Trend-Based Asset Flow in Technical Analysis and Securities Marketing

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ABSTRACT

This paper generalizes the asset flow model of the dynamics of equity prices to multiple groups of investors with distinct strategies and assesements of value. Applications include the closed end fund puzzle, government privatizations, marketing of initial and secondary offerings of equities. The generalized model is used to provide a theoretical foundation for the practice of technical analysis, in which price history and patterns are examined in order to obtain an indication of future prices.

The asset flow model, which is an extension of price adjustment due to disequilibria, tracks the finite assets of each group and involves a preference function that is governed by the price trend in addition to the fundamental value of the equity. The system, which consists of a system of ordinary differential equations, uses four parameters that characterize the extent to which investors' preferences are governed by trend versus fundamental value, and the time scales associated with each motivation. The evolution toward equilibrium if found to be much more complex than the monotonic change that is implied by standard price theories. Finally, the time scale for the return to equilibrium, a concept crucial to securities marketing, is considered in a precise quantitative context.

The asset flow approach provides a unified explanation for many of the basic pattern of technical analysis and market phenomena. In particular, the origin of a typical bubble can be explained on the basis of trend based investors entering a market hitherto dominated by value based investors. Thus, a channel of increasing prices yields to a "breakout from a trendline." The formation of triangle patterns in prices has its origin in the emergence of a second group of investors, with a different assesement of value, that offers fresh supply or demand near a particular price. The paper also considers applications to the marketing of securities, consumer preferences which are logically influenced by the popular trend (e.g. VHS versus Beta, or long-distance telephone companies) and (particularly three way) elections.

(sent to Beil Dec 5, 1995) **1. INTRODUCTION**

Nonclassical Phenomena in Equities Markets

The theories of financial markets that have largely dominated the academic studies of financial markets are generally based on the concept of random price fluctuations about a fundamental value [e.g., Fama (1970), Tirole (1982)]. A theoretical basis for this idea is that any deviation from a realistic value is reason for investors to purchase (due to undervaluation) or sell short (due to overvaluation) in the expectation that a profit can be made as other investors follow suit. The following three prerequisites appear to be part of any microeconomic model of this concept: (i) There are a group of investors with similar assessments of the value of the security. (ii) These investors have capital that exceeds those of less informed. (Or, alternatively they manage the capital of less informed investors who have learned of their investing skill through a perfectly accurate representation of managers' likely performance.) (iii) These investors not only want to maximize their returns but are also willing to rely on the self-interested maximization of others.

It will be evident in some of the examples discussed below that (iii) is a crucial component in that the deviation from true value typically does not in itself guarantee a profit in an equity. An overvalued security can become more overvalued and the short position must be covered before it exceeds margin requirement. The investor who shorts 1000 shares of a $1 stock is forced to buy back those shares (and contribute to further price elevation) if the stock subsequently trades at $2, or put up additional cash (under current rules). Consequently, taking such a short position involves relying upon other investors to take similar actions in self-interested maximization.

Although it would appear that investors would implement such a strategy, the laboratory experiments of Beard and Beil (1994) – to be discussed in Section 2 – indicate a reluctance of players to rely on the self-interested maximization of others even while they engage in self-interested optimization themselves, though they become more willing to do so when the cost of not maximizing becomes very high for the other player.

This line of reasoning is compatible with a formal analysis of "latent assets" by Brennan (1990) who discusses the value of information that is known by only part of the investors. He concludes (p. 728) that "in a market of small investors the rewards of information collection will depend in a non-monotonic fashion on the number of investors who collect information, since this will affect not only the frequency with which mispriced assets can be discovered, but also the speed at which the pricing discrepancies are eliminated and their discoverers rewarded". The time scale on which a return to fundamental value occurs is of paramount concern to marketing and investing in equities, since it affects the total return. The asset flow approach presented in this paper and its predecessors is an effort to address this issue in a quantitative setting.

The variation in the degree of validity conditions (i) - (iii) in different markets leads to the question of the *extent* to which markets are near fundamental value, or semi-strong efficient, meaning that public information cannot lead to profitable trading. A related question is the time scale on which a return to equilibrium occurs.

As with most idealizations, the efficient market hypothesis (in each of its forms) varies in the extent of its applicability depending on the validity of the underlying assumptions. In the Treasury bill market, for example, conditions (i) and (ii) appear to be valid due to the huge size of the market, the broad participation of financial institutions and the clear nature of the fundamental value. The validity of (iii) can be expected in this market since a player who buys a six month Treasury bill, even though he may need the money in one month, is relying on the self-interested maximization of other players who would be sacrificing a large and *certain* profit by not purchasing a discounted bill with five months left to maturity. At the opposite extreme would be the low capitalization stocks, emerging markets, etc., where bubbles and other deviations from fundamental value seem to occur most frequently. The low level market capitalization and trading volume tends to deter larger investors since the total potential profit, which may be large by percentage measurements, is small in comparison with the investor's assets, and, moreover, the profits may be very difficult to realize due to the limited liquidity. Relying upon the self-interested maximization of others apears to be most difficult when there is much uncertainty about their potential profits and even about the *existence* of such players!

We present below some recent examples of market phenomena that are not explained by random fluctuations about fundamental value.

(i) Many, though not all, common stocks of bankrupt companies are worthless, since the common shareholders legally are generally the last in a long line of creditors for whom there is insufficient funds. Usually even the price of the senior debt reflects a market expectation that the bond holders will receive almost nothing while the common stock, which will receive much less, continues to trade as though it had significant value. In some cases, e.g. Continental Air in 1992, the common stock had already been pronounced worthless by a bankruptcy court judge in accepting a reorganization plan. Nevertheless, the stock continued to trade and rally for weeks after the decision.

(ii) In some cases, large capitalization stocks that have been privatized by a government have risen to a market valuation that cannot be explained on a fundamental basis. For example, in 1989, as the Japanese government privatized the phone company, Nippon Telephone and Telegraph Co. (NTT) attained a market value that exceeded the entire West German market. The theories of efficient markets are not only confronted with the puzzle of valuation but of the possible influence of the sale process on the price.

(iii) Many of the world's markets have suffered severe drops that appear to exceed the change in fundamental value after long trends that exaggerated value. In addition to NTT, many other Japanese stocks soared to excess valuation by the end of 1989, and the Nikkei lost half of its value in eight months in 1990. Similarly Taiwan had seen the price/earnings ratios of many companies reach triple digits until the Weighted Index lost over 80% of its value in eight months.

(iv) Stock market crashes such as those of 1987, 1929 and the mini-crash of 1989 and subsequent rebounds appear to be inexplicable from a valuation perspective [see e.g. Kampuis (1989)] as is the large selloff in derivative and mortgage-backed securities in early 1994. The 1929, 1987 and derivative crash of 1994 may all be viewed as an exaggerated market response prompted by the Federal Reserve's rate increase and aggravated by the increased supply in the midst of declining demand for the securities.

(v) A puzzling phenomenon among closed-end funds is the large premium (50% to 100%) that has occurred particularly among country funds in which generated a high degree of investor interest.

(vi)The persisent discount from net asset value that afflicts the average closed-end fund (about 12% in January 1995) has been a puzzle for both practical and academic observers, since investors with cash can attempt to maximize their returns by buying $100 of stock for $88. Some recent articles have debated whether the origin of this discount is related to investor sentiment [Anderson and Born (1992), Lee *et al* (1991), Chen *et al* (1993), Chopra *et al* (1993)]

(vii) Secondary offerings of stock in major companies and closed end funds have resulted in lowering prices according to reliable statistical studies on Swiss stocks [Loderer and Zimmermann 1985] and common experience on Wall Street. Underwriting specialists generally acknowledge that additional supply of a closed-end fund or common stock will "weigh" on the market and attempt to estimate the amount of new supply that the market will bear. However, the efficient market hypothesis predicts that other investors would eliminate any discount by purchasing the shares. The removal of stock has essentially the opposite effect. Clearly, the efficient market is contingent upon a large supply of additional funds in these situations.

