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SIMULATION OF SLOPE FAILURE USING A MESHED BASED PARTITION OF UNITY METHOD

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ABSTRACT

A mesh based partition of unity method, known as the manifold method, is used in simulating the evolution of a slope failure. The problem configuration consists of a simple slope that has pre-existing tensile cracks along its crest. The slope failure is triggered by rainfall which raises water pressure in the crack. As the tensile stress around the crack tip increases, an existing crack grows and a failure surface is eventually developed. The maximum stress criterion is adopted in determining the crack growth and growth direction. After a failure surface is formed, the unstable soil mass, bounded by the failure surface, slides down the slope. This sliding process is also modeled.

Keywords: slope failure, crack growth, partition of unity

INTRODUCTION

The manifold method, which can be viewed as a mesh based partition of unity method, was originally developed by Shi (1992) as an extension to the discrete element method so that each element can have many more degrees of freedom. Because of this discrete element root, the manifold method can be used in integrating the continuum and the discrete analysis under one single framework. The method employs two sets of meshes in modeling a problem, one is called the physical mesh, the other mathematical mesh. The nodes and elements used in an analysis are obtained from these two sets of meshed. A physical mesh is a portrait of problem geometry that includes boundary, discontinuities, load application points and essential boundary points. A mathematical mesh is arbitrary and is used as in building local covers of the problem domain.

For the problem under study, the layout of the slope, the existing crack, the left and the bottom fixed boundaries constitute a physical mesh of the problem as depicted in Fig. 1(a). A triangular mesh of is used as the mathematical mesh of the problem. The mathematical mesh is required to cover at least every point of a problem domain. Generally, a mesh larger than necessary is often adopted, as shown in Fig. 1(b). These two meshes are then superimposed and nodes for the triangles that do not intersect the problem domain are trimmed. To avoid cluttering a drawing, nodes that are kept but lie outside a problem domain are not shown. That is why in Fig.

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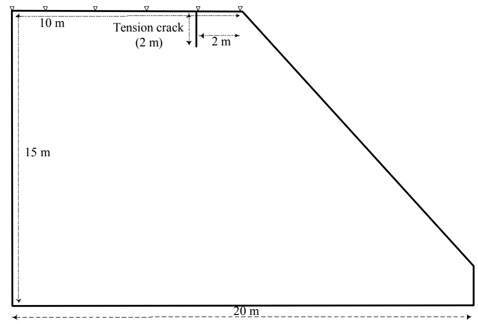
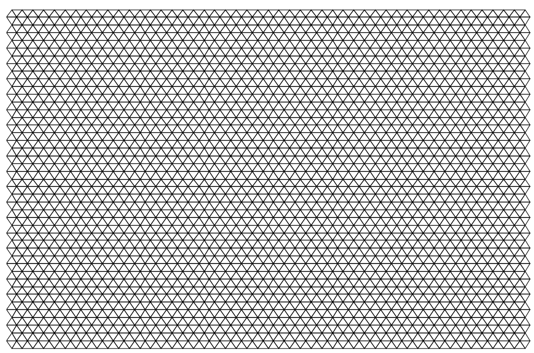
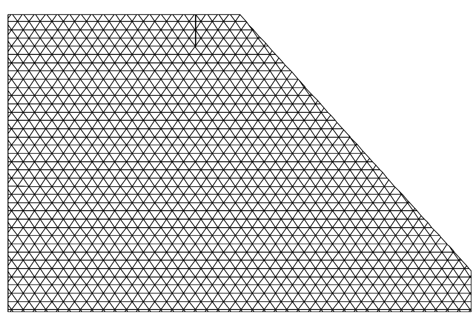
1(c) there are triangles that are only shown partially.



(a) Physical Mesh



(b) Mathematical Mesh



(c) A Superposition of two meshes

FIG. 1. The soil slope layout and the manifold meshes

An interpretation of manifold method using finite element analogy was provided by Lin (1995). The presentation herein, however, follows that of the partition of unity method. In the triangular mathematical mesh is used, a local cover, ω_α, associated with a node, α, is formed by the union of all the triangular areas share this common node. The collection $\bigcup_{\alpha=1}^{N}\omega_\alpha$ on a mathematical mesh with N nodes provides a finite covering of the domain of solution.

