ALGEBRA 2 – PROBLEM SET 8 – UNIVERSITY OF PITTSBURGH, FALL 2019

Revised version. Due on Friday October 25.
Problem 1 may be turned in on Monday October 28 if need be.

(1) (10 points) Let $I \subset R$ be an ideal in a commutative ring. Prove that the radical of I is equal to the intersection of all prime ideals containing I. Here is the statement in formulas:

$$\mathrm{rad}(I) = \bigcap_{\substack{R \supset P \supseteq I \\ P\text{: prime}}} P.$$

Remark: There are similar problems in the text. Section 7.4, problems 26 and 30-32.

(2) (8 points) Assume that k is an algebraically closed field. Prove that the localization of $k[x,y]/(x^2 - y^3)$ at the maximal ideal (x,y) is not a DVR. Conclude by explaining that $k[x,y]/(x^2 - y^3)$ is not a Dedekind domain.

(3) (2 points) Discuss what you would like to see reviewed for the midterm, or any other questions you have, with other other person in the class. Write down your conclusions/questions.

Extra problems: DF §15.3, Problems 15 and 17; DF §7.4, Problems 26, 30-32.