(viii) Many large companies hire advertising agencies to promote their stock in order to lift the share price and have done studies that measure a positive effect. According to the standard price theories this should have no effect.

These real world market examples are complemented by the controlled laboratory experiments such as those of Smith (1982), Plott and Agha (1982), Forsythe , Palfrey and Plott (1982), Porter and Smith (1989), (1994), Smith, Suchanek and Williams (1988a,b), Williams and Smith (1984) [see also Porter and Smith (1994) for other references] in which prices overshoot the fundamental value and subsequently crash.

An Asset Flow Perspective in Securities Marketing and Technical Analysis

These examples will be considered in the context of asset flow differential equations [Caginalp and Ermentrout (1990), (1991); Caginalp and Balenovich (1991), (1994)] which incorporate the ideas of a finite asset base, a trend based component of investor preference together with a valuation component. The models naturally incorporate the flow of additional stock or cash and can be generalized to include a variety of investor strategies and motivations. Moreover, once the parameters in the system of equations are calibrated for an investor population using a single experiment, the equations can then be utilized to obtain quantitative predictions for any hypothetical scenario or set of quantitiative assumptions such as the magnitude of assets of distinct groups. Current theories offer a limited amount of insight into the problems that confront practical securities marketing in that the theories tend to assume infinite arbitrage capital and infinite rationality. The securities dealers are all too aware of the issues raised explicitly in (vi)-(viii) above in which the question of how much supply the market will bear is paramount. From this perspective, a series of large offerings is perhaps closer to standard economic price theory involving consumer goods, since the demand depends upon a steady consumer need as much as the valuation of the security. This analysis is designed to provide a basic framework in which the flow of various components of supply and demand is tracked explicitly within the context of preferences that depend on valuation and the price trend.

In addition to the examples above there is a large body of knowledge known as technical analysis which attempts to identify the patterns on price charts that may offer an indication of whether a trend is likely to continue or terminate. Of course, such a possibility is ruled out by the (weak) efficient market hypothesis which maintains that prices alone have no predictive value, and so many academicians are quite skeptical of these ideas, while practitioners use them routinely in trading and marketing securities.

This paper demonstrates the applicability of the asset flow equations to issues in securities marketing and provides a generalization to participation by distinct groups of investors. The model provides a coherent and unified basis for the patterns of technical analysis, the concepts of resistance and breakout. The origin of bubbles, the discount in closed end funds and other market phenomena are explained within the same formulation. The evolution toward equilibrium if found to be much more complex than the monotonic change that standard price theories imply. The complexity is particularly evident in the presence of two groups with different assessments of fundamental value (Section 3). Finally, the time scale for the return to equilibrium, a concept crucial to securities marketing, is considered in a precise quantitative context.

2. THE MODEL AND BASIC IMPLICATIONS

The Theory of Price Adjustment

Modern theories of price adjustment [see e.g. Watson and Getz (1981)] stipulate that relative price change occurs in order to restore a balance between supply, S, and demand, D, which in turn depend on price, i.e.,

$$\frac{d}{dt}logP = \frac{1}{P}\frac{dP}{dt} = \tilde{F}(\frac{D(P)-S(P)}{S(P)}) = F(\frac{D(P)}{S(P)}) \tag{2.1}$$

where $D(P)/S(P)-1$ may be defined as the excess demand (normalized by supply). The function F has the properties

$$F(1) = 0 \qquad\qquad F^{'} > 0 \tag{2.2}$$

so that (2.2a) ensures equilibrium when $D(P) = S(P)$. With the standard assumptions that $D(P)$ and $S(P)$ are monotonic, condition (2.2b) then ensures that this equilibrium point is unique.

The vast majority of the phenomena discussed in the previous section cannot be explained on the basis of this formulation. In Caginalp and Ermentrout (1990) and Caginalp and Balenovich (1994) the basic theories were generalized by preserving as much of the foundation as possible, e.g. the structure of the price equation, while modifying some of the concepts that are in clear conflict with the experiments and phenomena. The laboratory experiments such as Porter and Smith (1989) exhibit a very strong autocorrelation [see also Caginalp and Balenovich (1994) Appendix B] that implies a trend dependency and is difficult to explain with supply and demand that depend only on price. The dependence of supply and demand on the trend implies the generalization of (2.1) to

$$\frac{d}{dt}(\log P) = F\left(\frac{D(P,P')}{S(P,P')}\right). \tag{2.3}$$

Note that equation (2.1) is a generalization of the more common linearized form $P' = (D/S - 1)$ (see discussion in Caginalp and Balenovich [1994]). That is, equation (2.1) can be derived from (2.3) by computing the functional (Frechet) derivative in the usual way (see, for example, Aczel & Dhombres [1989]).

A Supply/Demand Model

The main feature of this approach, which focuses on the flow of a finite supply of assets in the form of cash or a particular financial instrument, is to derive systems of differential equations in which supply and demand exhibit dependence on price trend as well as price itself.

The total demand for a stock is given by the amount of funds in cash multiplied by the rate, k (normalized so that it assumes values between 0 and 1), that investors place orders to purchase stock. With a similar description for the total supply of stock, one has

$$D = k(1-B) \qquad\qquad S = (1-k)B \tag{2.4}$$

An analysis of asset flow on the conservation of total capital implies

$$\frac{dB}{dt} = k(1-B) - (1-k)B + B(1-B)\frac{1}{P}\frac{dP}{dt} \tag{2.5}$$

This equation states that the fraction of assets in stocks changes in accordance with stock purchases, stock sales and stock appreciation respectively.

The notation $P_a(t)$ is used for the equilibrium point on the $P' = 0$ plane of the supply-demand curve so that $P_a(t)$ is the intrinsic value. If k depended only on the fundamental value $P_a(t)$, then one would have a generalization of price adjustment theory only in terms of the finiteness of assets and delay in taking action.

The rate k is specified through ζ, defined as investor sentiment, or preference for stock over cash. To understand the dependence of ζ on the history of price change, one considers the motivation of an investor who owns the security as it is undervalued but still declining. The choice available to this investor is either to sell or to wait in the expectation that those with cash will see the opportunity to maximize their profits by purchasing the undervalued security. The issue of distinguishing between self-maximizing behavior and *reliance* on optimizing behavior of others is considered in the experiments of Beard and Beil (1994) on the Rosenthal conjecture (1981) that showed the unwillingness of agents to rely on others' optimizing behavior. In these experiments, player A can choose a smaller payout that does not depend on player B, or the possibility of a higher payout that is contingent on player B making a choice that optimizes B's return (otherwise A gets no payoff). The experiments showed that A will accept the certain but smaller outcome that is independent of B. However, reinforcing a line of reasoning that is compatible with Myerson's (1978) "proper equilibrium" in which "mistakes" are related to the payoff consequences, Beard and Beil (1994) showed that player A will be less reluctant to depend on B when "deviations from maximality become more costly for B" (p. 257).

Applying these microeconomic principles to an undervalued $[P(t) < P_a(t)]$ but still declining security $[dP/dt < 0]$ in which players A own the security while players B have cash, the preference of A will depend upon the magnitude and duration of decline as well as the extent of the undervaluation. In particular, the longer and steeper the decline, the more evidence A receives about the unreliability of B. However, as the undervaluation increases, the Myerson aspect increases A's confidence in B and consequently A's preference for stock.

The total investor sentiment or preference function, ζ, as the sum of ζ_1 and ζ_2 where the former involves the trend and the latter valuation. In each case the basic motivation is summed with a weighting factor that declines as elapsed time increases. This leads to the equations:

$$k(\zeta) = \frac{1}{2}[1 + \tanh \zeta] \tag{2.6}$$

$$\frac{d\zeta_1}{dt} = c_1 \left[q_1 P^{-1}(t) \frac{dP(t)}{dt} - \zeta_1 \right], \qquad \frac{d\zeta_2}{dt} = c_2 \left[q_2 \frac{P_a(t) - P(t)}{P_a(t)} - \zeta_2 \right] \tag{2.7, 8}$$

Hence, equations (2.3)-(2.8) specify the complete system of ordinary differential equations which can easily be studied numerically on a personal computer with readily available software.