Just like the partition of unity method (Duarte and Oden, 1996; Babuska and Melenk ,1997) or the meshless method (Belytschko *et al*, 1996), the manifold method also uses the partition of unity function although it was originally called the weighting function. Following Duarte and Oden (1996), a partition of unity function, $\varphi_\alpha(\mathbf{x})$, subordinates to each cover satisfies the following,

$$\sum_{\alpha=1}^{N}\varphi_\alpha(\mathbf{x}) = 1 \quad for\ \mathbf{x} \in \Omega \tag{1}$$

$$\varphi_\alpha(\mathbf{x}) \in C_0^S(\omega_\alpha) \quad for\ 1 \le \alpha \le N; S \ge 0 \tag{2}$$

In this study, shape function is used as the partition of unity function and that a constant approximate function over each cover is employed as follows,

$$\hat{\mathbf{u}}_\alpha(\mathbf{x}) = u_\alpha \tag{3}$$

As such, an element is a overlapped areas of three covers. The solution approximation within an element, $\mathbf{u^h(x)}$, becomes

$$\mathbf{u^h}(\mathbf{x}) = \sum_{i=1}^{3}\varphi_i(\mathbf{x}) \cdot \hat{\mathbf{u}}_i(\mathbf{x}) \tag{4}$$

When a cover becomes disjoint due to discontinuity, a cover is partitioned which is equivalent to have more than one nodes at a point. Thus, in a crack propagation problem, instead of remeshing, the manifold method just adds additional covers, or nodes, to follow the growth of a crack. This requires that each involved disjoint cover be modified so that it only covers a partial area:

$$\varphi_i^{\,j}(\mathbf{x}) = \varphi_i(\mathbf{x}) \cdot \delta_i^j(\mathbf{x}) \tag{5}$$

where $\delta_i^j(\mathbf{x})$ is 1 on ϖ_i^j, and 0 elsewhere, and ϖ_i^j represents the jth division of a cover at node *i*. As a crack grows and if the new crack surface cuts through a cover, a new cover is added. In other words, new nodes are introduced as a crack propagates. Together with introduction of

$\delta_i^j(\mathbf{x})$, the approach can handle the introduced crack surface without remeshing.

MAXIMUM STRESS CRITERION

The maximum stress criterion by Erdogan and Sih (1963) is used in determining when and which direction a crack is propagated under a mixed mode scenario. This criterion is based on the following premises:

- Crack initial extension occurs at an orientation θ in which the tangential stress, σ_θ, is at a maximum.
- Crack initiation takes place when the maximum tangential stress reaches the tensile strength of the material, $\sigma_{\theta c}$.
- The orientation in which the crack extends is perpendicular to the direction of the maximum tangential stress.

In the present study, the material is considered linear elastic. The maximum stress criterion is applied by sampling the stress at a distance r_o away from the crack tip (Williams and Ewing, 1972). The r_o used in this study is set to be about 0.1*a*, where *a* is half length of the crack. The material strength, $\sigma_{\theta c}$, at r_o is determined from fracture toughness K_{IC}.

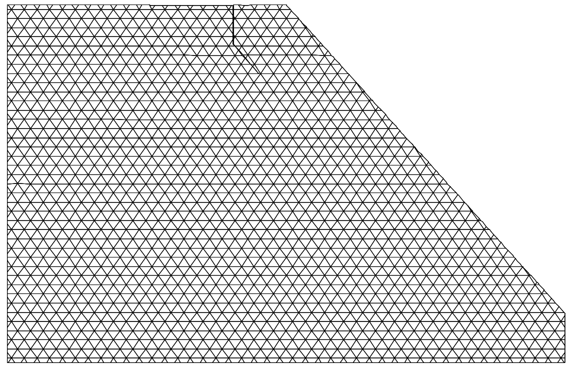
The criterion has been used successfully with the manifold method in modeling crack propagation that involves interaction across crack surfaces (Ku, 2001).

