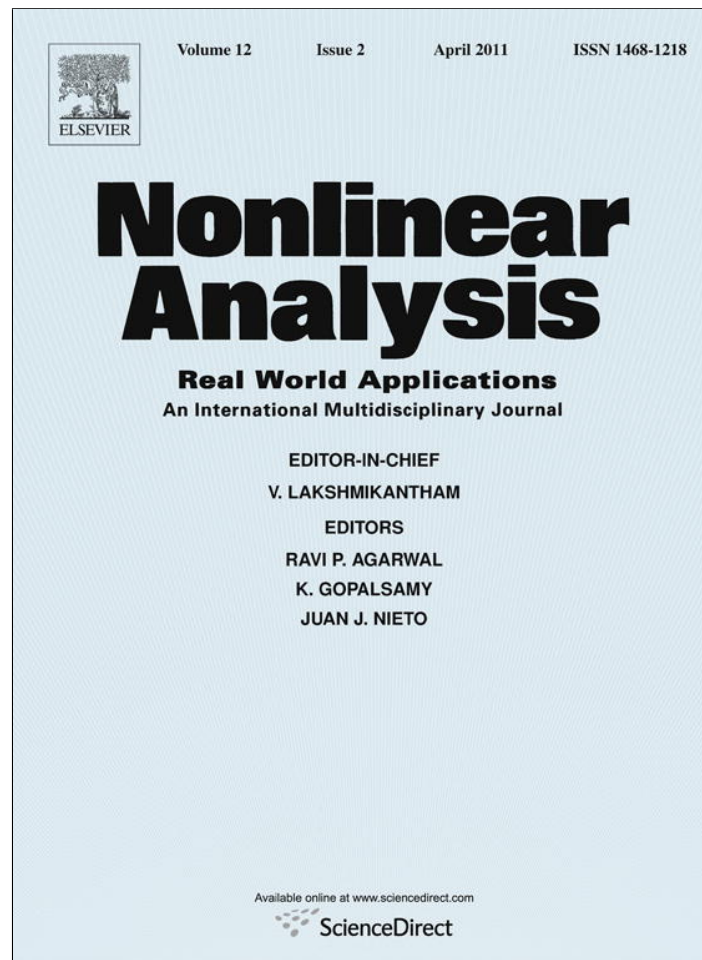


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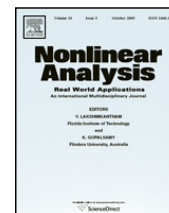
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# Nonlinear Analysis: Real World Applications

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## Nonlinearity in the dynamics of financial markets

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### ABSTRACT

A new set of methodologies extracts key nonlinearities in the dynamics of financial markets from data that would appear to be completely random with ordinary linear time series methods. The understanding acquired from this analysis forms a basis for modeling conflicting and competing motivations in market decisions. By standardizing the daily changes using the mean and standard deviation, it then becomes possible to compare the quantitative impact of very different variables such as price trend and valuation, and the nonlinear relationship between them. The analysis of a large data set of closing stock prices provides strong statistical evidence that relative daily price change is positively influenced by valuation, recent price trend, short term volatility, volume trend and the M2 money supply. However, there is a strong nonlinearity in the influence of the price trend, so that a significantly large recent uptrend has a negative influence on the subsequent day's relative price change. The nonlinearity is the key to understanding the conflicting role of price trend, since a single large data set exhibits both underreaction and overreaction in different regimes of the independent variables. The role of long term volatility is not a clear-cut risk/return inverse relation. But rather there is an ambiguous and complicated relationship between volatility and return. There is limited support for resistance when prices near the quarterly high. Mixed effects regressions are used after standardizing the data by subtracting the mean and dividing by one standard deviation individually for each of the 119 closed-end funds. A valuation variable is constructed in terms of the recent history of net asset value.

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### 1. Introduction

The dynamics of asset prices and the underlying motivations have been of great interest for both theorists and practitioners. A basic rationale for price movement is due to the changes in the value of the asset. In the absence of any insight into the motivations of investors and traders, one might stipulate that prices should fluctuate randomly about this basic valuation. One can regard this concept as a default hypothesis expressing a distinguished limit in which there is (a) unlimited information, that ensures all participants share the same notion of valuation, and (b) essentially infinite arbitrage capital, whereby informed investors vie with one another to quickly exploit any deviations from this valuation.

There is little doubt among practitioners that additional factors are at work in markets. The tumultuous financial markets of September and October 2008 are a dramatic reminder of the diverse forces driving markets. These recent upheavals capping a prolonged housing bubble are the most recent of a series of modern bubble/bust cycles. Among these are the internet/high-tech bubble of the late 1990s and the Japanese stock bubble of the late 1980s. Due to their sheer magnitude, these episodes have had an impact far beyond the immediate shareholders who lost trillions. The ensuing years of economic slowdowns and job losses were among the consequences of these boom/bust cycles.

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Yet it is surprising that only a relative handful of works focus on these phenomena, and finance textbooks hardly mention them. Papers that discuss motivations beyond valuation are often expository in nature, and rarely have direct contact with market data. As such it is easy for exponents of efficient market theories to dismiss them. In fact, it is often difficult to test hypotheses or theories of markets due to the randomness inherent in valuation changes (see [1]). Thus studies that examine stock prices for non-random behavior, such as trends, usually find only tiny effects (e.g. [2]).

An early study [3] highlighting the importance of trend examined two essentially identical closed-end funds, Future Germany Fund and Germany Fund. Closed-end funds, unlike open-end funds, trade as other company stocks on the exchanges (see, e.g. [4]). Thus the price can be higher or lower than the net asset value (NAV), and can vary independently of this value. In the case of these two clone funds, any change in valuation will be identical in both funds, and the ratio of valuation is constant in time. By considering the ratio of the trading prices of these funds as a time series they were able to extract all noise attributable to valuation. They found that tomorrow's price is not predicted well by today's price, as efficient markets would suggest. But rather, tomorrow's price is halfway between today's price and the price obtained by continuing the pure trend from yesterday to today.

The persistence of a trend can be viewed as an underreaction, as it suggests that there is a delay in reaching a particular price. On the other hand, the behavioral finance community has also stipulated that participants overreact to news, as recent information tends to overshadow previously established facts (see, e.g. [5]). Madura and Richie [6] considered a data set of daily opening and closing prices for AMEX-traded exchange traded funds (ETF) between August 1998 and August 2002. They found evidence that ETF stock prices reverse (i.e., exhibit overreaction) following extreme price changes of greater than  $\pm 5\%$ . They define overreaction as positive [negative] returns following negative [positive] price movements, while underreaction is positive [negative] returns following positive [negative] price movements. Overreaction has also been observed on a larger time scale (see, e.g. [7]).

Sturm [8] found evidence that if a stock with positive returns and/or increasing book value per share experiences a large drop in price of at least 10%, then it has a tendency to rebound, supporting the Madura and Richie findings. In addition, Duran and Caginalp [9] performed a large scale statistical study with 134,406 data points corresponding to daily stock prices for closed-end funds. They found that large discrepancies between the trading price and the net asset value of the fund led to significant price changes in the opposite direction of the discrepancy. Furthermore, the study established a precursor to these large changes suggesting oscillatory behavior.