The parameters q_1, q_2, c_1 and c_2 are the only parameters in the system in addition to the scaling of time. Increasing q_1 tends to increase the importance of trend based investing and the amplitude of oscillations. Increasing q_2 tends to drive prices closer to the fundamental value. While the value of c_2 does not usually have a very dramatic effect on the price evolution, the numerical studies indicate that large values of c_1 (i.e. focusing on short term trends) can lead to unstable oscillations. The model is not prejudiced *a priori* in terms of the extent to which markets are efficient. The full range of possibilities is in fact covered by the range of parameter values of q_1, c_1, q_2, c_2.

The numerical computations confirm that if the trend based coefficient is sufficiently small, then the price evolves rapidly toward $P_a(t)$ with little or no oscillations. This corresponds to a classical rational expectation model. As q_1 is increased, the damped oscillations increase in magnitude and frequency. Above a critical value of q_1 the oscillations become unstable in the sense that they increase in magnitude without bound. The behavior associated with the experimentally determined values of q_1 and q_2 falls in the damped oscillation regime. This pattern of behavior occurs for a broad range of functions $P_a(t)$, including functions with an abrupt drop. For some values of the parameters, the equations appear to exhibit chaotic behavior.

Implications for Microeconomics Experiments and Theory

A simplification of these equations (in which the time scales are long) involving only two parameters, F_1 and F_2 has been studied and compared with experiments in Caginalp and Balenovich (1994). The procedure consisted of using any one of the experiments to evaluate F_1 and F_2. The other experiments could then be compared with the model with no adjustable parameters. The statistical comparisons showed that the model, with these values of F_1 and F_2, was decisively more accurate than the efficient market hypothesis (statistical significance of $1 - 10^{-11}$), or the model with F_1 set to zero, thereby confirming the necessity for the trend based term (statistical significance of $1 - 2 \times 10^{-8}$). Similar results are obtained for the four parameter system.

This approach introduces the possibility of quantitative comparison between different experiments as well as theories. Given any experiment in which the fundamental value is clearly established, one can use a least squares procedure to evaluate the parameters of the model that describe it best. This then becomes a quantitative characterization of the investor population that can be compared with other subject groups. For example, two issues considered by Porter and Smith (1994) could be addressed in this manner. 1. Do different subject pools, e.g. students versus business people, have any quantitative difference? 2. As the subjects acquire experience as a group in the course of several experiments, how can one characterize the learning from trend based investing to value based?

Multiple Group Generalization: The principles used above can be generalized to a set of disparate investor groups labled $1, 2, \cdots, m$ with assets $x_1, x_2, \cdots, x_m$ such that the total assets, $\sum_{i=1}^{m} x_i = 1$, is normalized at unity. The assets of investor group j which are in stocks are denoted by B^j, with $B := \sum_{j=1}^{m} B^j$.

Each group has its own preference variables ζ_1^j and ζ_2^j, analogous to and its own transition ratios, $k^j(t)$, given by

$$k^j = \frac{1}{2}[1 + \tanh(\zeta_1^j + \zeta_2^j)] \tag{2.9}$$

so that

$$\frac{d\zeta_1^j}{dt} = c_1^j \left[q_1^j \frac{P_a(t) - P(t)}{P_a(t)} - \zeta_1^j \right] \qquad \frac{d\zeta_2^j}{dt} = c_2^j \left[q_2^j \frac{P_a(t) - P(t)}{P_a(t)} - \zeta_2^j \right] \tag{2.10}$$

The fraction of assets of group j which are invested in the stock is described by the analog of (2.5), namely,

$$\frac{dB^j}{dt} = k^j(1 - B^j) - (1 - k^j)B^j + B^j(1 - B^j)\frac{1}{P}\frac{dP}{dt} \tag{2.11}$$

The price equation is determined as in the single group case, since the change in price depends only on the ratio of buy/sell orders, irrespective of the origin of these orders, so that one has

$$\frac{d}{dt}(\log P) = \log \left[\frac{\Sigma k^j(1 - B^j)}{\Sigma(1 - k^j)B^j} \right]. \tag{2.12}$$

Hence, the $(3m + 1)$ ordinary differential equations, (2.10)-(2.12), together with the m algebraic equations (2.9) provide a mathematically complete system which can be studied numerically upon specification of the initial conditions. The two-parameter version is generalized in a similar manner.

While the equations appear to be complicated, they are rather simple to understand. For example, if two distinct groups are present then each one has its own preferences for the purchase or sale of the equity and its own assesment of value. If Group 1 values the stock at 20 and is trend oriented, then they will be using much of their cash to buy the stock when it is, say 10. On the other hand if Group 2 values it at only 15 and is value oriented, then strong selling will start at a price near 15. All groups go through the central clearing house so that the price equation involves the net supply and demand for the stock. Of course, the key aspect of the model and the experimental and world markets is that each group has finite assets. The question of which group prevails is ultimately determined by the domination of assets. As discussed in Section 3 the resolution of this dynamic question is far more complicated than a steady evolution toward the equilibrium price.

These issues are critical to securities marketing in that there are often several distinct groups that are very different in their investment strategies and time horizons. These models offer the possibility performing computations using various estimates of the assets of each group and allowing the scenarios to play out on the computer. An example with the Mexico Fund is discussed in detail in Section 3.

These equations can easily incorporate (i) the effects of additional cash that may be acquired by one or more groups, or (ii) the effect of additional supply of shares. An influx of cash into group j changes (2.16) by adding a term $^{-}dN(t)/dt$ that represents the flow of new cash per unit time. Similarly the effect of adding new stock to the system can be considered as a term that is added to the supply portion of (2.17). Since the model gives quantitative predictions for the price behavior when a sudden drop in value occurs, as discussed in the Introduction, a comparison between its predictions and those of the classical theories can be made if the appropriate experiments are performed. While the efficient market hypothesis would predict random fluctuations about the lower price, once the information is public, the asset flow perspective offers three possibilities depending on the parameters (established from previous experiments of a different type) governing the investor population. If the trend based coefficient is very small, the price will move smoothly down to the lower value with a slope that depends upon the time scale c_2^{-1}. For larger values of the trend based coefficient, q_1, the price will overshoot the lower value and gradually increase to a constant P(t). for still higher values of q_1, there will be damped ocsillations approaching constant P(t). Note that in the limiting two-parameter model the convergence is to the unique equilibrium which is $P_a(\infty)$. However, in the general model, the set of equilibria is infinite and the large time limit $P(\infty)$ depends upon the parameters of the system including the initial conditions. An interesting consequence of the numerics is that prices can even overshoot the *original* segment of $P_a(t)$ if q_1 is sufficiently large. An example of this in the stock exchange is the behavior of insurance stocks after the October 1989 California earthquake.

3. A THEORETICAL FOUNDATION FOR TECHNICAL ANALYSIS

Basic Ideas of Technical Analysis

Few topics have as wide a gulf between theoreticians in academia and practitioners in industry as does technical analysis. While most economic and financial scholars often ignore or downplay the role of conventional technical analysis, financial experts are often quite eager to implement it without regard to the nature of the economic assumptions inherent in the methods. The criticism generally has several forms. Some feel that the methods cannot provide any results because if they did, enough people would use them and thereby eliminate the inefficiency. This, of course, is an argument against any inefficiency. A discussion of the contrary view is presented by Grossman and Stiglitz (1980) who explain how informationally efficient markets are in fact impossible. Within the context of their model, they demonstrate that if it were true that prices reflect all available information, as efficient market theorists generally claim, then "informed traders could not earn a return on their information" and would stop paying for the information (p. 404). In one academic study it been has suggested that volume of shares traded could provide some insight into market decisions (Blume *et al* [1994]. Another set of objections concerns the lack of a microeconomic set of decision making justifications for the principles of technical analysis, which are usually presented as a list of patterns without much insight into their origin. This is addressed below by demonstrating that the asset flow models produce similar patterns with assumptions that are close to those of practitioners. The models, in turn, are derived from a microeconomic examination of the rational motivations of agents who are attempting to optimize their returns based upon their assessments of the strategies of others (Caginalp and Balenovich [1994] pp. 158-163). Of course, the agents are not "super-rational" in the sense that they are not willing to rely on the certainty of other agents' self-maximization (as discussed after equation (2.5)).

