

ALGEBRA 2 – PROBLEM SET 9 – UNIVERSITY OF PITTSBURGH, FALL 2019

Due on Friday November 15.

- (1) (5 points) DF §10.4, Problem 2.
- (2) (5 points) DF §10.4, Problem 3.
- (3) (10 points) State a form of the structure theorem for finitely generated modules over a PID. Applying this to the PID  $\mathbb{Z}$  and a finitely generated  $\mathbb{Z}$ -module  $M$ , write down a formula for

$$\dim_{\mathbb{F}_p} M \otimes_{\mathbb{Z}} \mathbb{F}_p$$

for an arbitrary prime  $p$ . The formula should be given in terms of data appearing in the expression of the structure theorem. Similarly, write down a formula for

$$\dim_{\mathbb{Q}} M \otimes_{\mathbb{Z}} \mathbb{Q}.$$

(You may of course use Theorem 17 of §10.4 and its corollaries, as well as anything else in §10.4.)

Terminology, FYI:

- When  $R$  is a commutative ring and  $P \subset R$  is a prime ideal, the associated “residue field of  $R$  at  $P$ ” is the field  $\text{Frac}(R/P)$ . Note that Problem 3 is asking for the dimensions of a finitely generated  $\mathbb{Z}$ -module after tensoring up to the residue fields of each prime ideal of  $\mathbb{Z}$ .

Extra problems: DF §15.3, Problems 4–9. Notice that Problem 9 finishes off the equality that we discussed at the end of class on Friday November 8.