The fact that some studies find evidence of overreaction while others demonstrate underreaction has led some to assert this as evidence that markets are efficient. Our perspective, however, is that perhaps there are nonlinear relationships between price movements. One of our key goals is to demonstrate this nonlinearity and to establish its form on an empirical and statistical basis. In particular, does an uptrend have a negative impact on daily returns after a particular threshold? And how does the valuation enter into that threshold? A second goal is to have an objective measure of the impact of various competing effects, e.g., price trend, valuation, etc.

In this paper we accomplish this by quantifying the effect of various factors on the daily relative price change. Given the unambiguous nature of the definition of a closed-end fund's valuation (namely, its NAV), we consider a data set comprised of 111,356 daily closing prices for 119 closed-end funds. We perform linear regressions with the relative change in daily price as the dependent variable and various decision-making factors as independent variables. Specifically, we consider (i) the recent and long term trend in fund price; (ii) the valuation of the fund; (iii) the M2 Money Supply; (iv) the recent and long term volatility of the stock price; (v) the recent trend in volume; and (vi) resistance, i.e., the idea that rising stock prices tend to slow their increase when approaching a recent high price, which acts as a barrier. As shown by the regressions of Caginalp and Ilieva [10], the incorporation of valuation in the appropriate form will greatly reduce the "noise" inherent in the stochastic nature of the valuation and enable the analysis of these other contributing factors.

By standardizing the independent variables, i.e., subtracting the mean values and dividing by the standard deviations, we are then able to easily compare the effects of each variable. Note that the methodology we employ to find quantitative measures of effect can be extended for use on ordinary stocks provided a valuation measure for these stocks is employed. In addition, this process is not limited to the above mentioned variables, but can be used to examine any decision-making factor provided it may be expressed as a variable. An important aspect of this study involves a judicious definition of valuation. Without a mechanism for extracting the valuation, the remaining terms would quite likely be masked by the "noise" inherent in changes in valuation (see Regressions 4 and 5 in [Appendix](#)).

## 2. The data set

We utilize a data set consisting of 111,356 records corresponding to 119 funds (28 Generalized, 62 Specialized, and 29 World funds). The records correspond to daily closing prices for the time period October 26, 1998 through January 30, 2008.<sup>1</sup>

### *Relative price change*

The basic quantity of interest is the Relative Price Change that we define as

$$R(t) = \frac{P(t) - P(t-1)}{P(t-1)},$$

<sup>1</sup> Due to the definition of certain independent variables, all funds were required to have at least one year's worth of data.

which is sometimes called the “return” for day  $t$ . The Relative Price Change for day  $t + 1$ , namely  $R(t + 1)$ , will be used as the dependent variable in the regressions.

With the exception of the M2 Money Supply, the following variables, which are utilized as independent variables in the regressions, are based upon the above mentioned daily closing prices and NAVs.

*Valuation*

Each of the funds included within our data set has both its price and NAV reported on a daily basis. It is, however, unusual for the fund to trade at its NAV. Typically, the fund trades at a persistent premium (price exceeds NAV) or discount (price is below the NAV). We define the Valuation,  $D(t)$ , of a closed-end fund to be the current day’s relative deviation between NAV and price,  $\frac{NAV(t)-P(t)}{NAV(t)}$ , minus a weighted average of this relative deviation over the previous 10 days:

$$D(t) = \frac{NAV(t) - P(t)}{NAV(t)} - \frac{1}{3.2318} \sum_{k=1}^{10} \frac{NAV(t - k) - P(t - k)}{NAV(t - k)} e^{-0.25k}$$

where  $NAV(t)$  is the fund’s net asset value on day  $t$  and  $P(t)$  is the trading price.

We consider this deviation because if a fund has been trading at a 10% discount but is currently at a 5% discount, then a valuation-based trader will probably not regard this fund as a bargain. To mitigate the effect of large deviations abruptly dropping out of the average, we utilize the scaling factor  $e^{-0.25k}$  and normalize it via the coefficient  $(3.2318)^{-1} = \sum e^{-0.25k}$ . This tends to smooth the effect of historical prices and also emphasizes recent events more strongly than past events.

*Price trend*

As noted in the Introduction, one possible motivation for buying a stock is that the price is in an uptrend (and analogously for selling). Since prices are fluctuating and changing direction frequently, the definition requires a choice of time scale, which we take as ten days, and a scaling factor that determines the weightings of recent days relative to earlier days. We use the exponential factor  $e^{-0.25k}$  for the weighting of the relative price change  $k$  days ago, as in the Valuation variable above. Thus with the normalization factor above, we define the Price Trend as

$$T(t) = \frac{1}{3.2318} \sum_{k=1}^{10} \frac{P(t - k + 1) - P(t - k)}{P(t - k)} e^{-0.25k}$$

For both the Price Trend and the Valuation variables, tests of robustness have shown similar results with a longer time scale (e.g., 25 days) and different weighting factors [10].

*M2 money supply*

Previous theoretical [11], experimental [12,13] and empirical [10] studies have shown that an increase in the money supply bolsters asset prices. We obtain the weekly M2<sup>2</sup> money data (not seasonally adjusted) for the time period of study from the Federal Reserve website [[www.federalreserve.gov/releases/h6/hist/](http://www.federalreserve.gov/releases/h6/hist/)]. We then performed a linear interpolation to obtain daily data. The M2 Money Supply variable is defined as the relative change in this statistic on a daily basis, i.e.,

$$M2(t) = \frac{M(t) - M(t - 1)}{M(t - 1)}$$

*Volatility*

Volatility is identified with the standard deviation of the return (daily relative stock price change). For our study we compute the standard deviation of the Relative Price Change variable,  $R(t)$ , for two time frames. We consider the Short Term Volatility as the standard deviation over the past eleven (including the current day,  $t$ ) days,  $STV(t) = \text{stddev}(R([t - 10, t]))$  and define the Long Term Volatility over the prior year,  $LTV = \text{stddev}(R([t - 251, t]))$ . To compute the standard deviation we take the square root of the unbiased estimator of the variance:  $\frac{1}{X} \sum_{i=t-X}^{t-1} (R(i) - \text{Mean}(R([t - X, t])))^2$ .

This definition mitigates the effect of the trend in the price on the volatility. It determines the deviation of the relative price change about the growth curve of the share price. So, if the relative price change is constant, then the price is an exponential function of time. Indeed, representing the relative price change in a limiting form such as  $\frac{1}{P} \frac{dP}{dt}$  yields the differential equation  $\frac{1}{P} \frac{dP}{dt} = C$  which implies  $P(t) = Ke^{Ct}$ , where  $K$  and  $C$  are constants. For example, if the stock price follows the pattern  $e^{0.02t}$  (i.e.,  $K = 1$  and  $C = 0.02$ ), then the Relative Price Change would be constant ( $e^{0.02} - 1 \simeq 0.02$ ) and the Volatility would be zero.