The results show that the major patterns of technical analysis can thus be obtained from a set of assumptions that consist essentially of price adjustment theory augmented with the concepts of (a) trend based investing (b) finite asset base, and for some patterns, (c) asymmetric assessment of value. The conclusions would not be significantly altered as a result of modifications of the model provided that these ingredients are preserved and that the parameters are evaluated with the same principles (e.g. calibrated from experiments). The asset flow approach is not only a unified perspective, but offers the potential for testing various hypotheses about current conditions. A practical application would be the estimation of the assets of each group, along with the characteristic parameters, and performing the numerical calculation to determine, for example, whether there is a breakout at the conclusion of a particular pattern, such as a symmetric triangle, defined below. In fact, in many cases the predictions of standard technical analysis are made well after the new trend has begun, thereby limiting its profitability. The asset flow approach has the potential for more quantitative, and earlier, conclusions.

The following is a brief summary of the basic principles or assumptions that standard references state as the justification for technical analysis [Edwards and McGee (1981), Myers (1989), Pring (1993)].

(1) Trends in prices tend to persist. This is essentially a momentum concept which means, in economic terms, that the supply/demand ratio is slowly varying (despite changing prices) unless there is a significant change in fundamentals or the sources of supply or demand. Note that this assumption may violate classical equilibrium economics in that a price rise is not expected to bring an immediate decline in demand or rise in supply. Thus, the validity of this key assumption of technical analysis appears to be contingent upon introducing price derivative dependence, in addition to price dependence, upon the demand function.

(2) Market action is repetitive: This assumption maintains that various patterns appear again and again in price charts. These patterns evolve as a consequence of investors' reactions to the change in their fortunes. Thus, the recurrence of various patterns is a manifestation of the tendency for people to behave similarly (or employ analogous strategies) in similar situations.

A description of the various price patterns can be found in most standard texts (e.g., [Myers (1989)]). From the perspective of the differential equations models, the major charts can be best understood by dividing the patterns into two categories, those that can be described by the presence of a single group of investors with similar information and motivations, and those which require at least two such groups. Furthermore, traditional explanations of charting tend to focus on the distinction between consolidation patterns (meaning the trend will resume after a brief respite) and reversal patterns (meaning the trend will terminate). While this distinction appears to be natural when the chief goal is to profit by predicting trends, it is not very basic since the origin of some consolidation patterns is quite close to related reversal patterns. In fact, one conclusion is that it appears to be difficult to predict the direction of prices (using *only* price histories) before they emerge from a complex pattern such as a symmetric triangle as discussed below.

Single Group Patterns

Trend lines or channels: One of the most common patterns or channels observed in financial patterns is the general movement of prices in either the upward or downward direction, roughly between two parallel lines as shown in Figure 1.

The differential equation model offers a very simple and natural perspective which explains the regular oscillations between the lower and upper part of the trend channel. An downward trend is thus a consequence of a steadily decreasing assessment of the value of the financial instrument as can be obtained by using for example $P_a(t) = -t + 40$, $P(0) = 38$, and the parameter values $q_1 = 950$, $q_2 = 100$, $c_1 = 0.001$, (???) $c_2 = 0.001$, in equations (2.3)-(2.8). Under these conditions, the price oscillates about $P_a(t)$, so that $P(t)$ varies between two lines parallel to $P_a(t)$ (see Figure 2). The width of this channel is dependent on the values of the parameters. In general, a higher trend based coefficient favors larger oscillations and a larger channel, while a higher value based coefficient favors a stronger link to the intrinsic value and therefore smaller oscillations.

A price path similar to a trendline also forms when the investing is largely trend based and very little attention is paid to fundamental value.

Rounding Top or Bottom: A reversal pattern which is simple but not very common occurs when the price gradually changes direction. This pattern is generated by assuming a $P_a(t)$ function which represents a simple turn-in-fortunes of the finanical instrument, namely a linearly increasing $P_a(t)$ for $t < t_0$ and a linearly decreasing $P_a(t)$ for $t > t_0$, for example, $P_a(t)= t + 5$ for $t < 20$ and $P_a(t) = -t + 45$ for $t > 20$, with $P(0) = 4$, $q_1 = 820$, $c_1 = 0.001$, $q_2 = 44$, $c_2 = 0.001$. For values of q_1 which are smaller those found in trendlines one obtains a rounded top. This is essentially what is to be expected since the rounded top is not observed frequently in charts and is associated with markets in which volatility is low and value based investing is the dominant strategy. Examples of such markets include high-quality utility stocks and bonds in which the value is easily measured in terms of the dividend paid versus the prevailing interest rates. Hence, under these conditions, one does not expect large oscillations near the tops or bottoms, unlike the head and shoulders or double top formations, to be discussed below, which are characterized by a trend based strategy resulting in more complex reversal patterns.

Key Reversals, V-formations and Spikes: Another set of reversal patterns is similar to the rounding top except that they occur more abruptly. A V-formation bottom has the appearance of a V (Figure 1), and analogously for a top. A spike is a sharp V in which the maximum is attained and abandoned quickly. A key reversal is more specific in that a maximum is attained on a particular day which ends near the lowest part of the day's trading range. Clearly, a V-formation will be obtained in a similar way to the rounding top if the value of q_1 is lowered and the value of q_2 is raised. Thus, using the same $P_a(t)$, $P(0)$, c_1 and c_2 as in the rounding top discussion but with $q_1 = 700$ and $q_2 = 1000$, one finds that prices track $P_a(t)$ very faithfully. This topping formation is closest to the principles of standard microeconomics. A key reversal pattern appears on a typical stock chart as a long vertical line (representing the day's range), which is higher than those to the left and right of it, with a slash near the bottom (representing the closing price) and this vertical line is higher than those to the left and right of it. Of course, if the price is plotted as a function of time (by minutes or hours) then the day of the key reversal typically contains a V-formation or a rounding top on a finer time scale, so that it has the same theoretical basis.

Head and Shoulders: A common reversal pattern with a more complex appearance, the head and shoulders (bottom) pattern appears as a major low flanked by a local minimum at either side (Figure 3). This is modelled with the same assumptions on $P_a(t)$ as in the rounding bottom, namely $P_a(t)$ is first linearly decreasing, then linearly increasing. Letting $P_a(t)= -x + 30$ for $0 < x < 15$ and $P_a(t) = x$ for $15 \leq x \leq 30$, with $c_1 = 0.005$, $c_2 = 0.05$, $q_1 = 210$ and $q_2 = 50$, one obtains a typical head and shoulders reversal (Figure 4).

Hence, this rather complex reversal pattern can be obtained as a consequence of a single group of investors with identical motivations and assessment of the value of the financial instrument. This may be in the sharp contrast to the initial appearance of a tug-of-war between disparate investor groups. Hence the conventional wisdom that market tops are generally complex formations, rather than a simple U-turn, is explained in terms of the coupling between natural market oscillations and the changing value of the company. In fact, a long standing idea of Wall Street is that when the trend becomes interrupted and begins to move into a region of large oscillations an slower average increases, then "distribution" has begun. This means that the more knowledgeable investors with large amounts of stock, and presumably among the early participants in the uptrend, have begun selling their shares into the rally. While this may be the origin of some topping formations, as in the wedge formations described below, it is clearly not a necessary condition in the formation of head and shoulders as well as the related multiple tops discussed later.

