firms' revenue more explicitly than in the standard linear-quadratic model where stockout costs do not depend on markups.

While inventories are an illustrative special case of the ideas emphasized in this paper, countercyclical variations in price markups should affect all aspects of a firm's behavior over the business cycle. Moreover, heterogeneous variations in price markups across sectors are likely to give rise to different sectoral responses to common shocks. This paper shows that this heterogeneity might have a profound influence on the inferences made by macroeconomists about the behavior not only of aggregate inventories, but also of labor and prices. Pursuing this issue further for these and other variables is an interesting avenue for future research.

[46] See, for example, the discussion in Blanchard (1987) and the references therein.

## Appendix A: Decision Rule for $A_{kt}$

In this appendix I report the coefficients of the linearized decision rules for the variable $A_{kt}$. The coefficients of equation (9) are found using the method of undetermined coefficients (see, e.g., Christiano, 2002). The relationship between these coefficients and the structural parameters of the model is as follows. Define:

$$\psi_3 \equiv \frac{\nu + \omega S}{\omega A} \frac{\phi S M}{A + \phi S M}$$

$$\psi_1 \equiv (\phi - 1)\psi_3 - 2 + \phi \frac{S}{A}$$

$$\psi_2 \equiv 1 - \phi \frac{S}{A} \qquad \psi_4 \equiv \psi_3 + \frac{S}{A} \qquad \psi_5 \equiv -\frac{S}{A}$$

where $A$ and $S$ denote the steady state values of $A_{kt}$ and $S_{kt}$ and are given by

$$A = \left(\frac{\phi\beta M}{1-\beta}\right)^{1/(1-\phi)}, \qquad S = A^{\phi}.$$

Also, define

$$\ell_1 \equiv \frac{-\psi_1 - \sqrt{\psi_1^2 - 4\psi_2}}{2} \qquad \ell_2 \equiv -\left(\eta - \frac{\psi_2}{\ell_1}\right)^{-1}\psi_3$$

$$\ell_3 \equiv -\left(\rho - \frac{\psi_2}{\ell_1}\right)^{-1}\left(\ell_1 \frac{\psi_5}{\psi_2} + \psi_4\right) \qquad \ell_4 \equiv \ell_1 \frac{\psi_5}{\psi_2}.$$

Then, the $\xi$ coefficients in equation (9) are

$$\xi_0 \equiv A(1 - \ell_1 - \ell_2 - \ell_3 - \ell_4)$$

$$\xi_1 \equiv \ell_1 \qquad \xi_2 \equiv \ell_2 \frac{A}{M} \qquad \xi_3 \equiv \ell_3 A \qquad \xi_4 \equiv \ell_4 A.$$

In particular, notice that if production occurs under constant returns to scale ($\omega \to 0$), then $\psi_1 \to -\infty$. With l'Hopital's rule, it is easy to show that $\ell_1 \to 0$. Thus, when $\omega = 0$, $\xi_1 = 0$ and $\xi_4 = 0$.

## Appendix B: Partial Adjustment Equations Implied by the Model

### *Inventories*

The $\zeta$ coefficients in the inventory equation (11) are:

$$\zeta_0 = \left(1 + \phi\frac{\xi_3}{A}\right)^{-1}(\xi_0 + \xi_3\phi + \xi_4\phi) \qquad \zeta_1 = \left(1 + \phi\frac{\xi_3}{A}\right)^{-1}\left(\xi_1 - \phi\frac{\xi_4}{A}\right)$$

$$\zeta_2 = \left(1 + \phi\frac{\xi_3}{A}\right)^{-1}\xi_2 \qquad \zeta_3 = \left(1 + \phi\frac{\xi_3}{A}\right)^{-1}\xi_3\frac{1}{S} - 1.$$

In particular, notice that if $\omega = 0$, since $\xi_1 = 0 = \xi_4 = 0$, the adjustment speed coefficient $\zeta_1$ is also equal to zero (i.e., adjustment is instantaneous).

### *Labor*

Define:

$$\varkappa_0 \equiv \frac{L}{S}(1 + \omega S) \qquad \varkappa_1 \equiv -L\omega S$$

where the steady state level of labor is given by $L = S^{1+\omega S}$. The coefficients of the partial adjustment equation for labor (equation 27) are:

$$\theta \equiv 1 - \zeta_1 \qquad \alpha^l \equiv \varkappa_0 \qquad \lambda^l \equiv (1-\zeta_1)^{-1}\varkappa_1(1+\zeta_3)$$

$$\chi^l \equiv -(1-\zeta_1)^{-1}\varkappa_1(\zeta_3 + \zeta_1) \qquad \iota^l \equiv (1-\zeta_1)^{-1}\varkappa_1\zeta_2.$$

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