*52 week price trend*

As with the Volatility variable, we are not only interested in the short term price trend, but also the longer term trend. As such, we determine the 52 Week Price Trend variable as follows: (i) fit a straight line to the past 252 Relative Price Change values (including the current day  $t$ ), i.e.,  $R([t - 251, t])$ ; (ii) take the slope of this line and multiply it by 252 for conversion to annual units. The resulting value is then denoted  $LT(t)$ .

<sup>2</sup> M2 includes: Currency, Traveler’s checks, demand and other checkable deposits, retail MMMFs (money market mutual funds), savings, and small time deposits. M2 is measured in trillions for this paper.

*Volume trend*

We treat volume and price similarly by considering weighted averages of their relative changes. The Volume Trend variable is thus defined as:

$$VT(t) = \frac{1}{3.2318} \sum_{k=1}^{10} \frac{Vol(t - k + 1) - Vol(t - k)}{Vol(t - k)} e^{-0.25k}$$

where  $Vol(t)$  is the trading volume for day<sup>3</sup>  $t$ .

*Resistance*

Previous studies (see, e.g. [14,15,10]) have considered the effect of proximity to a recent high on the stock price. To explore this further, we include a Resistance variable in the linear regressions. The recent quarterly high is defined by  $H(t) := \max(P(s))$  for  $s$  in  $[t - 63, t - 16]$ . The Resistance Indicator,  $Q(t)$ , is set if the following conditions are satisfied: (i) for  $s$  in  $[t - 15, t - 10]$ ,  $P(s) \leq 0.85H(t)$  and (ii)  $0.85H(t) \leq P(t) \leq H(t)$  (note that there is no condition on  $P(s)$  for  $s$  in  $[t - 9, t - 1]$ ). Thus, we interpret Resistance as having occurred on day  $t$  if the share price on day  $t$  is between 85% and 100% of the recent quarterly high.

*Time*

To ensure our results are not the artifacts of specific time periods, a historical time variable (denoted  $Time(t)$ ) is included as an independent regression variable. This variable represents the approximate number of months from the current record to the earliest record in the data set. It is calculated as follows: (i) determine the number of days from the current day,  $t$ , to the earliest date in the data set, October 26, 1998; (ii) divide this number by 21—the average number of working days in a month; (iii) take the floor of this number, i.e., round down to the nearest integer; and (iv) normalize this variable by dividing by the total approximate number of months in the data set, 143. We also consider the square ( $Time^2(t)$ ) and cube ( $Time^3(t)$ ) of this variable.

**Remarks.** 1. Theoretical studies have used the concept of a declining exponential in gauging investor sentiment and incorporated it into differential equations models (see e.g. [11]). There is experimental evidence [5] that individuals tend to emphasize recent events more heavily than earlier events in their decisions.

2. Traders have explained the concept of resistance by stating that investors who held the stock through the recent high may experience regret at not having reaped a profit. Thus, as the shares again approach this recent high, these investors seek to recover their perceived “paper losses” by selling; thereby lowering the stock price and preventing it from breaking through this price barrier. Recently, the concept of resistance has received academic attention in a study that indicates that the yearly return is influenced by the stock’s proximity to the yearly high [14], though another study [15] demonstrates some limitations of these findings and obtains more mixed results.

**3. Methodology**

The objective of this paper is to determine whether various factors such as Price Trend, Valuation, Money Supply, Volume Trend, and Volatility influence an asset’s Relative Price Change,  $R(t + 1) = \frac{P(t+1)-P(t)}{P(t)}$ , where  $P(t)$  is the asset price on day  $t$ . This is accomplished by executing several linear regressions with the relative price change as the dependent variable and subsets of the other factors as independent variables. The linear regression produces a regression coefficient,  $p$ -value, and  $t$ -value for each independent variable. The regression coefficient provides the “magnitude” of the effect, while the statistical significance ( $p$ -value and  $t$ -value) of the dependent variable determines whether the effect is truly present. This approach, however, has two shortcomings: (i) our data set consists of multiple stocks, each with its own individual attributes, and (ii) the scales of the variables vary over several orders of magnitude (e.g., the mean value for Price Trend is approximately 0.0004 while the mean value for the Volume Trend is 0.21).

Prior studies [10] have shown that the first issue can be circumvented by performing a mixed effects linear regression which accounts for the unique statistical characteristics of each fund.

The second issue can be addressed by standardizing the data, a methodology whereby all independent variables are placed on a common footing to facilitate comparisons of effect [16]. This is accomplished by subtracting the mean and dividing by the standard deviation. This makes the variables and resulting regression coefficients unitless by putting them on the scale of standard deviations. It also facilitates comparison, for example, of a two standard deviation event for one variable compared with another. However, as noted by Bring [17], a drawback of this approach to standardization is that the standard deviation unit may vary across groups—in our case funds within the data set. To mitigate this issue we standardize the coefficients by fund. That is, we compute the mean and standard deviation for each variable by fund, and then standardize each fund’s data with these values. Ultimately, as noted by Cohen et al. [16], the standardized regression coefficient “is often the most useful coefficient for answering questions about the influence of one variable on another, with or without other variables in the equation”.

<sup>3</sup> Note that the Volume may be zero on certain days. This would cause the Volume Trend to be infinite on those days. As such, we did not include any funds with zero Volume in our data set.

All of the independent variables have been standardized with the exception of the Resistance Indicator,  $Q(t)$ , and the time variable,  $\text{Time}(t)$ . The Resistance Indicator is a highly skewed binary variable in that it is either “set” (corresponding to 1) or “not set” (corresponding to 0) and less than 1% of the records satisfy the criteria. Consequently, standardizing this binary variable would distort the results.<sup>4</sup>

The Time variables are included in regressions to determine if the historical periods have an effect that could be disguised as one of the other variables. For example, Regression 2 below includes the same variables as Regression 1, but also incorporates the Time variables. Thus, any discrepancies in the output for the variables common to both regressions are due to the inclusion of the Time variables. While we are concerned with the statistical significance of the Time variables, their coefficients are not directly relevant to this study. Also, the Time variable is already scaled between zero and one. As such, we do not standardize this variable.

When utilizing standardized coefficients, one has the option to either standardize or not standardize the **dependent** variable. We choose not to standardize the Relative Price Change to facilitate the interpretation of results. For example, Regression 1 below shows the Price Trend variable to have a regression coefficient of 0.0012. In other words, a positive one standard deviation change in the recent trend will yield, on average, a 0.12% positive change in the daily relative price change (i.e., the Relative Price Change variable).

#### 4. Results

We perform a set of regressions, each one of the form:

$$R(t + 1) = \beta_0 + \sum_{i=1}^n \beta_i x_i$$

where  $x_i$  is one of the above independent variables or a product of these variables. Since the variables have been standardized (as discussed above),  $\beta_i$  may be interpreted as the standardized regression coefficient determined by the mixed effects linear regression. The intercept term,  $\beta_0$ , is present as we do not standardize the dependent variable, and it may be interpreted as the “drift” of classical finance. In other words, this is the average relative daily price change. The dependent variable is evaluated at day  $t + 1$  which indicates that it represents the following day's relative price change or return.