As an example which illustrates these ideas is the aggregate U.S. market in the second half of 1990. Soon after the Iraqi invasion of Kuwait in early August of 1990, the chances of the U.S. becoming involved in a war and the consequent recessionary effects on the U.S. economy implied a steady downward reevaluation of the intrinsic value of the major stocks. By October, the Federal Reserve's concern of a deepening recession led it to a more accommodative stance, so that the gradual decline in interest rates effectively provided a steadily increasing value for stocks (by reducing the value of competing fixed income instruments, and also by reducing corporate financing costs). At the same time the analysis of the possible war scenarios led to a general consensus that the outcome of a war would no be as grave as first thought for the U.S., since oil supplies would probably not be interrupted. Thus, the intrinsic value of the stock market, i.e., $P_a(t)$, can be represented by a V-shape. With this $P_a(t)$ in the equations, one obtains a $P(t)$ which is an inverted head and shoulders pattern similar the general pattern in late 1990. The introduction of a more complex peak for $P_a(t)$ would necessarily introduce the more complex topping out patterns (than the head and shoulders) which are often observed in financial markets.

Double and Triple Tops (or Bottoms): A pattern which bears some resemblance to a head and shoulders top occurs when two or three peaks (with similar peak values) occur successively as they attain essentially the same maximum. Triple tops and bottoms are less common than double. The basis of this pattern involves a $P_a(t)$ that is linearly increasing, then level and finally linearly decreasing, such as $P_a(t) = x + 5$ for $t \leq 15$, $P_a(t)$=20 for $15 < t < 20$, and $P_a = -x + 40$ for $t \geq 20$. Using the same values of $P(0)$, c_1 and c_2 as those in the V-formations with parameters $q_1 = 950$ and $q_2 = 400$ one obtains a price with a double top. The features of $P_a(t)$ and the extent of initial undervaluation or overvaluation in $P(t)$ determine the nature and number of oscillations near the peak. In particular, if $P_a(t)$ is symmetric about the peak, then there are the following parameters: the magnitude of the slope in the first and last parts of $P_a(t)$, the length of the level part, the magnitude of $t_0 - t_i$ (where t_0 is the midpoint of $P_a(t)$ and t_i is the initial point), and $P_a(t_i) - P(t_i)$.

The different shapes such as the double top and the head and shoulders can arise from identical $P_a(t)$ with slightly differing undervaluation (in timing or extent). In fact, one can obtain a one-parameter family of topping patterns, including these two, by simply varying the timing of the undervaluation and thereby the timing of the oscillations with respect to the peak of the V-shaped $P_a(t)$. It is also clear why the triple top is rare in markets, since a V-shaped $P_a(t)$ does not result in such a top with a wide range of parameters and initial undervaluations. A triple top can be attained with a $P_a(t)$ function which has a longer flat top, and using parameters in which the trend based coefficient is rather small.

Rising (or Falling) Wedge: This is a rather common reversal pattern that differs in a subtle way from the pennant and symmetric triangle formations which is discussed as part of the two group patterns below. The price rises into the wedge which has positive slope (i.e., rising wedge) as opposed to the symmetrical triangle or pennant which have zero or negative slope respectively. The negative prediction rendered by the rising wedge is evident upon a comparison with a rising channel. The rising wedge is essentially a deteriorating channel as the progressive new highs are unable to maintain the same ratios as the new lows (on the bottom trendline), thereby indicating that selling occurs earlier than one would expect, and sending a signal that a topping out process may be underway. Hence the model can be used to produce a rising wedge by assuming a single group of investors with a convex downward $P_a(t)$. The oscillations appear to narrow as the price gradually increases, but the price pattern is not consistent with a steadily increasing $P_a(t)$, and a reversal is signalled.

Two-group Patterns

Symmetrical Triangles, Pennants and Flags: A common pattern in charts appears as a series of oscillations with diminishing amplitude. A pennant is formed by the two lines which join the successive tops and bottoms respectively. If the two slopes have equal magnitude and different sign, then the pattern is called a symmetric triangle (Figure 5). In general, pennants are regarded as consolidation formations which appear at regular intervals as a trend takes a pause. However, they also occur as reversal formations as shown in Figures 6 and 7. A breakout usually occurs before the vertex is reached as the trend is resumed. Volume is often diminishing until shortly before the breakout point, at which time it increases dramatically.

The basic origin of these formations is the presence of two or more groups with distinct assessments of value. The pattern is essentially a series of oscillations about a particular value that represents the assessment of one group on fundamental value. The issue of whether the pattern is a consolidation or a reversal is ultimately decided by the exhaustion of either cash or stock on the part of the net buyers or sellers, and is not obvious from the immediate appearance of the triangle, though a more careful examination of prices may provide a better indication.

An example of a symmetric triangle that is a consolidation can be created by distributing 80 % of the initial assets to Group 1 which assesses true value at $P_a^{(1)} = 8$, while Group 2 values it at $P_a^{(2)} = 4$ With the other paramaters set at $c_1^{(1)} = 0.001$, $q_1^{(1)} = 950$, $c_2^{(1)}$ =0.001, and $q_2^{(1)} = 100$ for Group 1 and $c_1^{(2)} = 0.001$, $q_1^{(2)} = 900$, $c_2^{(2)} = 0.001$ and $q_2^{(2)} = 1000$ for Group 2 one obtains the formation shown in Figure 8. The essential features of configuration do not change if the q_1, q_2, c_1 and c_2 are varied widely. The amplitude of the oscillations in the triangle depends chiefly on the trend based coefficient of the group whose assessment of value is withing the triangle, and secondarily on the group with the higher assessment.

The continuation triangle can be changed to a reversal pattern (at the higher value) by exchanging the fraction of assets from one group to the other.

The flag is a consolidation pattern which is similar to the pennant in which the oscillation occurs between two parallel lines until a breakout occurs.

Breakout from trendlines: The piercing of a trendline in a channnel is generally a very significant event in technical analysis.

The significance of the breakout is related to the importance of the trendline that is broken, measured by the duration of the trend. From the asset flow perspective, a sufficient condition for an upside breakout of an increasing channel is the entry of a trend oriented group into a financial instrument that has been increasing in price due to value based investors who are focusing on improving fundamentals. Let Group 1 be the value based one that produces the channel (described earlier) and Group 2 the investors with a higher trend based coefficient and/or a higher assessment of value. Furthermore, suppose that Group 2 acquires cash during the trendline formation. The result is that a break occurs in the upper trendline and the channel gives way to a bubble (Figure 9).

Comparing this microscopic explanation with a typical bubble, e.g. Indonesia in the late 1980's (Figure 10), one sees that a clear lower trendline is in effect during the entire process. This is formed by the more sophisticated and value-oriented investors who are bidding in accordance with a value that is steadily increasing at a sustainable rate due to continued growth and increasing earnings. The steady increase in price, however, draws the attention of potential investors who may have little or no experience. These investors are not only mesmerized by daily price changes, but they tend to confuse projections of what the stock should be trading say three years from now with what one should pay for it today. For example, if the stock has been increasing at 20% per year, the price is currently at \$10 and the investor plans to use the money in three years, the idea of paying \$15 now seems reasonable to those investors. However, they have a finite supply of cash and ultimately the cash is inadequate to buy the increasing amounts of stock that Group 1 is selling due to the overevaluation. As supply of stock overwhelms demand the price begins to fall and Group 2 now begins to sell for the same reason that one induced them to buy – the trend in stock price – and the bubble bursts. Note that Group 1 is also somewhat trend oriented, and in fact it would be irrational to sell now if it appears that some investors will pay more for it later.