MODELING CONSIDERATION

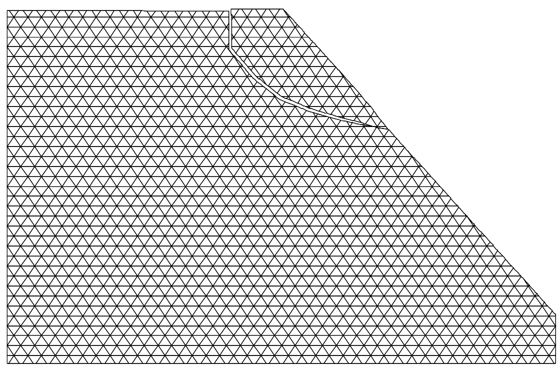
The case under study is a soil slope 15 m high with a 2-m deep initial tension crack located on the top of the slope. The soil is considered to be homogeneous and has a unit weight of 23.6 kN/m^3. The elastic modulus for the soil is 71,820 kPa, and the Poisson ratio is 0.3. Any newly fractured failure surface will have a low cohesive strength of 3.59 kPa, and a friction angle of 28 degrees. A K_{IC} of 91.54 kPa-$cm^{1/2}$ is considered.

The failure event studied is one that causes by a rise of 2m in ground water table, i.e., the rise of ground water table from the bottom of the crack to the ground surface. The ground water exerts pressure on the crack surface. Each fresh crack growth length is fixed at a size three times that of an element, or 1.2 m in this study. The penalty method is used in imposing the no penetration constrains across the block boundary. The implicit algorithm used is adapted from Shi (1989).

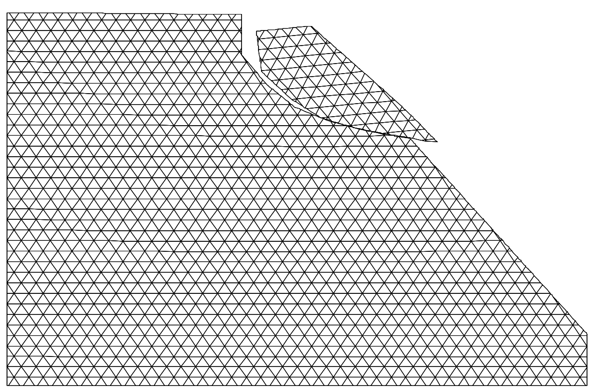
Evolution of slope failure is initially computed using pseudo-dynamic analysis, in which inertia force is made artificially small. When a failure surface extends the boundary of a slope and forms a separate body, the dynamic analysis is kicked in. Fig. 2 presents a snapshot of the evolving stages in the progression of the slope failure.



(a) Initial crack growth



(b) A fully developed sliding plane



(c) **A snapshot during sliding**

Fig. 2 Evolution of a Slope Failure

CONCLUSIONS

The manifold method, together with the linear elastic fracture mechanics are used in simulating the evolution of a slope failure. The advantages of an integrated discrete-continuum approach is clearly demonstrated in that an analysis can continue even after a continuum split into discrete objects. This may be important for assessing the impact of failure and estimating a post-failure configuration.

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REFERENCES

Erdogan, F. and Sih, G. C. (1963), "On the Crack Extension Path in Plates under Loading and Transverse Shear," *Journal of Basic Engineering*, Vol. 85, pp. 519-527.

Babuska, I. and Melenk J. M. (1997) The partition of unity method. *Int. J. Numer. Meth. Engng* , 40:727-758.

Belytschko T., Krongauz Y., Organ D., Fleming M., Krysl P. (1996) Meshless methods : an overview and recent developments, *Comput. Methods Appl. Mech. Engrg*., **139**: 3- 47.

Duarte, C. A. M. and Oden, J. T. (1996) An h-p adaptive method using clouds, *Comput. Methods Appl. Mech. Engrg*. **139**: 237-262

Ku C-Y (2001) Numerical Modeling of Jointed Rock Masses based on the Manifold Method, Doctoral Dissertation, Dept of Civ. & Environ. Eng., University of Pittsburgh.

Lin, J-S. (1995) Continuous and discontinuous analysis using the manifold method, Proceeding *Working Forum on the Manifold Analysis, Vol I*, 1-20, Geotechnical Lab, US Army Engineers Waterways Experiment Station

Shi, G. H. (1992), Modeling rock joints and blocks by manifold method, Proceedings of the *33rd U. S. Rock Mechanics Symposium*, San Ta Fe, New Mexico, pp 639-648.

Shi, G. H. (1989) Block System Modeling by Discontinuous Deformation Analysis, Doctoral Dissertation, Dept. of Civil Eng, University of California, Berkeley.

Williams, J. G. and Ewing, P. D., (1972) "Fracture Under Complex Stress – The Angled Crack Program," *International Journal of Fracture*, 8, pp. 441-446

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