Note that for each regression the total number of observations is 80,351 corresponding to 108 funds. The number of observations included in the regressions does not equal the total number of records in the data set because the calculations for some (long term) variables required data from the previous year. For example, the computation of the Long Term Price Trend and Long Term Volatility records required the previous 251 data points.

*Regression 1.* We consider the Relative Price Change regressed against all of the above mentioned variables with the exception of the Time variables. This regression has the form:

$$R(t + 1) = \beta_0 + \beta_1 T(t) + \beta_2 D(t) + \beta_3 M2(t) + \beta_4 STV(t) + \beta_5 LTV(t) + \beta_6 LTT(t) + \beta_7 VT(t) + \beta_8 Q(t).$$

The Valuation, Price Trend, Money Supply, Short Term Volatility, and Volume Trend terms are all highly statistically significant with positive coefficients. The coefficient of Price Trend indicates that a one standard deviation change in the Price Trend will induce a 0.12% change in the Relative Price Change in the same direction, i.e., if the Price Trend increases, then so will the Relative Price Change (provided all other independent variables are unchanged). The standardization allows a comparison of impact of the different independent variables. Thus, the Price Trend coefficient is approximately one half the magnitude of the Valuation coefficient. The M2 Money Supply variable has a coefficient that is approximately half the magnitude of the Price Trend. This supports theoretical [11], experimental [12,13] and empirical [10] studies which suggest that an influx of cash will bolster the trading price. These studies have helped in resolving the paradox of experimental bubbles (see, e.g. [18,19]). The volatility variables are quite interesting in that both are statistically significant, but while the Long Term Volatility has a negative coefficient, the Short Term Volatility coefficient is positive. Thus, recent volatility in the fund price tends to raise the price, while longer term volatility has a negative effect on the price. The Long Term Trend variable is only marginally statistically significant. However, its small negative coefficient agrees with the findings of Poterba and Summers [2] of stock price regression to the mean over longer time frames. The Volume Trend is also statistically significant with a positive coefficient that is approximately one quarter the magnitude of the Price Trend coefficient. This confirms a widely held belief among traders that rising volume is associated with rising prices. Finally, we note that the Resistance variable has a negative coefficient which indicates that when the Resistance criteria are satisfied, the price is

<sup>4</sup> Gellman [21] notes that we run the risk of overstating the importance of such a binary variable relative to the other standardized variables if our standardization procedure only divides by one standard deviation. He considers the possibility of division by two standard deviations. He indicates that a binary variable with equal probabilities has a mean of 0.5 and a standard deviation of 0.5. Then, the difference between a “0” and a “1” on the original (unstandardized) scale is actually 2 standard deviations. Now, suppose the binary variable is “highly skewed”, which our Resistance variable is—only approximately 1% of the records meet the criteria (so, the mean is 0.01 and the standard deviation is approximately  $(0.01 * 0.99)^{\frac{1}{2}}$ ). Then, the difference between a “0” and a “1” on the original (unstandardized) scale is actually approximately 9.8 standard deviations. Thus, Gelman suggests that by not standardizing the Resistance variable, we might actually overstate its importance because standardization in this case will actually yield larger values, which in turn will result in smaller regression coefficients.

**Table 1**  
Regression 1 results.

Term	Value	Std. error	t-value	p-value
(Intercept)	0.000417343	0.0000478596	8.72015	<0.0001
Price Trend	0.001184948	0.0000578973	20.46639	<0.0001
Valuation	0.002658618	0.0000547312	48.57592	<0.0001
M2 Money Supply	0.000470137	0.0000478630	9.82257	<0.0001
Short Term Volatility	0.000520087	0.0000555387	9.36440	<0.0001
Long Term Volatility	−0.000138966	0.0000530404	−2.62000	0.0088
Long Term Trend	−0.000050647	0.0000539793	−0.93827	0.3481
Volume Trend	0.000310028	0.0000481547	6.43816	<0.0001
Resistance	−0.000708118	0.0006023445	−1.17560	0.2398

Degrees of freedom<sup>5</sup>: 80,235.

**Table 2**  
Regression 2 results.

Term	Value	Std. error	t-value	p-value
(Intercept)	0.00137697	0.000378576	3.63724	0.0003
Price Trend	0.00115478	0.000057919	19.93789	<0.0001
Valuation	0.00262333	0.000054807	47.86512	<0.0001
M2 Money Supply	0.00047234	0.000047831	9.87515	<0.0001
Short Term Volatility	0.00061354	0.000056140	10.92873	<0.0001
Long Term Volatility	0.00014445	0.000060335	2.39407	0.0167
Long Term Trend	−0.00014979	0.000054925	−2.72721	0.0064
Volume Trend	0.00031614	0.000048187	6.56077	<0.0001
Resistance	−0.00066603	0.000602981	−1.10456	0.2694
Time	−0.01588736	0.002819201	−5.63541	<0.0001
Time <sup>2</sup>	0.04328314	0.005828924	7.42558	<0.0001
Time <sup>3</sup>	−0.02985772	0.003508966	−8.50898	<0.0001

Degrees of freedom: 80,232.

pushed downward. However, the statistical significance of this variable is marginal, probably due to the small number of records in the data set that met these criteria (518 out of 80,351). The Intercept term is both statistically significant and positive (Table 1).

Regression 2. The Time variables are included to ascertain whether any of the results from Regression 1 are artifacts of a particular era. This regression has the form:

$$R(t + 1) = \beta_0 + \beta_1T(t) + \beta_2D(t) + \beta_3M2(t) + \beta_4STV(t) + \beta_5LTV(t) + \beta_6LIT(t) + \beta_7VT(t) + \beta_8Q(t) + \beta_9Time(t) + \beta_{10}Time^2(t) + \beta_{11}Time^3(t).$$

Each Time variable is statistically significant (Table 2). The inclusion of these variables had little effect on the coefficients of the Price Trend, Valuation, M2 Money Supply, and Resistance (see Table 3) and slightly more impact on the Short Term Volatility coefficient. However, the Long Term Volatility and Long Term Trend terms were significantly impacted by the inclusion of the Time variables. This is to be expected as the definition of these variables includes data for the entire previous year. Furthermore, the data set only includes 10 years of data, which essentially amounts to 10 data points. Hence, a few years in which the broad market experienced a highly volatile year followed by a less volatile year, but one in which stock prices are increasing, could explain the change in sign (from negative to positive) of the Long Term Volatility coefficient. The Long Term Trend variable also changed from not statistically significant (p-value of 0.3481) to statistically significant (p-value of 0.0064) with a small negative effect.