An Example in Detail: The Symmetric Triangle as a Reversal Pattern

An examination of the Mexico Fund from late 1993 to early 1994 shows a rapid rise from mid-November until January. A long, symmetric triangle is then formed and prices break out of this triangle in mid-February and begin a steady downtrend (Figure 7). The Mexico Fund is a closed-end fund that invests in a diversified manner in Mexican equities, and is affected by news that impacts Mexico. These include the U.S. House passage of the Nafta Act in mid-November (where the outcome was in doubt to many), the peasant rebellion on January 1, 1994, and the assassination of the leading candidate for President on March 23, all of which were unprecedented in modern Mexican history. While the rise certainly appears to be precipitated by Nafta, neither the rebellion nor the assassination appears as the immediate cause of the downtrend that begins in mid-February. In fact, this date marked a small scandal in Brazil that drove a number of Latin American markets down, perhaps with the explanation that investors realized that political uncertainty remains a possibility in Latin America. From a fundamental point of view, this explanation is no more convincing than the factor of (roughly) two variation in the price of this stock in a short time period. Other deviations from classical theory include the variations in the discount/premium (20% discount to 5% premium) and the drop in the stock during the September rights offering that added more shares.

An explanation of this price pattern of this stock can be provided on the basis of two groups with differing assessments of value (i.e., asymmetric information). As far back as June 1994, published reports indicated that some large investors felt certain of Nafta's passage and estimated that the Mexican Bolsa could rise from 1900 to 2400 with the passage. This would probably translate to a value of about 32 or 33 on the Mexico Fund. This establishes a fundamental value for this group. On the other hand, another group of investors, who probably had not thought of investing in Mexico until the intense debate on Nafta, probably were much more conscious of the trend, and their vague assessment of value probably placed the fundamental value at a somewhat higher number. An examination of volume shows that volume increased significantly in the months after the House passage and returned to previous levels afterwards.

It may also be reasonable to assume that the sophisticated group will have greater assets or at least will have inventory to sell after the enthusiasm of post-Nafta buying has faded.

Toward this end, one can make a very simple set of assumptions and examine the extent to which they can be varied without altering the qualitative nature of the pattern. Assigning 60 % of the initial assets to Group 1 which assesses true value at $P_a^{(1)} = 32$, while Group 2 values it at $P_a^{(2)} = 39$. With the other paramaters set at $c_1^{(1)} = 1.00$, $q_1^{(1)} = 0.54$, $c_2^{(1)} =0.001$, and $q_2^{(1)} = 900$ for Group 1 and $c_1^{(2)} = 1.00$, $q_1^{(2)} = 0.01$, $c_2^{(2)} = 0.40$ and $q_2^{(2)} = 800$ for Group 2.

The resulting price evolution is displayed in Figure 11. Hence, both groups initially buy due to both underevaluation and positive trend. As the price becomes very high, however, group 2 begins some selling. After a few oscillations the price appears to be very stable and just below 40. Gradually, group 2 sells as group 1 buys, both due to valuation. Perhaps the most interesting point occurs when a sharp drop-off appears as almost a discontinuity in the derivative of price. At this point, group 1 has exhausted their supply of cash to the point where their buying is inadequate to sustain the price. Once the decline begins, however, they participate in the selling due to the trend and the steep decline sets in.

Continuing the computations one finds that a similar cycle is repeated many times although one of the oscillations disappears (Figure 12). In time, the oscillations become much smaller and the price reaches a steady state that depends on the parameters and initial conditions (Figure 13). The attainment of this price is considerably more complicated than in the classical adjustment. An interesting feature of this process emerges in the plot of the fraction of the total assets owned by each group. Throughout this complicated process group 2's assets increase almost steadily from 60% to almost 100% while group 1 gradually loses its share.

Also, it appears that the most important factor in determining the steady state price is the fraction of assets of each group rather than the extent to which it emphasizes a value based investment strategy. The group with greater assets dominates the determination of the eventual price and so its strategy pays off more than the other. Furthermore, the numerics also indicate how a quasi-equilibium can be formed at a price that appears to satisfy both groups – in the case of Figure 14 just below 40 – so that one group sells as the other buys, although this price cannot be maintained because one group exhausts its supply of cash. Sophisticated traders say that a period with low volatility often precedes a big move (up or down).

From this perspective, it is clear that this phenomenon is caused by one group exhausting its supply of cash/stock before the other, in a manner that is reminiscent of a simple field battle of foot soldiers. The battle appears to be a stalemate as the front moves very slowly, until one of the armies is decimated and the front then advances rapidly. Consequently, if one can make the assumption that two distinct groups are involved in trading and both have a significant trend component, then a period with no oscillations indicates that the trading price is satisfactory to both groups from a value perspective. In fact, numerical studies with a lower trend coefficient for both groups displays no oscillations near the two true value prices but a similar oscillation between the two values until a steady state is reached.

4. DISCUSSION AND CONCLUSIONS

Resolution of some nonclassical phenomena

By augmenting the classical price theory with the concepts of trend based investing and the flow of finte assets, one can explain a broad spectrum of phenomena.

(i) Consider a worthless stock which begins to rally as unknowledgeable investors buy due to takeover rumors that they (mistakenly) believe will raise the price to the stock which currently trades for \$1. More informed investors may sell this stock short in the expectation that the stock will eventually attain its true value of \$0. However an investor who has a total of \$1000 in his brokerage account and sells short 1000 shares is aware that he must begin purchasing back shares if the price moves above \$2. In fact, the brokerage will do this unilaterally if he does not (if no additional cash is brought into the account). If the funds of the unknowledgeable investors exceed those of the short sellers and others selling into the rally, then the price will move up from \$1. The investor who sold short at \$1 must be very concerned with the trend since a focus solely on fundamental value (of \$0) will quite possibly lead to a loss. Furthermore, if the price does move sufficiently higher then the short seller is motivated to close his position by purchasing the shares and thereby contributing to further lifting of the price. Even in this very obvious valuation, the knowledgeable investor is motivated to have a preference that depends on a price derivative or trend that he knows expresses the preferences of unknowledgeable investors. Hence one may have a bubble (as in the laboratory experiments) and a subsequent crash which may be prompted due to various reasons. One of these would be a published report that the stock is worthless, thereby eliminating almost all of the buyers, and changing the price derivative that induces others to sell solely for that reason. Another, but slower mechanism, is that eventually the supply of cash of the uninformed investors is exhausted.

(ii) The large bubbles that have occurred during a government's sale of stock (e.g. NTT) in a privatization program seem paradoxical even in view of the fact that the government is increasing supply. However, the huge amount of free advertising that is available to a government tends to increase the demand among the public, often in excess of supply. As series of offerings in which the initial subscribers have always seen immediate (paper) profits that are widely reported tend to bring a large demand at the next such sale. As in the other examples the resources of the new entrants to a particular financial instrument are ultimately overwhelmed by those of the more established group that is more concerned with economic fundamentals, and prices fall. In summary, the bubbles that arise during highly publicized government offerings are initiated by a dramatic enlargement of demand, which creates a trend, followed by further rising prices due to trend based investing. The implications for efficient market theories and the trend based asset flow are quite different in terms of public policy. While the efficient market hypothesis would maintain that prices are at fair value, the asset flow analysis idicates that careful consideration must be given to balance the supply of new stock with the new demand created by publicity, particularly since a similar amount of publicity will be created by the bubble's demise, thereby choking off additional demand for other new issues.

In time many of these bubbles collapsed and more recent privatizations such as the Japanese Tobacco sale were quite unsuccessful from the outset. The NTT bubble and bust can be compared with the learning process that occurs within a single bubble experiment of Porter and Smith (1989), while the failure of the privatizations of subsequent years is analogous to the learning that is manifest in subsequent experiments for a group of subjects.

