

## FLASH CRASH REVISITED

**O**n May 6, 2010 the 600-point drop in the Dow became known as the “Flash Crash,” and speculation on its cause continues. Meanwhile, as the WSJ reports (May 5, 2011), there have been similar flash crashes in commodity markets, also in the absence of significant fundamental changes in valuation. These include:

- The 5 percent drop within minutes in the US dollar versus Japanese yen on March 16, 2011.
- The 13 percent drop in cocoa futures in seconds before rebounding on March 1, 2011.
- The 6 percent drop in sugar in February 2011.

Since many of these market moves have set records (for percentage drop per unit time), one can ask if we have seen a series of unusual events (i.e., coincidence) or if there has been a change in markets in recent years. If we consider the latter possibility, then the following changes are probably relevant:

1. There is much more algorithmic computer trading, which means a greater fraction of trading is done on a faster time scale.
2. There are more electronic markets and fewer traditional market makers who are supposed to dampen wide swings.
3. The emphasis has been more on short-term trading profits rather than long-term. This necessarily means more momentum investing, since value investing requires a great deal more time and patience.
4. Investors are in a quandary in that real interest rates are negative. In other words, anyone who

# Are Flash Crashes Caused by Instabilities Arising from Rapid Trading?

Gunduz Caginalp, Mark DeSantis, and David Swigon of the Mathematics Department, University of Pittsburgh examine recent record-breaking market moves

does not want to take on risk is forced to lose 4 or 5 percent of purchasing power for health care, tuition, etc. Long-term investing in US equities has not been profitable for many investors during the past decade. This means that there are many more dollars in the speculative markets.

Taken together, these factors create an environment that can be rather unstable in the short term. Let’s illustrate what could be happening in a financial market. The price has been trading near \$100 as short-term traders are using a variety of algorithms to profit from short-term trades. Meanwhile, there is another group of (value) investors who are buying on price dips, e.g., when the price drops to \$99 they buy as a long-term investment. However, contrary to the assumptions of the efficient market theories, they have finite resources, as do the short-term traders.

At some point the market may reach a point where the value buyers have not necessarily exhausted their cash supply, but have reached a critical value relative to the short-term traders. This means prices begin to slip, which in turn triggers more

selling by the algorithms. After all, the latter are not long-term investors. When the market moves too far in the “wrong” direction, they need to bail out – and quickly! But since the cash of the value buyers is already below this critical point of the asset level of the short-term traders, there is a dearth of buyers. It invokes the joke about the investor who buys more and more of a low-volume stock, and subsequently asks his broker to sell his position. “To whom?” the broker asks.

This explanation leaves open the questions of when this phenomenon will occur, and why it does not happen more often. Our asset flow approach contains the basic components lacking in the classical theories, and can be used to understand the issue of stability:

1. There may be different groups of investors with different motivations, e.g., focus on value, trend, etc., and different assessments of the fundamental value,  $P_a$ .
2. There is a time scale,  $d_1$ , associated with the trend-based investors, and a time scale,  $d_2$ , associated with the value-based investors.

3. In addition to the trading price per share, and the value per share, there is a third quantity,  $L$ , which is defined as the total amount of cash divided by the total number of shares.
4. Each group is endowed with finite cash and shares.

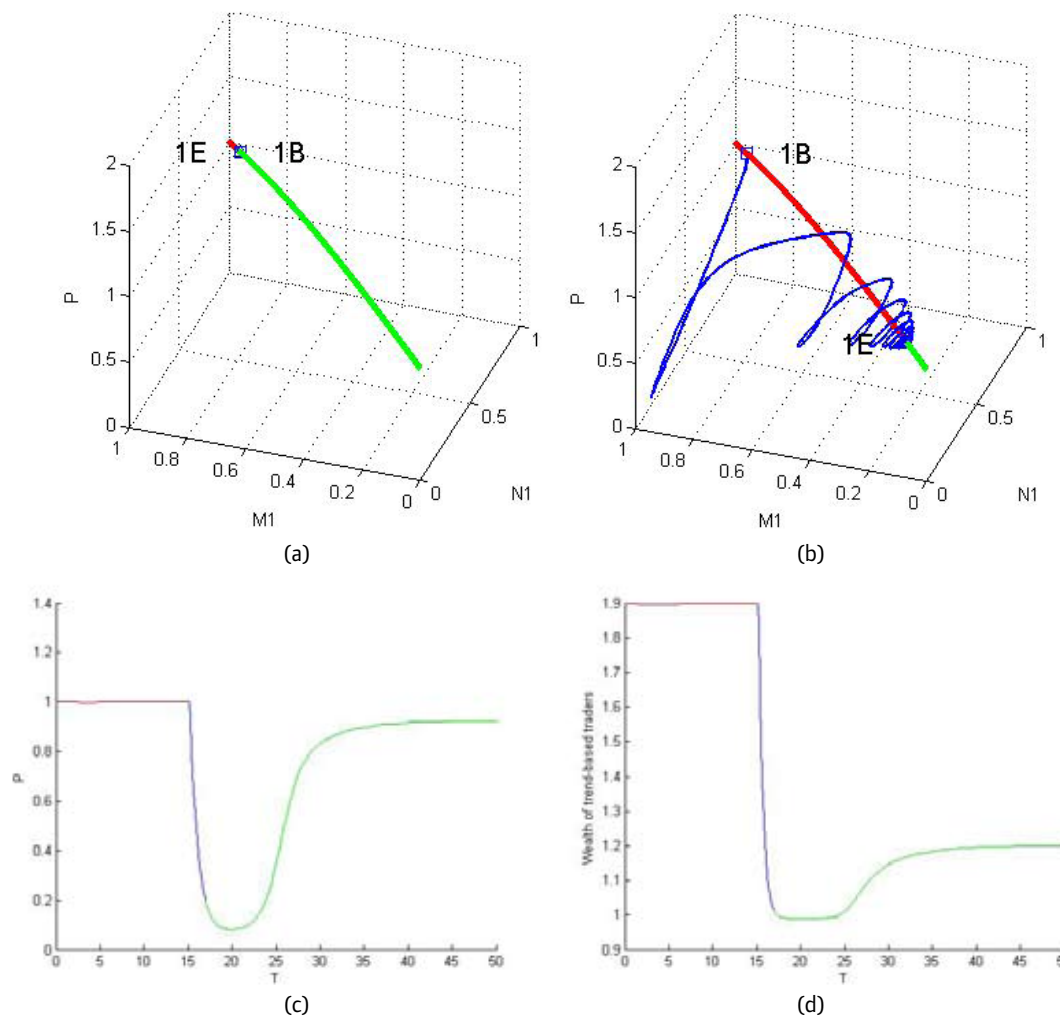
A key difference that emerges between this theory and classical theory is that there is no unique equilibrium that is essentially the intrinsic value. Rather, there is a line of equilibria that typically ranges between  $L$  and  $P_a$ , part of which is stable and part unstable. We have been studying these equilibria and their stability. An important destabilizing factor is a short time scale ( $d_1$ ) for momentum trading. This factor may be the key to these flash crashes.