Regression 3. In order to explore the nonlinearity in the Price Trend and Valuation variables, the interactions (multiplicative products) up to third order of these variables are included in the regression. Regression 6, which is presented in the Appendix, includes the additional variables that we exclude here for simplicity. There are relatively minor differences between the two regressions. Note that the Price Trend and Valuation variables are standardized (in both regressions), while the interactions are products of the standardized variables (i.e., the interaction variables are not standardized). This regression has the form (Table 4):

$$R(t + 1) = \beta_0 + \beta_1T(t) + \beta_2D(t) + \beta_3T^2(t) + \beta_4T^3(t) + \beta_5D^2(t) + \beta_6D^3(t) + \beta_7T(t)D(t) + \beta_8T^2(t)D(t) + \beta_9T(t)D^2(t).$$

<sup>5</sup> The number of degrees of freedom for each regression is dependent upon the number of independent variables. As such, this statistic is included for each regression.

**Table 3**  
Comparison of coefficients between Regressions 1 and 2.

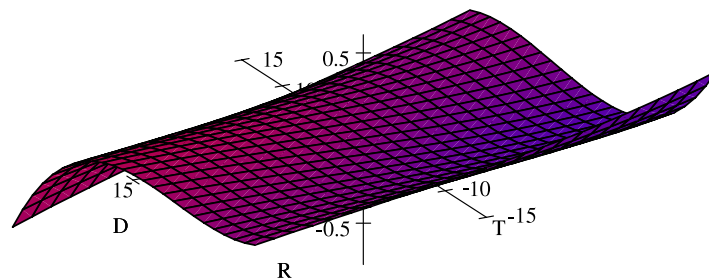
Variable	Regression 1 coefficient	Regression 2 coefficient	% Change
(Intercept)	0.000417343	0.00137697	229.94
Price Trend	0.001184948	0.00115478	-2.55
Valuation	0.002658618	0.00262333	-1.33
M2 Money Supply	0.000470137	0.00047234	0.47
Short Term Volatility	0.000520087	0.00061354	17.97
Long Term Volatility	-0.000138966	0.00014445	-203.95
Long Term Trend	-0.000050647	-0.00014979	195.75
Volume Trend	0.000310028	0.00031614	1.97
Resistance	-0.000708118	-0.00066603	-5.94
Time	NA	-0.01588736	NA
Time <sup>2</sup>	NA	0.04328314	NA
Time <sup>3</sup>	NA	-0.02985772	NA

The relative change in magnitudes of the Intercept, Long Term Volatility, and Long Term Trend coefficients are significant (greater than 20%). Also, note that the Long Term Volatility coefficient changed from negative to positive.

**Table 4**  
Regression 3 results.

Term	Value	Std. error	t-value	p-value
(Intercept)	0.000265656	0.00005434210	4.88858	<0.0001
Price Trend	0.001668129	0.00006798784	24.53570	<0.0001
Valuation	0.002911161	0.00006232596	46.70865	<0.0001
Price Trend <sup>2</sup>	-0.000117148	0.00003880772	-3.01868	0.0025
Price Trend <sup>3</sup>	-0.000105260	0.00001123276	-9.37080	<0.0001
Valuation <sup>2</sup>	0.000064430	0.00001803873	3.57174	0.0004
Valuation <sup>3</sup>	-0.000007622	0.00000160542	-4.74795	<0.0001
Price Trend * Valuation	-0.000268997	0.00004854760	-5.54089	<0.0001
Price Trend <sup>2</sup> * Valuation	-0.000087231	0.00001771199	-4.92498	<0.0001
Price Trend * Valuation <sup>2</sup>	-0.000008461	0.00000977158	-0.86592	0.3865

Degrees of freedom: 80,234.



**Fig. 1.** The plot was produced using the coefficient values from Regression 3. These coefficients define a cubic polynomial in two variables, Price Trend (*T*) and Valuation (*D*). The surface describes the effect on the following day's Relative Price Change (*R*), and exhibits the nonlinear relationship between *D*, *T* and *R*. Overreaction is evident as a large positive trend can result in a negative Relative Price Change (analogously for negative trend). The precise point at which the magnitude changes sign depends nonlinearly on the valuation.

With the exception of the interaction of Price Trend with the square of the Valuation, all of the independent variables are statistically significant. Using these results, we define a function representing the Relative Price Change as a function of the Price Trend and Valuation:

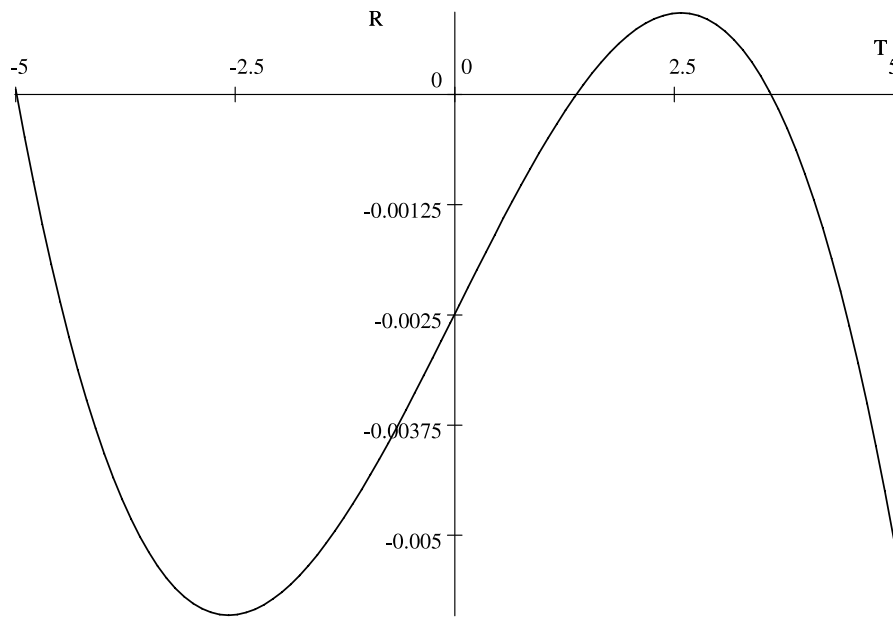
$$R(T, D) = 0.0003 + 0.0017T + 0.0029D - 0.0001T^2 - 0.0001T^3 + 0.0001D^2 - 0.00001D^3 - 0.0003TD - 0.0001T^2D - 0.00001TD^2.$$

Plotting this function yields the three dimensional surface depicted in Fig. 1.