(iii) Many of the world's bubbles appear to have an origin that is modelled in the upside breakout from a rising channel (e.g. Jakarta in Figure 10) as follows. A rising trend develops in response to steadily rising corporate earnings as the dominance by investors focused on economic fundamentals maintains the growth of stock prices to value, e.g. at 20% per year. The steady and seemingly certain increase of share prices attracts the attention of a new and less knowledgeable group of investors who bring a fresh supply of cash, a vague notion of value and a strong attachment to the trend. The value based investors are also aware of the trend, and as in the Mexico example, do not seem inclined to sell when a powerful rally has begun. However, once the new group begins to run out of funds, the rally begins to falter and the 'buying is exhausted.' The value based investors sell out more quickly simply because the value part of their preference is heavily negative at the height of the bubble while the trend part is just mildly positive as the rally begins to peter out, hence the sum is negative. On the other hand the new investors have a positive preference at and slightly *after* the peak. Once the trend has clearly reversed, the preference of both groups is negative and the rout cannot end until the price is at or below fundamental value and the value based investors resume buying. Hence, an examination of long uptrends (3 or 4 years) and upside breakouts in the absence of dramatic economic improvements should be viewed cautiously by new investors and public policy makers.

(iv) The large declines or crashes in the US and other markets in 1929 and 1987 have been discussed in Caginalp and Balenovich (1991), (1994). Using a constant value for $P_a(t)$ (since economic fundamentals are relatively unchanged in one week), and the same parameters that were calibrated from the experiments, one obtains good agreement with the time evolution for the first few days. In particular the results explain the large rebound that falls in between the top and bottom prices, and is difficult to explain from an equilibrium perspective. A similar analysis has been applied to the large premiums that evolved on the country closed-end funds, (v). In both 1929 and 1987 the longer term behavior may be described as reaching an overvalued state before being confronted by a full percentage discount rate hike that effectively lowered the value of stocks at an overextended time.

(vi) The persistence of discounts in the average closed-end fund has an immediate explanation from the asset flow perspective. The funds are initially priced about 6 to 10% above net asset value (NAV) in order to cover costs. Also, the attempts to sell to a broad base of investors before the offering may lead to a smaller group of potential buyers *after* the offering. There is also the possibility that the underwriters are left with some stock that must be sold shortly after the offering. The price is essentially determined but the underwriters (e.g. short selling at the outset is generally disallowed) so that a premium can be imposed. However, in the absence of a stellar performance of the NAV the value base buying and selling short will initiate a downward trend. The trend based investing will then result in crossing below the NAV and remaining below it.

A sample evolution with is shown in Figure 15, where $P_a(t)$ is \$10.50. The initial price is \$10.50, while $B(0) = 0.56$ so that more than half of the potential market is saturated. The other parameters are $q_1 = 175$, $c_1 = .0005$, $q_2 = 1$, $c_2 = .001$. The stock falls soon after opening and reaches a steady state of only \$9.30 which is an 11.4% discount, even without an initial overvaluation. In fact if one simply analyzes the dynamical equilibrium of equations (2.4)-(2.6), (2.9)-(2.11) it is clear that one must have $k = B$ and $\zeta_2 = q_2(1 - P/P_a)$ in order to have constant price as t approaches infinity. Then $k = (1 + \zeta_2)/2$ implies that $B > 0.5$. Thus, the equilibrium price, which depends upon initial conditions and parameters, will be lower than the NAV because of the initial overvaluation and an 'excess' fraction of investors already invested in the fund. The remedy to this problem would be an industry wide shift toward paying for costs over a long time period rather than through the initial offering, and to sell less aggressively *before* the offering and continue marketing steadily after the opening. This perspective applies to initial public offerings (IPO's) in general, though the intrincis value is not as clear. From the asset flow perspective, two factors contribute to rapid run-up of IPO's: (i) oversubscription of the IPO, leading to significant buying at the open; (ii) the absence of short selling before the IPO, and usually for some time after its issue.

The beginning stages of IPO trading has been examined by Welch (1992) who concludes from a game theoretic model that investors can ignore their private information and imitate other investors, producing "cascades" of successive investor trades particularly if the trading is very thin. Stated in the terminology of the asset flow model, investors acquire a large trend based coefficient since investors are looking toward others' reactions to asses valuation.

(vii) The addition of supply of stock clearly suppresses the price in the asset flow formulation. The situation is the opposite that of the addition of new cash which is one of the factors that initiates a bubble. The effect of supply is evident in recent trading in the local stock exhanges of the Peoples' Republic of China. On August 1, 1994 the government announced a ban on the issuing of new stock and propelled stock prices upward by about 100% in a single week, as investors recognized the obvious effect this would have on the market. Of course, in a rapidly growing economy, there will always be more cash entering into the stock market as people save and invest some to the money they earn. Note that under the (very reasonable) assumption that the potential value of Chinese shares did not increase as a result of this decision, which after all *restricts* the options of the corporations and thereby establishes the value inequality, as the corporations' optimization problem has an additional constraint imposed on it that may reduce the availability of increased financing. An application of the efficient valuation theories would then result in share prices that are at best unchanged after the decision.

(viii) The aspect of promotions that increase the demand by enlarging the supply of new cash has already been discussed in connection with the privatization programs. Of course, large scale advertising and promotional incentives are usually a key part of secondary offerings that help balance the new supply with new demand. Determination of the precise effect on price of promotions contingent upon estimating the increase in the supply of cash. The evolution in price can then be estimated using the asset flow equations.

(ix) An asset flow model in which a hypothetical amount of supply of stock is added to the market that is within a particular evoulution of price can be very useful in timing secondary stock issues. Conventional technical analysis does not allow for this possibility.

On the Role of Experiments in Understanding the Foundations of Markets

The robustness of the bubble experiments of Porter and Smith (1990), (1994) and Smith, Suchanek and Williams (1988) and their qualitative similarity to world market bubbles (including very recent ones) have had a profound effect on the microeconomic theory of markets. Particularly significant is the experimental result of Porter and Smith in which there is no uncertainty about the expected dividend payout so that any uncertainty must be due to the anticipated actions of others. This leads to the fundamental issue of whether players are willing to rely on the self-optimization of others who rely on others, etc., leading to a particular Nash equilibrium. The experiments of Beard and Beil (1994) resolve this question in its simplest game theoretic setting (two players, two choices) and cast doubt on game theoretic models in which absolute reliance on others' self-optimization is required. In particular, a game theoretic justification of a perfectly efficient market appears to require substantial reliance on others by a significant portion of investors. The model used in this paper has derivation from an extensive game with imperfect information, recall and reliance on others (Caginalp and Balenovich [1994]. Since the price equation depends upon the supply/demand ratio, the observed change in price at one stage provides an indication to players as to the reliablility of the self-optimization of others. In other words, if the price declines while it is undervalued, the stockholders receive some information that many cashholders are not engaging in self-optimization with an infinite horizon. At the next stage of the game then they may be more reluctant to rely on others' self-optimization. It would be interesting to test this idea as a multi-stage experimental game to see whether it is the basis of trend based behavior. This point is also material to the surprising observation of Smith, Suchanek and Williams (1988a) p. 28, that excess bids are a leading indicator of a burst in the bubble. An explanation of this can also be found as part of the extensive game suggested above. Since the bid/ask is public information, the potential sellers know that there are a number of other sellers each of whom is wondering how much the others will lower their asking price the next time. Although it may be in each of the potential sellers' self-interest to not lower it by much, the unreliability of others' self-interest will mean that at least some of them will lower it very significantly, analogously to the prisoner's dilemma and establish a much lower price. At the subsequent stage, the information on the extent to which the sellers have lowered the asking price reinforces further the lack of others' reliability and the drop continues.

An important conclusion that one can derive from the experiments on bubbles and game theory is that the application of microeconomic theories to markets in the absence of microeconomic experiments can make a premature departure from the behavior of real markets as a result of assumptions that are not borne out. Even if one constructs an extensive game that is completely realistic and universally accepted, there will be numerous parameters that will lead to drastically different temporal evolution with different choices, and there is no mechanism to decide what the parameters should be. Ultimately, many of today's puzzles must be settled at least partially by experiment since the financial world is not only complex but immutable. For example, any explanation for the discount in closed-end funds, including ours, is difficult to test in the absence of experiments, since it relies on parameters, e.g. the additional costs involved in the offering, that are not varied.