If a point is a stable equilibrium, one can create conditions (within this simple model) that will render it unstable. For example, if there is an increase in the cash supply of the momentum traders, it can move the equilibrium point into the unstable region. Similarly, if the momentum traders, as a group, have focused on a shorter time horizon, the equilibrium can shift to an unstable one, meaning that a small perturbation

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can move the market abruptly. We illustrate these with a set of computations in which there is a particular set of parameters including the time scale ( $d_1$ ) for the trend-based traders. This set of parameters is in a stable equilibrium, so that price (versus time) is essentially flat. Upon modifying only the time scale,  $d_1$ , to a smaller value, we move this stable equilibrium to an unstable one on the new line of equilibria. This means that a small perturbation from this value (caused by a small change in investor sentiment, for example) will lead to an “excursion” that may be far from this line of equilibria, before settling into a new equilibrium. Once there is a severe drop, much of the program trading ceases, effectively slowing down the average time scale of the momentum traders. We illustrate this with a computational example below. From a practical perspective, the magnitude of this excursion is manifested in terms of the maximum percentage drop.

Of course, a significant change in valuation can also cause this shift. But in each of the flash crashes mentioned above, there was no indication of such a value shift. Within the setting of classical finance, an abrupt change in valuation would cause a similar change in trading price. Violent price movements could also be justified under the assumption that traders are trying to assess the significance of the valuation changes. Classical finance describes methods to understand changes in valuation, e.g., through changes in GDP, interest rates, inflation, etc. However, in each of these flash crashes, one can state with reasonable certainty that change in valuation was not the issue. One thus enters a regime in which a new discipline is required for any meaningful understanding.



**Figure 1.** Sub-figures 1(a) and 1(b) show the curve of equilibrium points that lies between the fundamental value,  $P_a = 0.8$ , and the liquidity value,  $L = 1$ . The M1 and N1 variables represent the total amount of cash and the total number of shares possessed by the trend-based traders. The cash and shares of the value-based (fundamental) traders may be determined by subtracting M1 and N1, respectively, from 1. Any point along this red/green curve corresponds to an equilibrium of the system, where red points correspond to unstable equilibria and green to stable. A sample solution trajectory that begins at the point labeled “1B” and ends at the point “1E” is also shown (in blue). In sub-figure 1(a) the trajectory has an initial price of 0.9811 which is near a stable (green) equilibrium price and remains close to this value. In sub-figure 1(b) the trajectory again starts with the price of 0.9811; however, in this scenario this price is near an unstable (red) equilibrium. The difference between these two scenarios is the value of the trend-based traders’ time scale,  $d_1$ . In sub-figure 1(a) this value is 8, while in

sub-figure 1(b) this value is  $1/5$ . Shortening the time scale changed the equilibrium price from stable to unstable. Sub-figure 1(c) displays the price (divided by the initial price),  $P$ , versus time,  $T$ . The price starts at 0.9811 (scaled to 1 in the plot), which is near a stable equilibrium as  $d_1$  is set to 8. At  $T = 15$  the value of  $d_1$  is decreased to  $1/5$ . The price curve is essentially flat (red curve) until this time scale is shortened, at which point the price decreases (blue curve). At  $T = 17$  the value of  $d_1$  is reset to its initial value of 8 and the price returns to a new equilibrium value slightly below the original equilibrium. Sub-figure 1(d) was constructed in the same manner as sub-figure 1(c) except it tracks the wealth of the trend-based traders.

Decreasing their time scale of interest ( $d_1$  from 8 to  $1/5$ ) caused a decrease in their wealth. Upon returning their time scale,  $d_1$ , to its original value of 8, the wealth of the trend-based traders increased slightly before settling at a new equilibrium significantly less than their initial wealth.