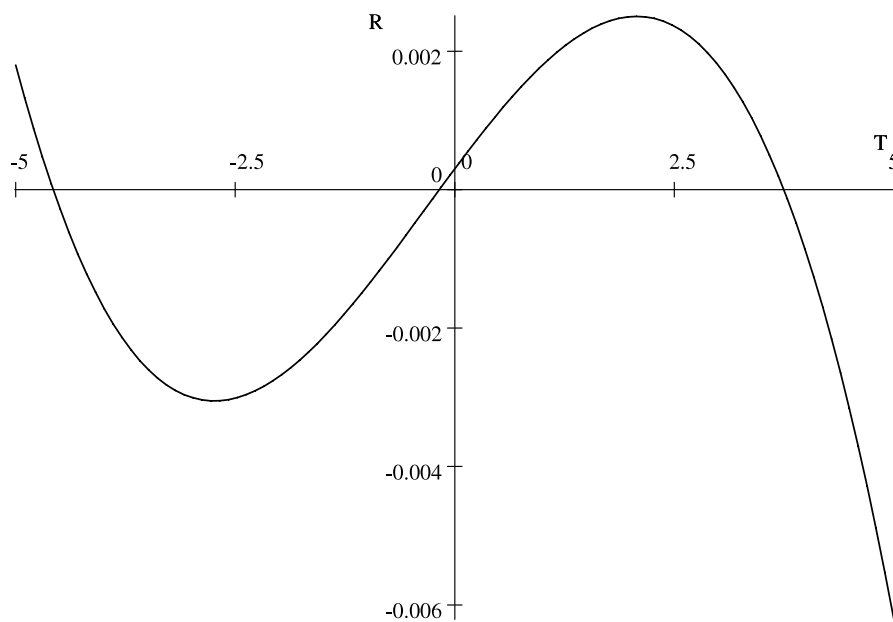
To better understand the nonlinear relationship between Price Trend and Valuation we consider the cross-sections of the above graph for  $D = -1, 0,$  and  $1$ .

$$R(T, -1) = 0.0003 + 0.0017T + 0.0029(-1) - 0.0001T^2 - 0.0001T^3 + 0.0001(-1)^2 - 0.00001(-1)^3 - 0.0003T(-1) - 0.0001T^2(-1) - 0.00001T(-1)^2.$$

Fig. 2 displays the cross-section of the surface in Fig. 1 with the valuation held constant at  $D = -1$ . This cubic function intersects the *T*-axis at  $T = -4.9891, 1.3847,$  and  $3.6044$ . So, for a negative one standard deviation change in the Valuation variable, the Relative Price Change will also be negative for  $-4.9891 < T < 1.3847$  and  $T > 3.6044$ . But, for  $1.3847 < T < 3.6044$ , the Relative Price Change will be positive implying that the Price Trend will have a greater impact



**Fig. 2.** With the Valuation held constant at  $-1$ , the Relative Price Change may be represented as a cubic function of the Price Trend variable,  $T$ . For a large enough positive change in the Price Trend,  $T > 3.6044$ , the next day's Relative Price Change is negative. Analogously, for a Price Trend  $< -4.9891$  the next day's Relative Price Change is positive.

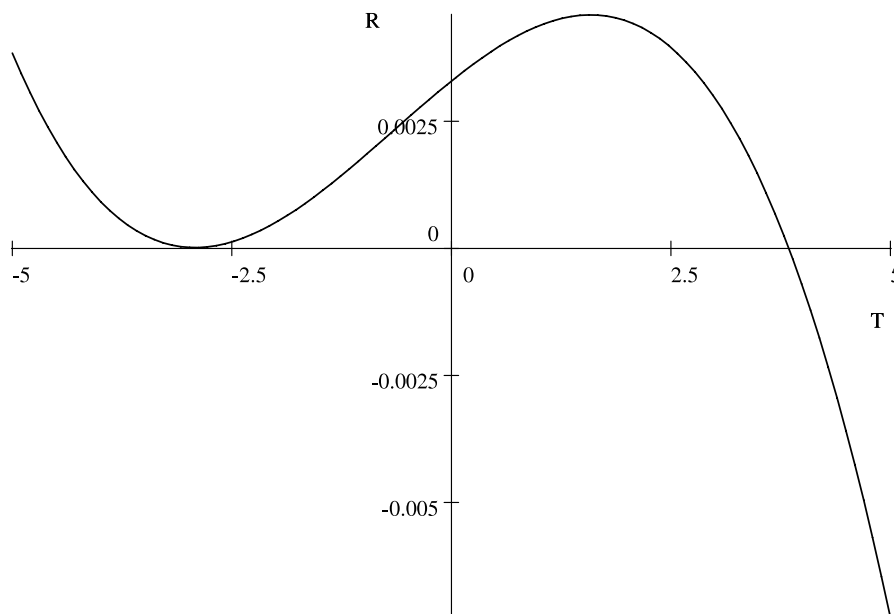


**Fig. 3.** A cross-section of Fig. 1 is represented with the Valuation variable held constant,  $D = 0$ . As in Fig. 2, overreaction is present if the Price Trend variable,  $T$ , is large enough in magnitude, i.e., for  $T > 3.7486$  the next day's Relative Price Change is negative, while for  $T < -4.5736$ , the next day's Relative Price Change is positive. Hence an uptrend has a positive influence on price change for  $T$  satisfying  $-4.5736 < T < 3.7486$ .

on the price than the Valuation. Approximately 92% of the Price Trend values will actually be less than 1.3847 standard deviations.

$$R(T, 0) = 0.0003 + 0.0017T + 0.0029(0) - 0.0001T^2 - 0.0001T^3.$$

Fig. 3 corresponds to the cross-section of Fig. 1 with  $D = 0$ . If the Price Trend is positive, then the Relative Price Change is also positive up to 3.7486 standard deviations. However, if the Price Trend is greater than 3.7486 standard deviations, the Relative Price Change is negative. Thus, we see that a large (and unusual) change in the Price Trend produces a negative Relative Price Change supporting the theory of overreaction. Similarly, a negative Price Trend yields a negative Relative Price Change unless the change in the Price Trend is less than  $-4.5736$  standard deviations. Thus, there is evidence for underreaction for  $-4.5736 < T < 3.7486$  (approximately 99.9921% of the time) and evidence for overreaction when large (either positive or negative) changes in the Price Trend occur.



**Fig. 4.** A cross-section of Fig. 1 is represented with the Valuation variable held constant,  $D = 1$ . Thus, we have the next day's Relative Price Change,  $R$ , as a function of the Price Trend,  $T$ . If  $T > 3.8435$ , then the next day's Relative Price Change is negative, thereby providing evidence to support overreaction in stock prices. However, notice that for  $T < 3.8435$ , the Relative Price Change is always positive. Thus, a one standard deviation change in the Valuation variable is large enough to counteract the effects of any negative Price Trend.

Part of the curve lies above the negative  $T$  axis for  $-0.17498 < T < 0$  due to the drift (intercept) term from the regression.

$$R(T, 1) = 0.0003 + 0.0017T + 0.0029(1) - 0.0001T^2 - 0.0001T^3 + 0.0001(1)^2 - 0.00001(1)^3 - 0.0003T(1) - 0.0001T^2(1) - 0.00001T(1)^2.$$

With the Valuation fixed at 1 in Fig. 4, we see that the cross-section only crosses the  $T$ -axis once at  $T = 3.8435$ . Thus, a negative Price Trend of any size does not have enough of an impact on the Relative Price Change to counteract the positive one standard deviation change in Valuation. However, a positive change in Price Trend of more than 3.8435 standard deviations results in a negative Relative Price Change and exhibits overreaction.