Implications for Technical Analysis

This approach provides a coherent explanation of not only the patterns of technical analysis but also of the limitaions involved in their application. One of the difficulties that affects technical analysis is that the rules and conclusions seem somewhat arbitrary and consequently unscientific. From the perspective of this model the validity of technical analysis is contingent upon only two factors: a finite asset base and the influence of trend based investing. The vast majority of the conclusions would not be altered with a somewhat different approach (e.g., discrete difference equations or a different form for the flow rate k) that nevertheless preserves these properties. This (or any similar) methodology elevates technical analysis from a set of arbitrary assumptions and conclusions to a set of implications that are implied by the unique solution of the differential equations. Given a set of parameters (trend based coefficient, etc.) that describe an investor population, one can obtain the implications of a particular configuration of price development as the unique solution of the equations.

In view of the simplicity of assumptions and the limited intelligence involved in the model, it is somewhat surprising that these patterns emerge so naturally. In the symmetric triangle pattern, for example, one can imagine a complex set of strategies evolving in time as each local maximum or minimum is attained. The numerical results, however, seem to suggest the information included in the price trend and the extent of under/over-valuation is a suitable representation of the average of a wide spectrum of strategies.

There are two additional research problems that arise from this analysis. First, can patterns be detected in a market through statistical and computer testing, and if so, do they have predictive value? One would need to define the patterns in an algorithmic way that corresponds to human experts, as done by Kamijo and Tanigawa (1993) for some patterns. The next step would be the precise identification of the rules for deciding the trading action. Of course, a related issue is the possibility that these patterns can be created in a laboratory, perhaps using experienced participants. A second issue is the problem of deducing fundamental values, say $P_a^{(1)}$ and $P_a^{(2)}$, and assets of each group, given $P(t)$. This is an inverse problem similar to inverse scattering.

Applications to Securities Marketing

One of the most important aspects of markets to the practitioner involves the behavior of the investors and prices before the market has settled into an equilibrium price. Hence equilibrium economics does not address the significant questions that arise in large market changes whether or not they are attributable to fundamentals. As shown in the simulation with two (value based) groups with distinct assessments of value (Figure 14), an abrupt change can occur with no change in fundamentals, little prior volatility and little trend based investing. Asymmetric information implies a complex group dynamics and psychology of group behavior. It is clear from the examples discussed that this interaction cannot be understood without a model of the asset flow. The models presented can be used as a basis for numerical experiments in order to obtain an understanding of a market situation. With modern software, this can easily be done by practioners who are not necessarily specialists in differential equations or their numerical computation.

With this procedure, it is possible to determine a time scale for the return to equilibrium, as a function of the fraction of assets of each group, that is very difficult to estimate with other methods.

The analysis of two groups also gives some insight into trading patterns of intermediate term traders who are generally more inclined to be short equities despite the fact that equities tend to rise in the long run. While it is difficult to determine when a new group will enter the market to create a speculative bubble, it is relatively easy to know the time scale on which the bubble will return to fundamental value or below. In the terminology of traders, the ownership of the stock has been transferred from "strong hands" to "weak hands" as the bubble develops. A trader who takes a short position is likely to benefit in a relatively short time provided the price rise does not force a closing of the position at a higher price. In order to avoid this, some traders use a rule such as a 15 or 20% decline from the peak, at which time they believe the "weak hands" will begin selling. In most cases, e.g. the Indonesia example in Figure 10, the bubble cannot withstand a very sharp decline, since the trend based coefficient becomes negative in addition to a value based coefficient that is already very negative. Simply stated, the bubbles usually cannot last long since the self-feeding mechanism of rising prices requires capital for its maintainance. Unless there is a steady influx of new investors, the finiteness of assets forces a termination.

The asset flow model as presented in this paper is completely deterministic though it can be generalized to a stochastic systems of differential equations to incorporate the randomness in markets. For the practitioner, a deterministic system is ideally suited for obtaining the expected value in a particular situation. This is in contrast to many financial models in which the is randomness.

The difference between the asset flow approach and the theory of fluctutations about fundamental value are dramatic in the context of a bubble. As the bubble nears its top, the less sophisticated group has a large share of the stock and is very concerned with the short term trend. Of course, the fundamental value, $P_a(t)$, undergoes fluctuations due to world events. Near the peak of the bubble, any small drop is likely to develop into a rout as the trend based investors are not only prone to do the selling, but they have ample stock to sell. Meanwhile the value oriented investors are not likely to buy until the stock has gone much lower. Thus the asset flow perspective applied to Mexican stocks in December 1993 (Figure 7) clearly indicates that any adverse event will lead to disproportionately large selling and drop in prices. The absence of volatility is a source of confidence in the valuation theory with noise, but may be a cautionary signal from the asset flow theory.

Applications of Asset Flow Concepts to Consumer Choices, Elections and Social Trends

As a particular application one can consider a consumer preference of the VHS-Betamax type, in the initial stages of development, in which the consumer has a clear interest in being with the majority and consequently has a self-interest in observing and being influenced by the trend. While this type of activity is also being modelled using a lattice statistical mechanics idea in which agents prefer to align themselves with their neighbors [Bell 1993], the asset flow model would correspond to a "mean field approximation" that is far simpler to study mathem**a**tically. "Mean field" means roughly that each agent effectively interacts with the average or aggregate of all others. While the mean field is an approximation for physical systems such as magnetism, it is arguably a more accurate description of the type of economic system described above since the typical consumer is certainly aware of the aggregate preference. To be more specific, consider the consumer who is considering a VCR purchase. Even if all of his neighbors have bought a Betamax, he is aware through the plethora of statistics and polls that the VHS is more prevalent and that he may eventually have difficulty in renting Betamax tapes. A somewhat different situation arises if the preference of long-distance telephone services in which choosing the same service as his friends leads to a discount. In this case the lattice model is presumably more accurate though the results should be identical in the limit of large number of agents. In both the lattice and the mean field, the consumer has a self-interest in having a trend dependent preference.

A second example involves elections in which a trend may influence voter preference. Consider, for concreteness, a three-way election in which A and B are similar while C is distinct. The voters who favor A/B need to be aware of not only the relative percentages in the polls but also of the trends, since there is some lag time between the last poll and the election, so that they may vote for the more probable of A or B, based partly on the trend, that is likely to win over C. Of course, the trend effect may also be present in a two-way election as voters would like to 'go with the winner,' though the motivation here is not as rational as it is in a three-way race. The asset flow approach is likely to be useful in any preference selection system in which the 'supply' is finite. For example, in the decisions involved in individual liberties versus security concerns, there is a finite limit in both directions, so that any inclination for individuals to make choices based on a trend would be described by a similar formulation. Since the two key ingredients in the system are the trend based preference and the finiteness of assets (or voters, consumers, individual rights, etc.), it is elementary to consider the broad range of possible applications. Agents are likely to be influenced by the trend whenever there is a lag time between their choice and their information .

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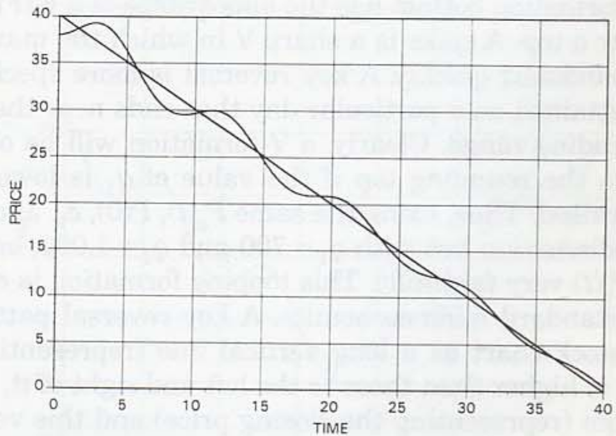

Figure 2 DOWNWARD TRENDLINE. The price oscillates about a linearly decreasing $P_a(t)$ that represents steadily diminishing fundamentals. The magnitude of the oscillations depends on the trend-based coefficient.