Fig. 5 depicts the relationship between Relative Price Change and Valuation (holding Price Trend constant). This relationship is essentially linear suggesting that an increase in value is always positive for stock prices:

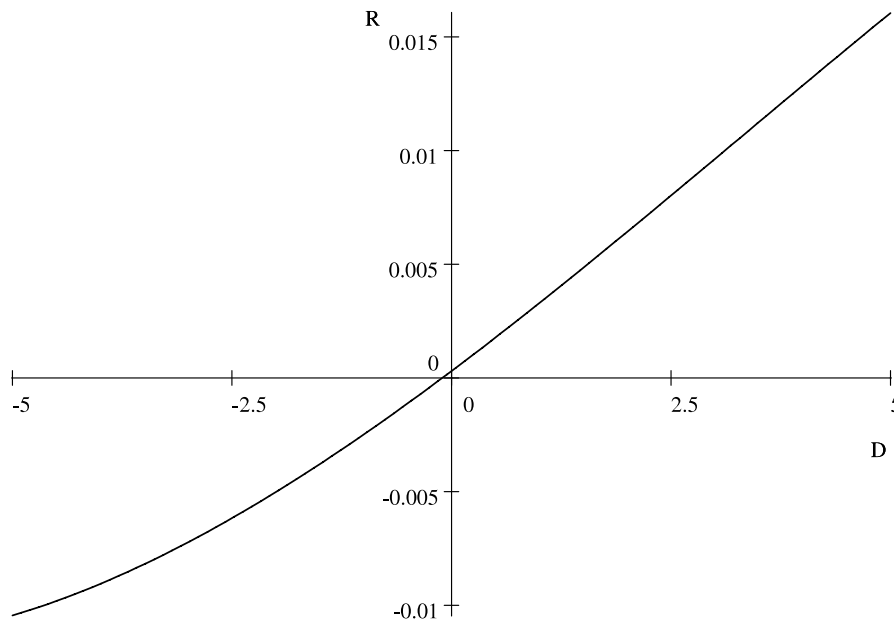
$$R(0, D) = 0.0003 + 0.0017(0) + 0.0029D - 0.0001(0)^2 - 0.0001(0)^3 + 0.0001D^2 - 0.00001D^3 - 0.0003(0)D - 0.0001(0)^2D - 0.00001(0)D^2.$$

Fig. 6 displays a contour plot of the Relative Price Change function,  $R(T, D)$ , for  $R = -0.01, 0$ , and  $0.01$ , which correspond to relative price changes of  $\pm 1\%$  and  $0\%$ . These provide an intriguing view of the nonlinear relationship between the Valuation and Price Trend variables.

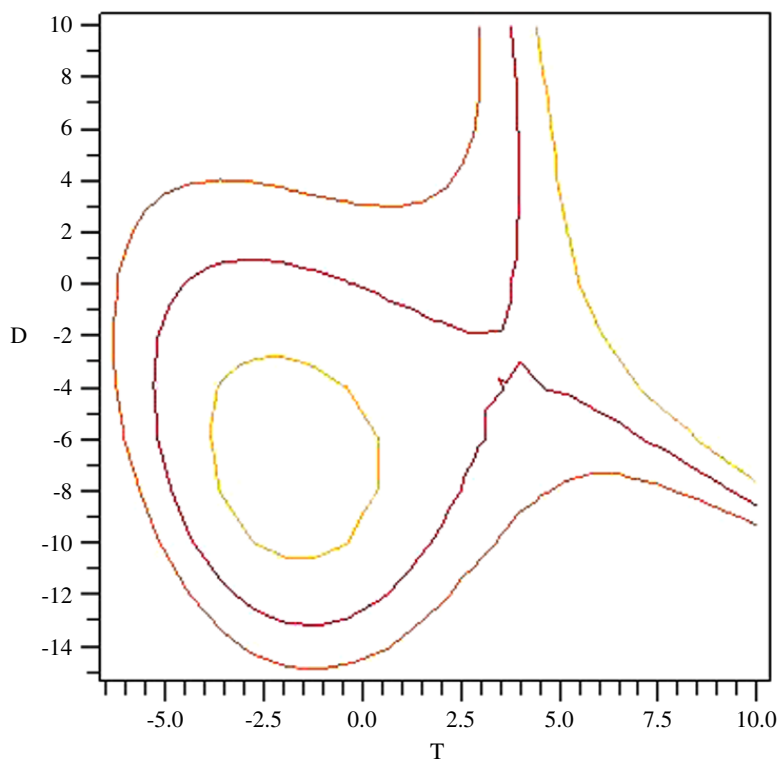
### 5. Conclusion

We have presented an empirical methodology that is capable of testing almost any quantitative hypothesis involving dynamics of asset prices. We have utilized the mixed effects regressions on a set of independent variables: valuation, short and long term trends, short and long term volatility, the M2 money supply, volume trend and resistance. The results exhibit strong statistical support for the assertion that the short term price trend is a factor that tends to increase trading prices. The magnitude of this effect is almost half of that for valuation. Unlike some previous studies in which raw data were analyzed, displaying a tiny effect for trend, our study shows that the effect of the trend is very important. This is largely due to a methodology in which the changes in valuation (amounting to noise) are considered in a multi-regression in an appropriate form. Similarly, positive statistically significant coefficients were found for short term volatility, volume trend and the money supply. The latter confirms the assertions of the asset flow theory (supported by experiments and empirical studies cited in Section 2) that additional cash fuels trading price increases.

The positive coefficient obtained for short term volatility is surprising in the context of classical finance since the inverse risk–reward relationship stipulates that high volatility should be interpreted as greater risk that would diminish the price that traders would pay for the stock. The positive coefficient for short term volatility may be explained by the hypothesis that traders are attracted to higher volatility as it offers the opportunity for greater profits. As more capital is attracted, the increased level of cash (as shown with the money supply and prior studies) would tend to bolster prices. The role of long term volatility is quite small compared to trend. Also, it is more ambiguous and complicated. While we obtain a negative



**Fig. 5.** A cross-section of Fig. 1 is represented with the Price Trend variable held constant,  $T = 0$ . This shows that the Relative Price Change variable,  $R$ , is essentially linear in the Valuation variable,  $D$ .



**Fig. 6.** Using the coefficient values from Regression 3, one can express the following day's Relative Price Change,  $R$ , as a cubic function of two variables, the Price Trend,  $T$ , and the Valuation,  $D$ . This figure is a contour plot of this function. The curves represent level sets on which  $R$  is held fixed at  $\pm 1\%$  and  $0\%$ . The innermost curve, i.e. the union of the inner loop and the curve closest to the right side of the frame, corresponds to  $R = -1$ , the middle curve to  $R = 0$ , and the outermost curve to  $R = 1$ .

coefficient (provided the historical time is not taken into account) consistent with the expectations of classical finance, as shown in Regression 1, we also find that the coefficient is positive in Regression 2, where the time variables are included. However, the coefficient has only about one-tenth of the magnitude of the short term trend. A basic classical finance tenet is that investors are rational and seek to avoid risk and increase return (see e.g. [20]). This is followed by the assumption that risk can be identified with volatility. Our study shows that the only interpretation in which this classical concept is upheld is that a time period exhibiting high volatility is followed by a period of slightly more negative price changes.

**Table 5**  
Regression 4 results.

Term	Value	Std. error	t-value	p-value
(Intercept)	0.0004126711	0.00004845682	8.516265	<0.0001
Price Trend	−0.0001618137	0.00004847659	−3.337976	0.0008

Degrees of freedom: 80,242.

The methodology uses a technique of standardizing the data prior to executing the linear regressions. This has the salient feature that it allows direct comparison of the distinct hypothesized factors. In particular, it allows us to go beyond the question of whether these effects are actually present to the possibility of quantifying, for the first time, their relative importance. In particular the aggregate effect of short term trend, short term volatility and M2 money supply is comparable to that of the valuation.

Another feature of our methodology is that the data are standardized with respect to individual stocks. This tends to mitigate the distortion introduced by the large variations in magnitudes of the independent variables. As a practical application, one should be able to obtain more accurate results by performing the regressions on only one stock of interest at each time. Although the statistical significance and scientific impact would be diminished, the practical results for prediction would be enhanced.

While our sample is inadequate to make a strong statistical assertion on resistance (the tendency for prices to move down while nearing a recent high), the fact that the size of the coefficient is comparable to most of the other variables (except short term trend and valuation) suggests that a larger sample could result in establishing this as an important factor.

The growing evidence for factors influencing asset market dynamics may appear, upon cursory analysis, to be contradictory. For example, there are studies that demonstrate the presence of underreaction, exhibited by the continuation of a price trend. There are also studies showing that overreaction is present as prices reverse course. The statistical analysis involving 111,356 data points supports our assertion that the presence of both under- and overreaction is a manifestation of the underlying nonlinearity of trader motivations. In particular, Fig. 1, which displays the daily return as a function of trend and valuation, shows that while an uptrend is positive for stocks in one region (e.g., the uptrend is not too large and the valuation range is far from zero), it may be negative in another region (e.g., when the trend is very large or the valuation change is large). Thus, nonlinearity is the key to understanding competing motivations.

By incorporating the squared and cubic price trend and valuation terms as well as the interactions (up to third order) of these two key variables, we are able to express the relative price change as a nonlinear function of the price trend and valuation. Plotting this function in various ways (3d and 2d with valuation constant, as well as level sets) illuminates the nonlinear relationship between these variables and renders a quantitative and empirical explanation for how the competing effects of overreaction and underreaction can coexist within the same data set. For example, it yields the precise information that a positive change in valuation at the level of one standard deviation will not be counteracted, on average, by any short term trend. However, a similar negative change in valuation will be balanced by a positive trend at the level of 1.38 standard deviations.

**Appendix. Additional regressions**

As noted above, the key to obtaining meaningful coefficients for the independent variables involves formulating a suitable definition for the valuation. To illustrate this point, suppose that we perform a linear regression in the manner of most financial studies, i.e., without making any attempt to subtract out the valuation. If we consider the most significant of the remaining variables, namely, Price Trend, then we obtain the relation below.

Regression 4. As a baseline we consider the linear regression with the single independent variable, Price Trend:

$$R(t + 1) = \beta_0 + \beta_1 T(t).$$

The regression results are included in Table 5.

From this we see that although the Price Trend is statistically significant, the coefficient has one-tenth of the magnitude of the previous regressions, and the opposite sign. Without accounting for changes in valuation, a one standard deviation change in the Price Trend variable corresponds to a change of −0.00016 in the Relative Price Change.

Regression 5. By incorporating the Valuation variable in the regression, namely,

$$R(t + 1) = \beta_0 + \beta_1 T(t) + \beta_2 D(t),$$

we can readily demonstrate that the true effect of the Price Trend is extracted. Indeed the regression results are summarized in Table 6.

Hence, the Price Trend is now much more significant (t-value 20.25 versus −3.34), positive, and approximately 10 times larger in magnitude. By accounting for the Valuation, we find that the trend in price is statistically significant and has roughly half the effect of Valuation on the Relative Price Change.

From these two regressions, we can conclude that ignoring the changes in value (as most studies have done in the past) leads to coefficients for Price Trend that are essentially useless in terms of practical trading.

**Table 6**  
Regression 5 results.

Term	Value	Std. error	t-value	p-value
(Intercept)	0.000412679	0.00004775848	8.64096	<0.0001
Price Trend	0.001103913	0.00005451764	20.24873	<0.0001
Valuation	0.002630504	0.00005452487	48.24412	<0.0001

Degrees of freedom: 80,241.

**Table 7**  
Regression 6 results.

Term	Value	Std. error	t-value	p-value
(Intercept)	0.00139316	0.000378472	3.681	0.0002
Price Trend	0.00168829	0.00007044	23.96784	<0.0001
Valuation	0.00290006	0.0000625270	46.38096	<0.0001
M2 Money Supply	0.00047243	0.0000477520	9.89339	<0.0001
Short Term Volatility	0.00057453	0.0000591470	9.71363	<0.0001
Long Term Volatility	0.00015736	0.0000602920	2.60991	0.0091
Long Term Trend	-0.0001755	0.0000548500	-3.19962	0.0014
Volume Trend	0.00026413	0.0000487820	5.41459	<0.0001
Price Trend <sup>2</sup>	-0.00022655	0.0000413210	-5.48258	<0.0001
Price Trend <sup>3</sup>	-0.00012016	0.0000114110	-10.53101	<0.0001
Valuation <sup>2</sup>	0.00005617	0.0000180640	3.10918	0.0019
Valuation <sup>3</sup>	-0.0000079	0.0000016040	-4.92458	<0.0001
Price Trend * Valuation	-0.00026923	0.0000488830	-5.50767	<0.0001
Price Trend <sup>2</sup> * Valuation	-0.00009215	0.0000177630	-5.18788	<0.0001
Price Trend * Valuation <sup>2</sup>	-0.00000651	0.0000097640	-0.66662	0.505
Time	-0.01582821	0.0028159320	-5.62095	<0.0001
Time <sup>2</sup>	0.04268599	0.0058229880	7.3306	<0.0001
Time <sup>3</sup>	-0.02936503	0.0035057630	-8.37622	<0.0001

Degrees of freedom: 80,226.

**Regression 6.** To further explore the nonlinear relationship between Price Trend and Valuation we consider a regression that incorporates all of the significant variables from Regression 2 as well as the Price Trend and Valuation interaction terms. This regression has the form:

$$R(t + 1) = \beta_0 + \beta_1 T(t) + \beta_2 D(t) + \beta_3 M2(t) + \beta_4 STV(t) + \beta_5 LTV(t) + \beta_6 LTT(t) + \beta_7 VT(t) + \beta_8 T^2(t) + \beta_9 T^3(t) + \beta_{10} D^2(t) + \beta_{11} D^3(t) + \beta_{12} T(t)D(t) + \beta_{13} T^2(t)D(t) + \beta_{14} T(t)D^2(t) + \beta_{15} Time(t) + \beta_{16} Time^2(t) + \beta_{17} Time^3(t).$$

Comparing the results in Table 7 with those from Regression 3, we find that the Intercept, Price Trend<sup>2</sup>, and Price Trend \* Valuation<sup>2</sup> terms are the only variables with significant (i.e., greater than 20%) relative changes in the magnitudes of their coefficients.

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